Pacific Earthquake Engineering Research
Approach to Random Vibration Theory (RVT)
for NGA-East

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Outline

Motivation for Using RVT

Introduction to RVT

Peak Factor Formulations
  Vanmarcke Peak Distribution
  Discussion

Processing Time Series
  Effective Amplitude
  Time Series Processing

Conclusion
Motivation for Using RVT

- Response spectrum has been the standard for:
  1. engineering design and evaluation
  2. ground motion models
- Response spectrum depends on characteristics of single-degree-of-freedom oscillators – response at $f_n$ depends on lower frequencies
- Using Fourier amplitude spectra simplifies some problems:
  1. Application of ”factors” (e.g., $\kappa_0$, site amplification, etc.)
  2. Limited bandwidth of recorded motions
- RVT offers a method for calculation of mean response spectrum
Random vibration theory defines a motion by:

1. Frequency content
   - Fourier amplitude spectrum, $Y(f)$ (seismological community)
   - Power-spectral density (structural community)
   - (indirectly) Response spectrum

2. Stationary duration – time with constant statistical properties

Expected values in the time domain are computed using extreme value statistics, which is used to define the peak factor relationship.

Response of a system (site response, oscillator response, structural response) can be computed by applying the appropriate transfer function.
RVT Calculation Steps

1. Apply transfer function (optional)
2. Compute root-mean-squared response ($y_{\text{rms}}$):

$$y_{\text{rms}} = \sqrt{m_0/T_{\text{gm}}}$$

$T_{\text{gm}}$ is the ground motion duration
$m_0$ is first spectral moment computed by:

$$m_k = 2 \int_0^{\infty} (2\pi f)^k |Y(f)|^2 df$$

3. Compute the peak factor ($p_f = y_{\text{max}}/y_{\text{rms}}$)
4. Compute time domain peak value ($y_{\text{max}} = p_f \cdot y_{\text{rms}}$)
Peak Factor Formulation

- Defines the distribution of peaks based on spectral moments of the ground motion

- A number of proposed peak factor formulations:
  2. Davenport (1964)
  3. Vanmarcke (1975)
  5. Toro and McGuire (1987)

- General assumptions:
  - band-limited white Gaussian noise with zero mean
  - stationary stochastic process over duration interval
  - random phase angles

- Cartwright & Longuet-Higgins (56) assume statistical independence of peaks (Poisson process)

- Vanmarcke (76) extended peak factor formulations to include the potential for temporal clustering
Vanmarcke (1975) Peak Distribution

Cumulative distribution of peak values defined by:

\[ F_x(x) = \left[ 1 - \exp \left( \frac{-x^2}{2} \right) \right] \]

\[ \cdot \exp \left\{ -N_z \left[ 1 - \exp \left( -\sqrt{\frac{\pi}{2}} \cdot \delta_e \cdot x \right) \right] \right\} \]

where \( \delta_e \) is defined as:

\[ \delta_e = \delta^{1+b} = \left[ 1 - \frac{m_1^2}{m_0 \cdot m_2} \right]^{(1+b)/2} \]

and \( b \) is empirically calculated to be 0.2

Expected peak factor computed by:

\[ E[x] = \int_0^\infty [1 - F_x(x)] dx \]
Comparison of V75 and DK80 Peak Factors

DK80 is simplifies V75 with in minor differences at low $N_z$, which are important at long periods
Kottke and Rathje (2013) observed differences between TS and RVT (CLH56) site response. On-going research indicates improved agreement using D80 RVT.
Discussion of Peak Factors

- CLH56 assumes statistical independence between local peaks which is a significant approximation.
- CLH56 shortcomings can be partially addressed through empirical duration modification (see Boore and Thompson, 2012), but fail to address underlying statistical deficiencies.
- V75 is more theoretically robust, but not as widely used within seismology.
- Empirical duration modifications will be developed for V75.
- CLH56 and V75 are both recommended, but V75 is preferred.
Processing Time Series for RVT

