Summary

Accurate predictions of the intensity and variability of ground motions from future large earthquakes depend strongly on our ability to simulate realistic models of the earthquake source. In order to make these simulations more accurate, we have developed a procedure to generate physically consistent earthquake rupture models. We term these models pseudo-dynamic because they are kinematic models that are designed to emulate important characteristics of fully dynamic rupture models. The user-specified parameters to generate pseudo-dynamic rupture models are the earthquake magnitude and hypocenter, with the option of specifying the fault dimensions as well. We construct pseudo-dynamic models by first generating a slip distribution as a realization of a spatial random field that is consistent in its overall scaling and spatial variability with slip distributions observed in past earthquakes [Mai and Beroza, 2000; Mai and Beroza, 2002]. We compute the static stress drop associated with this slip distribution, which in turn is used to assign the temporal evolution of slip (rupture velocity, rise time, slip-velocity function) by applying a series of empirical relationships derived from the analysis of a set of spontaneous rupture models. A simple energy-budget calculation is used to discard source models that are not realizable as spontaneous ruptures.

Our approach is based on the findings of Guatteri et al. [2003], incorporating ideas developed by Mai et al. [2001] and Guatteri et al. [2002] for a source characterization that adequately describe the dynamics of rupture without having to do full dynamic simulations. The pseudo-dynamic approach circumvents the limits imposed by the extreme computational demand of generating fully dynamic rupture models for simulating multiple realizations of a scenario earthquake for strong-ground motion prediction. While the relationships between source parameters described in this paper are significant simplifications of the true complexity of the physics of the rupture process, they help identify important interactions between source properties that are relevant for strong ground motion prediction.

Introduction

Statistical models have long been proposed to characterize heterogeneity in earthquake slip and to relate it to observable characteristics of earthquake ground motion [Haskell, 1966; Aki, 1967]. Andrews [1980b] presented a theoretical formulation for this problem for heterogeneous faults that has formed the foundation for most subsequent efforts [Herrero and Bernard, 1994; Frankel, 1991; Zeng et al., 1994]. Andrews [1980b] showed that a two-dimensional slip function $D(x,z)$ with a fractal dimension $D = 2$ results in a far-field spectral decay of displacement as $\omega^{-2}$. The fractal dimension $D$ is related to the wavenumber decay of the two-dimensional Fourier spectrum of the slip function, $D(k)$ (where $k$ is the wavenumber); for $D = 2$, the wavenumber spectra decays as $k^2$. Using this relation, Herrero & Bernard [1994] propose the “k-square” model that introduces a source-size dependent length scale $L_s$ as well as a wavenumber-dependent behavior of the rise time distribution. Zeng et al. [1994] develop the composite source model based on the model of Frankel [1991] in which elementary circular sources with a fractal size distribution are summed to form the complete two-dimensional slip function. An explicit assumption in these
models is that stress drop is scale-independent. Moreover, none of these methods directly takes into account slip heterogeneity as imaged using seismic data in finite-source models.

In order to characterize the slip complexity of past earthquakes, Somerville et al. [1999] adopt a deterministic approach to correlate the size and number of asperities with seismic moment for a set of finite-source rupture models. Mai and Beroza [2002], in contrast, use a spatial random-field model to describe the complexity of earthquake slip distributions. They find that a von Karman autocorrelation function for which the correlation length increases with source dimension most closely represents the spectral properties of existing earthquake slip models. Increasing magnitude and hence increasing source dimensions with constant correlation lengths would mean that the rupture would be comprised of many isolated high-slip (high stress-drop) asperities, which would show extremely large slip (and stress drop) in order to accommodate the seismic moment. Unless the correlation length grows with earthquake size, these areas of positive stress drop will be too weakly connected for the rupture to propagate spontaneously because they will be separated by large areas of negative stress drop, and therefore negative elastostatic energy, which will act to stop the rupture.

Guatteri et al. [2002] find that including spatial and temporal variations in slip, slip rise time, and rupture propagation that are consistent with dynamic rupture models exerts a strong influence on near-source ground motion and has the potential to improve strong ground motion prediction. Their results lead to a feasible approach to specify the variability in the rupture time distribution in kinematic models consistently with dynamic source properties. Mai et al. [2001] and Guatteri et al. [2002] calculate ground motion from a “first-generation” set of pseudo-dynamic models, demonstrating the potential of such a procedure. In this paper, we focus on the generation of the source models by expanding and improving upon the previous “first-generation” pseudo-dynamic source characterization, and leave the ground motion calculation for future applications.

**Pseudo-Dynamic Source Characterization**

In this paper we define as a pseudo-dynamic source model a kinematic model in which the relevant source parameters (slip, rupture velocity and slip-velocity function) are specified in such a way that they emulate both slip distributions of past earthquakes and the temporal behavior of spontaneously propagating dynamic rupture models. We developed the pseudo-dynamic approach (henceforth abbreviated as PD) by investigating the relationships between kinematic and dynamic source parameters for a set of dynamic rupture models representing strike-slip earthquakes with a magnitude range of $6.4 < M < 7.2$.

