ADAPTIVE ERROR COMPENSATION BASED ON ONLINE SYSTEM IDENTIFICATION FOR REAL-TIME SUBSTRUCTURE TESTING

Van Thuan Nguyen and Uwe Dorka

ABSTRACT

For development of testing facilities in Europe, the E-FAST project, a design study for a European Facility for Advanced Seismic Testing, is being carried out with the support of the European Union. Development and implementation of advanced algorithms for real-time substructure testing plays an important role in the design of the new facility. This paper presents two novel compensation methods that deal with destabilizing effects occurring in such tests: The unbalanced force at the end of a time step especially when implicit integration algorithms are used and the phase lag in hydraulic systems. Based on online system identification, the compensating value is estimated; the error of estimation is fed into an adaptive mechanism which adjusts system parameters and minimizes errors. For demonstration, substructure tests using a virtual 2-DOF structure combined with a real hydraulic testing system have been performed. The effectiveness of the compensations on accuracy and stability of substructure tests are discussed.

INTRODUCTION

E-FAST (European Facility for Advanced Seismic Testing) is a design study for a new European testing facility performed in collaboration between five leading European institutions in earthquake engineering. (Marazzi and Molina 2009, http://efast.eknowrisk.eu/EFAST/). Major features of the new European facility shall be high performance, large capacity, great flexibility, strong integration with other facilities and advanced networking in Europe and worldwide.

Advances on simulation and testing methods as well as new hardware developments are reviewed and studied for implementation in the new facility. In this context, current challenges in testing and simulation are studied to arrive at a concept for the new facility which is open to future developments.

Therefore, substructure testing plays a key role in its development and various advanced methods are studied in this project to define performance criteria for E-FAST’s hardware. These are: substructure testing using shaking tables, combining shaking tables with other facilities and geographically distributed tests. One important issue in this context is the compensation of destabilizing effects during such tests resulting from possible force equilibrium errors at the end of each integration time step and the phase lag in hydraulic systems. Two novel compensation methods dealing with these effects in an adaptive manner are presented here.

1 Steel and Composite Section, Dept. of Civil Engineering, University of Kassel, Germany
UNBALANCED FORCE COMPENSATION

When an implicit integration scheme is used in a substructure algorithm, the equilibrium equation at the next step is given in Eq. (1).

\[ M \ddot{u}^{(i+1)} + C \dot{u}^{(i+1)} + Ku^{(i+1)} = f_l^{(i+1)} + f_c^{(i+1)} \tag{1} \]

\( M, C \) and \( K \) are the mass, damping and stiffness matrices of the numerical substructure. \( \ddot{u}^{(i+1)}, \dot{u}^{(i+1)} \) and \( u^{(i+1)} \) are acceleration, velocity and displacement vectors of the numerical substructure respectively. \( f_l^{(i+1)} \) is the loading vector of the numerical substructure and \( f_c \) is the vector of coupling forces between the experimental and numerical substructures, the index \( (i+1) \) denotes the next integration step.

Because the coupling force at the next step is unknown, each substructure algorithm with implicit integration uses an approximation to reach equilibrium at this time step. The \( \alpha \)-OS (Nakashima 1990) and P-C methods (Ghaboussi et al. 2005) for example apply a technique of prediction and correction based on estimating the structural stiffness. Another approach is feeding the measured coupling force back into the algorithm during the time step. Thewalt and Mahin (Thewalt and Mahin, 1987) did this in analog form by feeding the force measurement through an amplifier with properly adjusted gain directly into the drive signal. Dorka (Dorka 1990) implemented the first digital version by feeding the force measurement into the algorithm at discrete sub-steps.