- Fourier amplitude spectra (FAS) computed from time series will be used to develop ground motion model of FAS
- Need to develop an orientation independent FAS to represent the two horizontal components from as-recorded ground motions
- Reduce frequency spacing to a reasonable number while maintaining consistent characteristics for RVT
Effective Amplitude

Effective spectra defined as:

\[
EA(f) = \sqrt{\frac{1}{2} [FA_{H1}(f)^2 + FA_{H2}(f)^2]}
\]

Average is performed on the \( FA^2 \) to maintain power (Boore, 2003)
## Time Series Characteristics

<table>
<thead>
<tr>
<th></th>
<th>Type 1</th>
<th>Type 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling rate (1/sec)</td>
<td>10, 20, &amp; 40</td>
<td>50, 100, &amp; 200</td>
</tr>
<tr>
<td>Time step, $\Delta t$ (sec)</td>
<td>0.1, 0.05, &amp; 0.025</td>
<td>0.02, 0.01, &amp; 0.005</td>
</tr>
<tr>
<td>Duration (sec)</td>
<td>3276.8</td>
<td>2621.44</td>
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<tr>
<td>$2^n$</td>
<td>15, 16, &amp; 17</td>
<td>17, 18, &amp; 19</td>
</tr>
<tr>
<td>Freq. step, $\Delta f$ (Hz)</td>
<td>0.00030158</td>
<td>0.0003815</td>
</tr>
</tbody>
</table>

- Time series were classified and zero padded to achieve consistent frequency increments.
- Large number of frequencies which need to be reduced for ground motion model development.
Evaluation of Time Series Processing

- Selected the Konno and Ohmachi (1998) smoothing window
- Smoothing windows were evaluated to ensure no change in RVT characteristics:
  1. $m_0$
  2. $\delta = \sqrt{1 - m_1^2/(m_0 \cdot m_2)}$
  3. $f_z = \sqrt{m_2/m_0}/\pi$
  4. $f_e = \sqrt{m_4/m_2}/\pi$
- Considered:
  1. Number of points per decade
  2. Width of smoothing operator
### Results of TS Evaluation at 0.2 sec

<table>
<thead>
<tr>
<th>Points per Decade</th>
<th>Operator Width</th>
<th>$m_0$</th>
<th>$\delta$</th>
<th>$f_z$</th>
<th>$f_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>1/15</td>
<td>21%</td>
<td>15%</td>
<td>87%</td>
<td>92%</td>
</tr>
<tr>
<td>30</td>
<td>1/30</td>
<td>12%</td>
<td>10%</td>
<td>66%</td>
<td>78%</td>
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<tr>
<td>50</td>
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<td>25%</td>
<td>24%</td>
<td>89%</td>
<td>94%</td>
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<tr>
<td>50</td>
<td>1/50</td>
<td>14%</td>
<td>13%</td>
<td>73%</td>
<td>83%</td>
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<td>100</td>
<td>1/30</td>
<td>99%</td>
<td>98%</td>
<td>100%</td>
<td>100%</td>
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<tr>
<td>100</td>
<td>1/100</td>
<td>17%</td>
<td>15%</td>
<td>79%</td>
<td>88%</td>
</tr>
</tbody>
</table>
Examples of Smoothed TS

Effective Amplitude Spectra (EAS) and smoothed and down-sampled Effective Amplitude Spectra (S-DS EAS). The red dashed vertical lines represent the high-pass (HP) and low-pass filters (LP) of the record.
Conclusion

- Range of peak factor formulations were considered for response spectra calculation and beyond.
- Vanmarcke (1975) is the preferred RVT peak factor formulation due to its ability to accurately model both response spectra, as well as other responses.
- Effective amplitude spectra will be computed by the average power of two horizontal components (i.e.,
  \[ EA = \sqrt{\frac{FA_1^2 + FA_2^2}{2}} \]
- Standard time series processing methodology evaluated against RVT parameters.
- Recommended processing approach is a Konno and Ohmachi (98) operator with a width of 1/30\(^{th}\) of a decade, and 100 frequency points per decade.