The complexity of the spatial slip distribution in our PD models is characterized as a spatial random field [Mai and Beroza, 2002] that is stochastically consistent with slip models for past earthquakes. The starting point of the PD model consists of generating a spatial distribution of slip as a realization of a spatial random field.

The characterization of the temporal slip evolution in finite-source rupture models for past earthquakes is limited by a lack of resolution of the variability of temporal slip parameters, such as rupture velocity and slip rise time. The fundamental idea of our approach is that we can overcome this limitation by assuming that dynamic rupture modeling can provide physical constraints to the temporal slip evolution characterization, given a spatial slip distribution. We therefore perform spontaneous dynamic rupture simulations to develop the relationships between kinematic and dynamic rupture parameters to constrain the temporal rupture characteristics. The following section describes our dynamic modeling approach and the various steps toward defining physically consistent rupture velocity, rise time and slip-velocity functions.

**Initial Dynamic Rupture Modeling**

The first PD source characterization was developed by Mai et al. [2001] and improved by Guatteri et al. [2002]. Both their results were based on the set of 6 stochastic-dynamic models
developed by Guatteri et al. [2003] computed using a Boundary Integral Method [Boatwright and Quin, 1986; Das and Kostrov, 1987; Quin and Das, 1989]. These models were discretized into a grid size of 0.75 km, allowing for a maximum frequency of about 2 Hz (for this method), and a slip-weakening fault constitutive relationship was assumed [Andrews, 1976; Day, 1982] (Figure 1). Table 1 summarizes and defines the relevant dynamic parameters together with a brief explanation of how we assign or calculate them to generate our set of stochastic-dynamic models.

**Figure 1**: Slip-weakening model showing the relevant energy contributions: elastostatic energy $E_{el}$ (left), fracture energy $E_{fr}$ and relaxation work $E_{rx}$ (center). The right panel displays the case of negative stress drop $\Delta \tau$ for which fracture energy is still defined. See Table 1 for further explanations. $\tau_1$ is the final stress, which may be different from the dynamic frictional level.

**TABLE 1**: Relevant dynamic parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>General Information</th>
<th>In this study</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\sigma_n$</td>
<td>Normal stress</td>
<td>150 MPa</td>
<td>Uniform over the fault</td>
</tr>
<tr>
<td>2. $\mu_d$</td>
<td>Coefficient of dynamic friction</td>
<td>0.43</td>
<td>Uniform over the fault</td>
</tr>
<tr>
<td>3. $\tau_f$</td>
<td>Dynamic resistance (frictional stress level)</td>
<td>$\mu_d \sigma_n$</td>
<td>Uniform over the fault</td>
</tr>
<tr>
<td>4. $\Delta \tau$</td>
<td>Stress drop</td>
<td>$\tau_0 - \tau_1$</td>
<td>Calculated from slip distribution (eq. 3)</td>
</tr>
<tr>
<td>5. $\tau_0$</td>
<td>Initial shear stress</td>
<td>$\Delta \tau + \tau_f$</td>
<td></td>
</tr>
<tr>
<td>6. $D_c$</td>
<td>Slip-weakening distance</td>
<td>Sampled from $[0.25 - 0.4]$ m for the whole fault</td>
<td>Assigned to lower bound where $\Delta \tau &lt; 0$</td>
</tr>
<tr>
<td>7. $E_{fp}$</td>
<td>Apparent fracture energy (also termed $G_c$)</td>
<td>$\frac{1}{2} D_c (\tau_f - \tau_1)$</td>
<td>Simulated from stress drop distribution using (eq. 6)</td>
</tr>
<tr>
<td>8. $\tau_s - \tau_0$</td>
<td>Strength excess $\tau_{ex}$</td>
<td>Calculated from $E_{fp}$ and $D_c$ values</td>
<td>Lower bound $\tau_{ex} = 0.5$ where $\Delta \tau &lt; 0$. Upper bound given by $S_{max}$</td>
</tr>
<tr>
<td>9. $S$</td>
<td>Strength parameter</td>
<td>$(\tau_s - \tau_0) / \Delta \tau$</td>
<td>Upper bound $S_{max} = 2.5$</td>
</tr>
<tr>
<td>10. $\tau_1$</td>
<td>Static resistance (upper yield stress)</td>
<td>$\mu_d \sigma_n$</td>
<td>Calculated from (eq. 8) and (eq. 5)</td>
</tr>
<tr>
<td>11. $\mu_s$</td>
<td>Coefficient of static friction</td>
<td>$\tau_s / \sigma_n$</td>
<td></td>
</tr>
<tr>
<td>12. $E_{el}$</td>
<td>Elastostatic energy</td>
<td>$-\frac{1}{2} \text{slip}_{max} (\tau_f - \tau_0)$</td>
<td></td>
</tr>
<tr>
<td>13. $E_{rs}$</td>
<td>Radiated seismic energy</td>
<td>$E_{el} - E_{fp}$; $E_{rs} = 0$ if the frictional stress $\tau_f$ is equal to the final stress $\tau_1$</td>
<td>Assumption: static stress drop is equal to the dynamic stress drop</td>
</tr>
</tbody>
</table>
We assume the same velocity profile as in Guatteri et al. [2003] derived from Boore and Joyner [1997]. We force the rupture to nucleate at a given hypocenter location and to propagate at a constant rupture speed within a nucleation area whose size is calculated according to Day [1982].