In digital feedback, the equilibrium equation at the next integration step is given in Eq. (2).

\[ M \ddot{u}^{(i+1)} + C \dot{u}^{(i+1)} + Ku^{(i+1)} = f_l^{(i+1)} + f_c^{(i+1,k-1)} \tag{2} \]
The index \( i \) denotes the integration step and \( k \) the number of sub-steps within the time step. Note that at the end of a time step, an unbalanced force may occur which can be computed using Eq. (3). This can have a destabilizing effect and therefore should be compensated in the next time step \( \Delta f^{(i+1)} \) in Eq. (4). Dorka (Dorka 1990) proposed a PID error minimization (Eq. 5) and applied it successfully in an aerospace application where a 2DOF payload model specimen was used in combination with a numerical model of the Ariane IV rocket (Dorka et al. 1998).

\[
\begin{align*}
 f_u^{(i+1)} &= (f_i^{(i+1)} + f_c^{(i+1)}) - (Mu^{(i+1)} + Cu^{(i+1)} + Ku^{(i+1)}) \\
 M_{u^{(i+1)}} + C_{u^{(i+1)}} + Ku^{(i+1)} &= f_i^{(i+1)} + f_c^{(i+1,k-1)} + \Delta f^{(i+1)} \\
 \Delta f^{(i+1)} &= P \left[ f_u^{(i+1)} + \sum_{i=1}^{i} f_u^{(i)} + \frac{D}{\Delta t} \left( f_u^{(i)} - f_u^{(i-1)} \right) \right]
\end{align*}
\]

To allow for the compensation to adapt automatically to changing testing environments, Nguyen (Nguyen and Dorka, 2007) introduced a compensation based on online system identification. This requires recursive identification in the time domain and a data model capable of adjusting rapidly to changes in the data structure. Additionally, the compensation must be computationally efficient to be used in real-time substructure testing and should not be susceptible to noise.

A discussion on possible recursive identification methods and data models that may be used for unbalanced force compensation in substructure testing is given in (Nguyen, 2009). The recursive Pseudo Linear Regression (PLR) method with forgetting factor \( \lambda \) (Söderström et al. 1989, Ljung...
1999) has been selected here because of its generally stable asymptotic behavior, rapid convergence and computational efficiency. Although stability cannot be guaranteed under all circumstances, it can be assessed beforehand with standard methods.

As data model, the ARMAX model (Söderström et al. 1989) is used, because it is very accurate, can handle noise effectively and the computational cost is moderate.

With this, the formulation of the unbalanced force compensation is given in pseudo-regressive form in Eq. (6) ~ (11).

\[
y(i+1) = \Delta f^{i(1)} \\
x(i) = u_c^i \\
e(i) = f_u^i \\
y(i+1) = \phi^T (i+1) \phi \\
\phi(i+1) = \left\{ -y(i) - y(i-1) \ldots - y(i+1-n) \ x(i) \ x(i-1) \ldots x(i-n) \ e(i) \ e(i-1) \ldots e(i-n) \right\}^T \\
\phi = \left\{ a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n, c_1, c_2, \ldots, c_n \right\}^T
\]

where, \(x(i+1)\) and \(y(i+1)\) are input and output of the data, \(u_c^i\) is the computed displacement, \(\phi(i)\) is the vector of regression variables, \(e(i)\) is error of the compensation as the remaining unbalanced force at current step, \(n\) is the order of the data model, \(\phi\) is the vector of adaptive parameters, the indexes \((i)\) and \((i+1)\) denote current and the next integration steps.

The input to the data model is chosen as the computed displacement \(u_c^i\) (Eq. 7) because of two reasons. Firstly, the force \(\Delta f^{i(1)}\) (as the output of the data model) depends strongly on the displacement. For certain applications with velocity- and acceleration depending forces, the input can be a combination of displacement and its time derivatives (multi-variable model). Secondly, using the computed value of displacement can take advantage of separating noise and input signals. The advantage is that the identification mechanism works more effectively in term of convergence and error minimization.

**PHASE LAG COMPENSATION BASED ON ONLINE SYSTEM IDENTIFICATION**

The problem of phase lag or time delay of hydraulic actuators in real-time substructure tests has been discussed in many publications (see for example Horiuchi et al. 1999, 2001; Darby et al. 2001, Wallace et al. 2005; Spencer et al. 2007). Phase lag does not only induce errors but negative damping which may lead to instability during a test. A typical phase lag has a time delay between 8 ms to 40 ms (Horiuchi et al. 1999, Stoten et al. 2001, Spencer et al. 2007) which is very large compared to a usual 10 ms time step in civil engineering applications. Phase lag compensation is therefore a critical issue in real-time substructure tests in civil engineering.

For a discussion on different phase lag compensation methods that have been suggested to date, see (Nguyen, 2009). They have been developed for specific testing environments (even specific testing equipment) but cannot be generalized. To broaden the application of phase lag compensation to changing testing environments, a compensator with adaptive capabilities that works in a wide range of frequencies is needed. Therefore, the novel phase lag compensation described here is based on online system identification. It uses the ARMAX data model in...
connection with the recursive PLR method for the same reasons as given for the unbalanced force compensation.

The ideal control signal which leads to zero phase lag between response and target displacement is defined in (Eq. 12). Its phase must be ahead of the target displacement $u_{ii}^{jj}$.

$$\Delta u_{ii}^{jj} = u_{(ii)}^{jj} + \Delta u_{ii}^{jj}$$

$\Delta u_{ii}^{jj}$ is the compensating displacement and is estimated using historical data of displacement and/or its time derivatives as input to the ARMAX data model, the indexes $(ii)$ and $(jj)$ denote identification step and control step.

![Fig. 2. Hydraulic control with time lag compensation based on system identification](image)

The phase lag compensation has two different time intervals for identification of data model parameters and estimation of the compensating value. This is necessary since $\Delta u_{ii}^{jj}$ must be updated at each control step but the data model parameters should not be updated within a time interval that is shorter than the time delay. The critical time interval $Dt$ for parameter updating is thus given in Eq. (13).

$$Dt = k_{lag} \cdot dt \geq \delta t$$

$k_{lag}$ is the required number of control steps for identification, $dt$ is the control step time interval (typically 2 ms or less) and $\delta t$ is the maximum value for time delay in the considered range of frequencies.

The scheme for estimating the model parameters and compensating displacements is shown in Fig. 3. The data for updating the parameters are shown as points with mark “□” while the data to estimate compensating displacement are shown with mark “●”. Using the current parameters, the compensating displacement is calculated at each control step. At control step $jj$ ($jj \leq k_{lag}$), the data to estimate $\Delta u_{ii}^{jj}$ are the past values before ($l.k_{lag}$) control steps (with $l = 1, 2, \ldots, n$; $n$ is the order of the data model). At each control step $jj$, the parameters of the data model are calculated using a linear transition between the two last values of a parameter (Eq. 14).
The data model to estimate compensating displacement is given in Eq. (15) ~ (17).

\[
\Delta u_i^j = \left[ \phi_i^j \right]^T \cdot \theta_i^j
\]  

(15)

\[
\phi_i^j = \left\{ - \Delta u_i^{j-1}, - \Delta u_i^{j-2}, \ldots, - \Delta u_i^{j-n}, u_i^{j-1}, u_i^{j-2}, \ldots, u_i^{j-n}, e_i^{j-1}, e_i^{j-2}, \ldots, e_i^{j-n} \right\}^T
\]  

(16)

\[
\theta = \left\{ a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n, c_1, c_2, \ldots, c_n \right\}^T
\]  

(17)

\( \phi \) is the vector of regression variables and \( \theta \) is the vector of model parameters. The parameter \( \theta \) is estimated by the recursive PLR method with forgetting factor \( \lambda \) ranging between 0.96 and 0.99 (Söderström et al. 1989, Ljung, 1999).

This phase lag compensation adapts to time-varying phase and amplitude errors. It therefore changes the transfer function of the hydraulic system according to the developments during the test (changes of stiffness in the specimen, appearance of higher modes etc.).