Our procedure to generate the new set of stochastic-dynamic models is very similar to that of Guatteri et al. [2003]. We first generate our target slip distribution using the method of Mai and Beroza [2002], from which we compute the corresponding static stress drop using the method of Andrews [1980]. Once we set the hypocenter location, instead of assigning the fracture energy distribution by the trial and error procedure as in Guatteri et al. [2003], we applied the empirical relationship between fracture energy and stress drop and crack length developed by Guatteri et al. [2002]. In a later section in this paper we describe this relationship in detail and discuss how we modified it to satisfy the assumption of sub-shear average rupture velocity over the entire fault plane for the stochastic-dynamic models.

The set of stochastic-dynamic rupture models are representative of strike-slip earthquakes with a magnitude range of 6.4 < M < 7.2. Table 1 summarizes the important features of the dynamic parameters, while Figure 2 schematically shows the roadmap of our PD procedure development. Table 2 summarizes the parameters used to define a PD model.
<table>
<thead>
<tr>
<th>TABLE 2: Kinematic, dynamic and pseudo-dynamic parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Slip</strong></td>
</tr>
<tr>
<td>$L_h$, distance from the hypocenter</td>
</tr>
<tr>
<td>$\Delta \tau$, stress drop</td>
</tr>
<tr>
<td>$G_c$, fracture energy</td>
</tr>
<tr>
<td>$v$, rupture velocity</td>
</tr>
<tr>
<td>$\tau_r$, rise time</td>
</tr>
<tr>
<td>SVF ($V_{max}$, $T_p$)</td>
</tr>
</tbody>
</table>

**Characterization of Spatial Slip Distribution**

The variability of slip in earthquakes maps directly into high frequency strong ground motion [Bernard and Madariaga, 1983; Spudich and Frazer, 1984]. At present, it is not possible to anticipate earthquake slip distributions deterministically, but we can anticipate the characteristics of slip slip variation in earthquakes of a given size if we have a valid stochastic description of the source. With this statistical description of the earthquake source in hand, we can generate many realizations of the slip distribution in a scenario earthquake of interest (with accompanying strong ground motions) without the need to know the slip distribution deterministically.

Here we use a spatial random-field model for earthquake slip to represent the slip distribution [Mai and Beroza, 2002], but other models characterizing spatially variable slip are possible [Andrews, 1980; Herrero and Bernard, 1994; Zeng et al., 1994; Sommerville et al., 1999]. A spatial random field is characterized either in space by its autocorrelation function, $C(r)$, or in the spectral domain by its power spectral density, $P(k)$, where $k$ is the wavenumber. Analyzing a set 44 finite-source rupture models, Mai and Beroza [2002] found that a von Karman auto-correlation function best represents the power spectral decay of complex earthquake slip, with correlation length increasing with increasing magnitude. The power spectral density $P(k)$ of the von Karman auto-correlation function is given by

$$ P(k) = \frac{4\pi H}{K_H(0)} \frac{a_x a_z}{(1 + k^2)^H}, $$

with $k = \sqrt{a_x^2 k_x^2 + a_z^2 k_z^2}$, $H$ being the Hurst exponent, and $K_H$ the modified Bessel function of the first kind (order $H$). The characteristic scale lengths are given by the correlation lengths, $a_x$ and $a_z$ in the along-strike and down-dip directions. The Hurst exponent, $H$, in the expression for the von Karman distribution describes the spectral decay at high wavenumbers. The spectral
decay of 44 source models studied by Mai and Beroza [2002] revealed that \( H = [0.8 \, -1.0] \), and that the correlation length scale with magnitude as:

\[
\begin{align*}
    a_x &\approx 2.0 + \frac{1}{3} L_{\text{eff}} & \log(a_x) &\approx -2.5 + \frac{1}{3} M_w \\
    a_z &\approx 1.0 + \frac{1}{3} W_{\text{eff}} & \log(a_z) &\approx -1.5 + \frac{1}{3} M_w 
\end{align*}
\]

(2)

where \( L_{\text{eff}} \), \( W_{\text{eff}} \) are the effective source dimensions as defined in Mai and Beroza [2000].