**VIRTUAL SUBSTRUCTURE TESTS**

To demonstrate the proposed compensations, virtual substructure tests were performed. In these tests, the algorithm, control and measurement system, hydraulic system and numerical substructure are the same as in a real test. Only the experimental substructure is “virtual” meaning that it is numerically simulated using appropriate time integration methods such as the Duhamel integral which is exact for linear systems. The displacements and coupling forces between numerical and experimental substructure are compared to those of the complete structural model. This reference solution is obtained using Duhamel integral and mode superposition.
An ADwin Pro II electronic system (Jäger Computegeteuerte Messtechnik, www.adwin.de) is used to run the substructure algorithm, perform all measurements necessary for control and control the servo valve of the hydraulic cylinder (Dynamic Servo Cylinder SLZ-250-400, SANDNER-Messtechnik GmbH, hydraulic servo valve model 550, Star Hydraulics Ltd.). To include typical measurement noise, a real noise signal from a free load cell (load cell 10 kN, Hottinger Baldwin Messtechnik GmbH) with its amplifier (Amplifier system MGCplus, Hottinger Baldwin Messtechnik GmbH) is added to the computed value of the coupling force. The time delay of the hydraulic system is between 12 ms to 16 ms.

The simulated structure consists of a SDOF numerical substructure with \( m = 1000 \) kg and \( k = 78956.84 \) N/m and a virtual SDOF substructure specimen with \( m_s = 500 \) kg and \( k_s = 39478.42 \) N/m. Rayleigh damping is applied with \( \alpha = 0.418879 \) and \( \beta = 0.00530516 \). The structure is subjected to 50% of the TAZ090 record of the Kobe earthquake.

Virtual substructure tests using (1) no compensation, (2) unbalanced force compensation only, (3) phase lag compensation only and (4) both compensation methods are performed to demonstrate the effectiveness of the two compensations.

Fig. 6 shows that the compensating force approaches the ideal value and the remaining unbalanced force \( f_{i+1} \) is not much larger than the noise level.
The response of the hydraulic system with phase lag compensation only is shown in Fig. 7. A close fit to the target displacement can be observed. The result is excellent for both large and small displacements as well as different velocities during the test.

The cases with and without phase lag compensation are compared in Fig. 8. The hysteresis loop of the system with compensation (right) has no visible area in contrast to the one without (left). It therefore reduces the negative damping due to phase lag almost to zero in this case.
In Fig. 9 the results of the virtual substructure tests using (1) no compensation, (2) unbalanced force compensation only, (3) phase lag compensation only and (4) both compensation methods are compared to the reference solution (5). Using only one of the compensations, the accuracy of the test is already improved significantly, as can be seen from the improvement of the amplitudes, especially at higher frequencies. The test with both compensations further improves accuracy and its response fits perfectly to the reference solution in this case.
SUMMARY AND CONCLUSIONS

Two novel methods for compensating unbalanced forces and phase lag in real-time substructure tests are presented. The methods are based on online system identification using a data model with recursive parameter identification. This makes them adaptable to time-varying test setups. In these methods, there is no need to estimate the parameters of the data model beforehand, because the recursive mechanism will adjust them repeatedly to the appropriate values. Their initial values can be set to zero.

Accuracy and stability of the methods depend on the accuracy of the chosen data models and the convergence of the selected system identification. The ARMAX model and PLR identification method were chosen for both compensators, since they provided the best performance.

The compensators were tested in a virtual substructure test using the complete testing equipment (hydraulics and measurement and control systems) and a SDOF numerical structure excited by the Kobe earthquake, but a virtual SDOF substructure specimen represented by a numerical model. The effectiveness of the two compensations was demonstrated for this case. In addition, noise uncorrelated to system displacements was removed effectively.

Further studies are under way within the E-FAST project to assess the effect of non-linear specimen behavior on the developed compensators.

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