We generate heterogeneous slip distributions for the selected target earthquake magnitudes using equations (1) and (2). Source dimensions for these earthquakes (6.4 < \( M \) < 7.2) are calculated following Mai and Beroza [2000]. Having calculated the power-spectral density \( P(k) \), the two-dimensional slip function is obtained by assuming a uniform-random phase and subsequent two-dimensional Fourier transformation under the requirement of Hermitian symmetry to ensure a purely real valued slip-function.

**Calculating Static Stress Drop from Slip Distribution**

The complete stress-time history can only be calculated by knowing the entire slip-time evolution of the rupture [e.g., Bouchon, 1997]; however, the goal in this study is to define a dynamically consistent slip-time evolution based on the final slip only. For this purpose we use the static stress drop, \( \Delta \tau \), associated with slip occurring on the rupture plane, as a fundamental parameter for characterizing the temporal slip evolution. We relate slip and stress to one another by a convolutional integral [Andrews, 1980] expressed as a multiplication in the wavenumber domain, as:

\[
\Delta \tau(k) = - K(k) \cdot D(k)
\]

(3)

where \( \tau(k) \) and \( D(k) \) denote the Fourier transforms of the two-dimensional stress drop \( \Delta \tau(x,z) \) and slip-function \( D(x,z) \), respectively. \( K(k) \) represents the static stiffness function that for crustal rocks can be approximated as \( K(k) = -\frac{1}{2} \mu \cdot k \). Hence, having simulated a two-dimensional slip distribution \( D(x,z) \), equation (3) can be used to compute the associated static stress drop \( \Delta \tau(x,z) \) by means of Fourier transformation of \( D(x,z) \) and subsequent inverse Fourier transformation of \( \tau(k) \).

**Characterization of Fracture Energy for Temporal Slip Evolution**

The characterization of fracture energy \( G_c \) is the critical step in our PD procedure for determining the temporal evolution of slip. Our PD approach is built on the following relationship derived by Andrews [1976] for a simple homogeneous anti-plane crack:

\[
1 - \frac{\nu^2}{\beta^2} = \frac{\pi^2}{R_c^2} \cdot (R_c / 2)^2 
\]

(4)

where \( R_c \) is the dimensionless parameter:

\[
R_c = \mu \cdot G_c / (\Delta \tau^2 \cdot L_h)
\]

(5)

\( \nu \) is the rupture speed, \( \beta \) is the shear wave speed, \( \mu \) is the shear modulus, \( \Delta \tau \) is the stress drop, and \( L_h \) is crack length. Guatteri et al. [2003] showed that relationship (4) applies approximately to 3-
D heterogeneous dynamic models suggesting that rupture velocity variability can be inferred from
the distribution of dynamic parameters over the fault plane.

**Rupture Velocity.** The first step towards a temporal slip evolution characterization is to
determine a distribution of rupture velocity that is physically consistent with a given slip
realization. Given a slip distribution, Equation (4) allows us to achieve this by determining the
corresponding stress drop distribution (Equation 3), calculating the crack length $L_h$ which we take
as the distance of each point on the fault to the hypocenter, and based on that, generating a
physically consistent distribution of fracture energy, $G_c$.

**Fracture Energy.** Different distributions of fracture energy may be consistent with a given slip
distribution and rupture propagation. Through a trial and error procedure Guatteri et al. [2003]
imposed the fracture energy distribution on their stochastic-dynamic models such that the
resulting rupture velocity would be sub-shear on average everywhere over the fault plane. In
order to avoid this time consuming trial and error approach, Guatteri et al. [2002] used their set
of stochastic-dynamic models to build an empirical relationship to generate a distribution of
fracture energy from a given slip model and hypocenter location. After some data exploration
they developed an empirical relationship based on the normal linear model where the conditional
expectation of $G_c$ is:

$$E(G_c|\beta, \Delta \tau, L_h) = \beta_0 + \beta_1 \Delta \tau L_h^{1/2},$$

where $\beta$ is the vector containing $\beta_0$ and $\beta_1$, the regression coefficients that they determined
through a least squares regression procedure. Figure 3 shows the fracture energy plotted as a
function of the respective predictor $\Delta \tau L_h^{1/2}$ (that is proportional to the stress intensity factor) for
all points over the fault plane for all the different dynamic models developed by Guatteri et al.
[2002].

Relationship (6) can be used either to assign a fracture energy distribution to compute
spontaneous rupture models and to apply Equation (5) to calculate a distribution of rupture
velocity in our PD procedure. Before using it for the latter application, we validated and modified
it as described below.

We first applied Equation (6) to derive a distribution of fracture energy to compute a new set
of stochastic-dynamic models. However, unlike Guatteri et al. [2003], in this study we allow the
dynamic models to have areas of negative stress drop (Figure 1). This difference implies that on

![Figure 3: Fracture energy values as a function of stress drop and crack distance. Values relative to different dynamic models described in Guatteri et al. [2003] are shown in different colors.](image)
average we need lower values of $G_c$ to maintain a sub-shear rupture velocity over the fault plane, because the negative stress drop areas increase the resistance of the fault to rupture. As a result, we find that lower values of regression coefficients $\beta_0$ and $\beta_1$ are needed than those found previously by Guatteri et al. [2002].

Furthermore, we found that two distinct sets of coefficients are needed, corresponding to rupture models with $M \leq 6.5$ and for $M > 6.5$, respectively. The two sets are $\beta_0 = 0.18$ and $\beta_1 = 0.0015$ for $M \leq 6.5$, and $\beta_0 = 2.7$ and $\beta_1 = 0.0021$ for $M > 6.5$. The difference $\beta_0$ stems from the difference in average stress drop values, while the almost identical slope $\beta_1$ indicates that the dependency of the stress intensity factor is preserved. We interpret this result as a consequence of lower stress drop values in our rupture models with $M \leq 6.5$ than those with a larger magnitude, with corresponding different values of available elastostatic energy $E_{el}$ (Figure 1, Table 1). This would imply that the coefficients of the fracture energy distribution in Equation (6) vary as a function of earthquake magnitude.

The possibility that the scaling coefficients for fracture energy $G_c$ are different for earthquakes below and above $M = 6.5$ has interesting implications as this is the magnitude range where standard scaling relations may break down [Shimazaki, 1986; Hanks and Bakun, 2003]. Once rupture width approaches the thickness of the seismogenic zone, and rupture length $L$ becomes larger than about twice rupture width $W$, ruptures grow in along-strike direction only, and the above defined crack length $L_h$ becomes progressively larger for larger/longer earthquakes, requiring higher values of fracture energy $G_c$.

Equation (6) is derived for points on the fault having a positive stress drop. For areas of negative stress drop, we assign a constant value of $G_c$ calculated as the area shown in Figure (1c) by assuming minimum allowed strength-excess and $D_c$ values (Table 1). Note that the use of Equation (6) provides a very simplistic parameterization of fracture energy given a slip distribution (and therefore a static stress drop distribution) and hypocenter. The resulting fracture energy distribution represents only one possible realization out of the space of physically plausible fracture energy distributions consistent with a given slip model. However, Equation (6) contributes to the intent of this paper to provide a simple tool to quickly generate a physically consistent rupture model.

Having assumed in our dynamic models, distributions of $D_c$ and strength excess that are fairly uniform and within assigned limits, the strong dependency of $G_c$ on stress drop in our modeling appears to be an artifact of our assumptions. As pointed out by Favreau and Archuleta [2003], however, it is plausible that larger values of stress drop are necessary to break stronger barriers of energy, or else the rupture would stop. The dependency found between $G_c$ and the stress intensity factor may be physically interpreted as corresponding to the increase of energy lost due to the occurrence of off-fault microcracking with rupture propagation distance [Andrews, 1976; Peck et al., 1985].

**Slip-Velocity Function Parameterization**

A realistic parameterization of the slip-velocity function (SVF) is a critical component of earthquake rupture modeling for strong motion prediction. As for the rupture velocity, the variability of the SVF is poorly constrained from waveform inversion procedures. In dynamic modeling the shape of the SVF at different fault points depends on several factors, such as local stress parameters and distance from the hypocenter [Day, 1982; Nakamura and Miyatake, 2000; Guatteri and Spudich, 2000; Guatteri et al., 2003]. Typical SVFs of dynamic models governed by a slip-weakening friction law are shown in Figure 4. Despite the considerable complexity seen in these slip-velocity functions, their time-dependence shows an approximate $t^{-1/2}$-decay, consistent with the quasi-dynamic Kostrov-type slip-velocity function proposed by Archuleta and Hartzell [1981]. Note that some points on the fault are characterized by a long tail of low slip-
velocity. From inversion of strong-ground motion, it is not clear whether such low-amplitude tails would be resolved.

![Slip-Velocity functions](image)

Figure 4: Slip-velocity functions for identical points on the fault (each column) at a depth of 9.5 km for four spontaneous-dynamic rupture models [Guatteri et al., 2003].

In this study, we define the rise time as the time from 5% to 95% of the total slip, i.e. being representative of the total slip duration. A comparison between the rise time inferred for real earthquakes from kinematic inversion procedure [Wald et al. 1994; Beroza and Spudich, 1988; Cotton and Campillo, 1995] and the rise time defined for dynamic SVF’s may not be very meaningful. In order to characterize the shape of the SVF (impulsive vs. smooth), we define an additional time parameter, the ‘time pulse’ $T_p$, as the time from 5% to 50% of the total slip. For very spiky SVF, $T_p$ will be much shorter than for a broad and smooth SVF, while the respective rise time may be of comparable size. We believe that $T_p$ is a more relevant time parameter for strong-ground motion prediction than the rise time, particularly at high frequencies.

In this study we propose a very simple approximation of the dynamic SVF using a small number of parameters in order both to provide a feasible parameterization within the PD procedure and to capture the relevant characteristics of the dynamic SVF for strong-ground motion prediction. Nakamura and Miyatake [2000] proposed a SVF parameterization that closely resembles the SVF obtained in dynamic modeling; their characterization is perhaps superior to our proposed approximation, but it is also rather complex to implement within our approach. Day [1982] also proposed very insightful relationships between source parameters describing the SVF in dynamic models. We make use of both the Day [1982] and the Nakamura and Miyatake [2000] results in formulating our parameterization. Table 2 lists the parameters defining the SVF at a given point on the fault: the maximum slip-velocity value $V_{max}$, the time pulse $T_p$, and the rise time $\tau$. 

Figure 5 shows our proposed parameterization of the SVF as a simple approximation of the SVF typical of dynamic models governed by a slip-weakening friction law (Figure 4). The SVF is composed of two overlapping triangles, T1 and T2 with a total base equal to the rise time, $\tau$. T1 is an isosceles triangle having an area $A$ equal to half of the local total slip $S$, height equal to $V_{max}$ and base equal to the pulse width $T_p$. T1 is the portion of the SVF that contributes the most to the seismic radiation. T2 is a rectangular triangle with height equal to $V_2 = c V_{max}$. The area of the non-overlapping part of T2 is equal to $A$. In this study we set $c = \frac{1}{2}$, but other values may be
chosen. For simplicity, let us assume that slip starts at $t = t_1 = 0$. By solving the following system:

$$
\begin{cases}
    cV_{\max} = \left( -\frac{2V_{\max}}{T_p} \right) t + 2V_{\max} \\
    t = t_2
\end{cases}
$$

we find $t_2$:

$$
t_2 = T_p \left( 1 - \frac{c}{2} \right),
$$

and for $V_2 = \frac{1}{2} V_{\max}$ it follows that $t_2 = \frac{1}{4} T_p$.

The following system of equations applies to our SVF parameterization (Figure 5):

$$
\begin{cases}
    \frac{\sqrt{2}}{2} V_{\max} T_p = A \\
    A = \frac{1}{2} V_2 (\tau_s - t_2) - \frac{1}{2} V_2 (T_p - t_2)
\end{cases}
$$

Figure 5. Slip-velocity function parameterization in our PD approach, characterized by two overlapping triangles, an isosceles triangle T1 and a rectangular triangle T2.

$V_{\max}$. The value of peak slip-velocity is mainly controlled by the local value of stress drop, of $D_c$, and distance from the hypocenter [Day, 1982; Andrews, 1985; Ohnaka and Yamashita, 1991; Nakamura and Miyatake, 2000; Guatteri and Spudich, 2000]. Because for a given stress drop distribution, different choices of $D_c$ result in different peak-slip velocities (a small $D_c$ results in a large $V_{\max}$, and a large $D_c$ in a low $V_{\max}$) [Guatteri and Spudich, 2000], we cannot provide a unique parameterization for $V_{\max}$ for a given slip distribution. However, the user has the option to set a value of $D_c$ within a given range and tune the SVF shape in a manner consistent with the frequency band of interest in ground motion simulation.

The analysis of our set of dynamic models suggests that the following relationship, modified from Day [1982], for $V_{\max}$ provides an adequate description of the $V_{\max}$ distribution observed in dynamic models:

$$
V_{\max} = 0.5 V_{\max,\text{Ref}} W \tau_p \nu (\beta \mu),
$$

where $W$ is the width of the fault, $\tau_p = \text{max}(\Delta \tau, \nu (\tau_s - \tau_d))$ and $V_{\max,\text{Ref}} = 0.9 D_c f_c$, where $D_c$ is a chosen value of slip-weakening distance, $\tau_s - \tau_d = 2G_c / D_c$, and $f_c$ is a frequency parameter defined in Ohnaka and Yamashita [1989]. The equation for $V_{\max,\text{Ref}}$ has been adapted from
Ohnaka and Yamashita [1989]. Our definition of \( \tau_p \) is based on the idea that, if the value of strength excess is very high compared to the stress drop, then it has a large effect on \( V_{\text{max}} \). In our parameterization, we found that \( w=0.6 \) for areas with \( \Delta \tau > 0 \) and \( w=1 \) for areas with \( \Delta \tau \leq 0 \) gives a satisfactory fit with the \( V_{\text{max}} \) distribution of the dynamic models.

**Pulse width** \( T_p \). As Figure 4 shows, short values of \( T_p \) correspond to large values of \( V_{\text{max}} \). Figure 6 displays the relationship between \( T_p \) and the value of total slip \( S \) divided by \( V_{\text{max}} \) for the points on the fault with a slip and slip-velocity larger than 50% of the maximum slip and maximum slip-velocity over the fault, respectively. A linear relationship provides a simple parameterization of a \( T_p \) given \( V_{\text{max}} \) and the total slip:

\[
T_p = \beta_1 \frac{S}{V_{\text{max}}},
\]

where \( \beta_1 = 0.84 \). Let \( V_{\text{ave}} \) be the average slip-velocity during \( T_p \), then \( 0.45 \, S = V_{\text{ave}} \, T_p \), implying that \( V_{\text{ave}} \, T_p \approx \frac{1}{2} \cdot V_{\text{max}} \, T_p \) that justifies our parameterization \( 0.5 \cdot S = 0.5 \cdot V_{\text{max}} \, T_p \). Note that this relationship is derived for areas of large slip and slip-velocity that are the areas of the fault with large seismic energy radiation, and hence contribute the most to strong-ground motions.

![Dynamic "Pulse Width" on Areas of Large Slip and Slip-Velocity](image)

**Slip Rise Time.** As for the characterization of the fracture energy distribution, Guattieri et al. [2003] built an empirical relationship for slip rise time based on the dynamic models, consistent with our stochastic-dynamic models. Assuming a slip-weakening friction model [Day, 1982, Andrews, 1985], the duration of slip at a given point on the fault is controlled mainly by the total fault rupture duration. Figure 7 shows the rise time values \( \tau_r \) plotted as a function of the difference between the total effective fault rupture duration, \( T_{\text{rup}} \), and the corresponding rupture time value, \( T_{\text{rup}} \). The total effective fault rupture duration is the maximum rupture time value among the fault boundary locations aligned with the hypocenter. Figure 7 suggests a normal linear model for an empirical relationship for rise time

\[
E(\tau_r | \beta, T_{\text{rup}}, T_{\text{rup}}) = \beta_0 + \beta_1 (T_{\text{rup}} - T_{\text{rup}}).
\]

Although we have derived an empirical relationship for \( \tau_r \) (Equation 12), we require that \( \tau_r \) be consistent with Equation (9). \( T_p \) is one of the most important time parameters in terms of its
influence on the calculated ground motion. Therefore, we follow a 2-step approach to assign all the parameters for SVF such that the consistency with the given slip distribution is satisfied. First, we calculate an initial guess for \( \tau_r \) from the empirical relationship for the purpose of imposing an upper bound to \( T_p \). Then, from Equation (10), we can solve for \( \tau_r \) as a function of \( T_p \) as follows:

\[
\tau_r = T_p + \frac{S}{cV_{\text{max}}}.
\]  

(13)

Figure 8 shows a comparison of PD SVF’s derived by applying the parameterization described above with the corresponding dynamic SVF’s.

![Figure 7. Slip rise time as a function of total fault rupture duration and rupture time. Values corresponding to different dynamic models are shown with different colors.](image7)

![Figure 8. Comparison between the dynamic SVF with the PD SVF parameterization proposed in this paper for two selected points on the fault for two different models. The rupture times of the dynamic model has been aligned to that of the PD model for better comparison.](image8)
Summary of PD-approach:

The basic steps involved in the development of a pseudo-dynamic source realization are the following:

1. Define the target earthquake magnitude and event type.
2. Assign or determine the fault dimensions [Wells and Coppersmith, 1994; Somerville et al, 1999; Mai and Beroza, 2000].
3. Set the hypocenter location.
5. Compute the corresponding stress drop distribution [Andrews, 1980].
7. Calculate rupture velocity and rupture time distributions using Equation (5).
8. Calculate \(V_{\text{max}}\) distribution from Equation (10).
9. Apply 2-step procedure to derive the distribution of \(T_p\) and \(\tau_r\) using Equations (11-13).

Figure 9 shows an example of a pseudo-dynamic source model for a M=6.5 strike-slip earthquake. Figure 10 shows a comparison of a fully dynamic source model and its corresponding pseudo-dynamic source characterization. The common constraints to the two source characterizations are the fault geometry, the hypocenter location, the slip distribution and the requirement of sub-shear rupture propagation.

Figure 9. Example representing the pseudo-dynamic procedure. The starting point is a slip realization generated as a spatial random field [Mai and Beroza, 2002]. The corresponding stress drop distribution is computed using the method of Andrews [1980].

Figure 10. Comparison between a fully dynamic rupture model (left) and a corresponding pseudo-dynamic source model (right). Notice that the main characteristics of the dynamic rupture are represented in the PD model, such as the variation in the rupture velocity, the areas with large and low peak slip-velocity, and the areas with short \(T_p\).
Discussion: The energy budget of earthquake rupture

The pseudo-dynamic source characterization developed here is not intended to be a perfect fit to the corresponding spontaneous dynamic rupture models, which are themselves only models of what actually happens in nature during earthquake rupture. The features of the earthquake source most important for realistic strong-ground motion simulations, however, are well described by our PD parameterization. The main correlation between source parameters can be described in plain English as follows: the distribution of stress drop is the main factor affecting the distribution of the other dynamic rupture parameters. The rupture velocity correlates with stress drop, \( V_{\text{max}} \) correlates with stress drop, and \( T_p \) and \( \tau_r \) are inversely proportional to stress drop. Finally, the position of the hypocenter affects the distribution of parameters, and certain hypocenter positions are not plausible as those would not lead to spontaneous rupture propagation. To further investigate this last statement, we analyzed the energy budget during earthquake rupture.

Plausible Models Based on Energy Budget and Hypocenter Location

It is a common and frustrating experience of many dynamic modelers to initiate spontaneous rupture calculations that subsequently abort before rupturing to the desired earthquake size [Nielsen and Olsen, 2000; Oglesby, 2002; R. Archuleta, pers. comm.; R. Harris, pers. comm.].

While the investigation of the underlying physical process distinguishing small and large earthquakes is well beyond the scope of this paper, we focus on identifying those target slip distributions that are not consistent with a given hypocenter location based on a simple energy budget calculation. Our approach helped us both to speed up the computation of successful spontaneous rupture models and to select physically plausible (realizable) PD models.

In describing the energy budget during earthquake rupture, we follow Favreau and Archuleta [2002]. Each point on the fault provides a seismic energy \( E_{rs} = E_{el} - E_{fr} - E_{rx} \), where the various terms are defined in Table 1. For simplicity, in this study we neglect the relaxation work \( E_{rx} \) spent at the arresting the rupture (Figure 1). As noted by Favreau and Archuleta [2002], the fault can be characterized by locally negative seismic energy density values, but its integral on the fault must be positive. Based on this physical requirement, we consider that, as a fundamental condition for its growth, the rupture must be propagating such that the integral of \( E_{rs} \) on the rupture area is remains always positive. In other words, while there can be local sinks of energy over the fault, an earthquake process cannot be described overall as a sink of energy. For a given earthquake magnitude, we might expect a more stringent constraint such as a minimum value of radiated seismic energy, but we leave this as an open question for future studies.

It is important to note that the placement of the hypocenter exerts a strong influence on the energy budget as it affects both the specific fracture energy parameterization, as well as the determination of the areas of the fault over which we incrementally integrate \( E_{rs} \). The distribution of elastostatic energy \( E_{el} \) can be calculated from the target slip model and the corresponding stress drop. The fracture energy distribution is assigned through Equation (6), and finally the seismic energy density distribution is calculated as shown in Table 1. Although our calculated energy budget strongly depends on our fracture energy parameterization, it provides a consistent approach with our dynamic source modeling and PD procedure.

The integrated seismic energy over the ruptured fault area should remain positive in order to allow rupture growth. In other words, if the integrated seismic energy becomes negative, there is no more energy available for the rupture to grow, and hence the rupture would stop. This is the idea proposed originally by Husseini [1977] and is consistent with the idea that a small earthquake is like a large earthquake that ran out of energy.

Figure 11 shows the energy budget calculation for a given slip distribution (Figure 11a; the hypocenter location is shown by the star). Figure 11e shows the seismic energy integrated over concentric areas around the hypocenter in order to mimic simplified rupture propagation. Notice that around the hypocenter there is an area that is a large sink of energy. In Figure 11* the
Figure 11. Simulated slip distribution for a M=7 strike-slip earthquake (top left) and the corresponding energy calculations for the PD-model. Note of $E_{rs}$ (bottom left) indicated incomplete rupture propagation.

Figure 11*. Dynamic model for the slip distribution shown in Figure 11. Note how the rupture terminated at an early stage, in accordance to what Figure 11 indicates.

Figure 12. Same slip distribution as shown in Figure 11, but different hypocenter location; the PD-energy calculations indicate that the rupture would propagate farther in this case.
Conclusions and Caveats

Dynamic rupture modeling has the advantage with respect to kinematic modeling of providing a physically self-consistent earthquake source characterization. Guatteri et al. [2003] showed that it leads to a realistic representation of ground motion time histories and to a realistic prediction of ground motion intensity from future earthquakes. In our work we have shown that a pseudo-dynamic source characterization has the potential to improve the source model design (physically based) in probable earthquake scenarios. Pseudo-dynamic modeling is a feasible approach for the simulation of suites of ground motion time histories, merging naturally with the probabilistic approach usually taken for seismic hazard analysis. However, the pseudo-dynamic procedure outline above has focused on strike-slip earthquakes in the magnitude range from 6.4 to 7.2. Future work will be devoted to investigate and develop a PD-model that also includes dip-slip earthquakes, and is potentially applicable over a wider magnitude range. At the time of writing this report, we have only limited experience in applying the PD-approach to large-magnitude earthquakes [Mai et al., 2001] that already suggest that at least size-dependent relationships for the fracture energy $G_c$ should apply. The application of our PD-model to earthquake scenarios that are clearly out of the range for which the current characterization has been developed warrants extra care and validation of the results.

References
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