A Constant Stress-Drop Model for Producing Broadband Synthetic Seismograms: Comparison with the Next Generation Attenuation Relations

by Arthur Frankel

Abstract Broadband (0.1–20 Hz) synthetic seismograms for finite-fault sources were produced for a model where stress drop is constant with seismic moment to see if they can match the magnitude dependence and distance decay of response spectral amplitudes found in the Next Generation Attenuation (NGA) relations recently developed from strong-motion data of crustal earthquakes in tectonically active regions. The broadband synthetics were constructed for earthquakes of $M_{5.5}$, $M_{6.5}$, and $M_{7.5}$ by combining deterministic synthetics for plane-layered models at low frequencies with stochastic synthetics at high frequencies. The stochastic portion used a source model where the Brune stress drop of 100 bars is constant with seismic moment. The deterministic synthetics were calculated using an average slip velocity, and hence, dynamic stress drop, on the fault that is uniform with magnitude. One novel aspect of this procedure is that the transition frequency between the deterministic and stochastic portions varied with magnitude, so that the transition frequency is inversely related to the rise time of slip on the fault. The spectral accelerations at 0.2, 1.0, and 3.0 sec periods from the synthetics generally agreed with those from the set of NGA relations for $M_{5.5}$–$M_{7.5}$ for distances of 2–100 km. At distances of 100–200 km some of the NGA relations for 0.2 sec spectral acceleration were substantially larger than the values of the synthetics for $M_{7.5}$ and $M_{6.5}$ earthquakes because these relations do not have a term accounting for $Q$. At 3 and 5 sec periods, the synthetics for $M_{7.5}$ earthquakes generally had larger spectral accelerations than the NGA relations, although there was large scatter in the results from the synthetics. The synthetics showed a sag in response spectra at close-in distances for $M_{5.5}$ between 0.3 and 0.7 sec that is not predicted from the NGA relations.

Introduction

One of the key products that seismologists can provide engineers is a set of accurate synthetic seismograms for future moderate and large earthquakes. While these synthetics are not expected to match the strong-motion records of future earthquakes wiggle for wiggle, they must have comparable duration, pulselike character, and frequency content to actual strong-motion records. These seismograms are used by engineers in the design of structures where it is important to assess the role of duration, phasing, coherent pulses, and changes in frequency content during shaking on building response. Synthetic seismograms can supplement the catalog of recorded strong-motion records by providing records where the data are sparse, such as for close-in distances from earthquakes above magnitude 7. The synthetics also allow the user to sample different rupture directivity and slip distributions that would not be available in recorded data. Thus, the synthetics may be better suited to characterize the variability in the building response to different earthquake scenarios.

In this article I present a method to compute broadband synthetic seismograms using a source model where the stress drop is constant with magnitude. Stress drop is a key parameter because it basically quantifies the level of high-frequency shaking for a given seismic moment. There are a multitude of stress-drop measures that can be used to characterize the source process. One measure is from the corner frequency and seismic moment and is often denoted as the Brune stress drop, based on the work of Brune (1970). This measure essentially relates the high-frequency level of the acceleration spectrum to the seismic moment. Another type of stress drop is the dynamic stress drop, which is proportional to the slip velocity on the fault during an earthquake (Boatwright, 1982). I will use both types of stress drop in the model for synthetic seismograms.
The procedure I describe combines long-period synthetics made using a plane-layered velocity model with short-period synthetics constructed from combining point-source stochastic seismograms derived from the procedure of Boore (1983) based on the stochastic model introduced by Hanks and McGuire (1981). Using these deterministic synthetics at longer periods is critical for reproducing the coherent ground-motion pulses observed in actual strong-motion records, especially for forward directivity, as well as the delay of long-period surface waves relative to S-waves. An empirical method was developed by Mavroeidis and Papageorgiou (2003) to characterize forward directivity pulses and has been applied to stochastic methods (Motazedian and Atkinson, 2005). However, this empirical method is not based on the physical properties of the rupture such as rupture velocity, hypocenter location, and slip distribution. Furthermore, the stochastic methods do not produce the correct arrival time of long-period surface waves after the initial S-wave.

In the method used here, the long- and short-period seismograms are summed over a fault plane with a physically reasonable rupture process and are then combined using a matched filter. There has been previous work using this combination, usually referred to as hybrid methods, such as Hartnell and Heaton (1995), Kamae et al. (1998), and Hartzell et al. (1999). Graves and Pitarka (2004) presented a hybrid method most similar to the one in this article. However, there are significant differences between my method and theirs, which will be described subsequently. One key new feature is that the transition frequency between the deterministic and stochastic seismograms varies with magnitude. Another different aspect of my method is the dependence of rise time on the local slip on the fault to maintain the average slip velocity. It is crucial to validate any synthetic seismogram method using data. A new set of attenuation relations, the Next Generation Attenuation (NGA) relations has recently been developed from a worldwide data set of strong ground-motion records from crustal earthquakes in tectonically active regions. Three of these relations, Chiou and Youngs (2008), Boore and Atkinson (2008), and Campbell and Bozorgnia (2008), have been incorporated into the latest update of the national seismic hazard maps (Petersen et al., 2008). Each NGA relation contains a different functional form relating median ground motions to the earthquake magnitude, source–receiver distance, and other parameters. I compare the response spectral amplitudes from the synthetics with those determined from four of the NGA relations (including Abrahamson and Silva, 2008), as one way to validate the procedure to make the synthetics. I did not use the Idriss (2008) NGA relations because $V_{S30}$ (average shear-wave velocity in the top 30 m) is not contained in their functional form. Most importantly, the comparison of the response spectral values from the synthetics with NGA indicates whether the NGA relations make physical sense. In other words, the NGA results consistent with ground motions found from a model of rupture on an extended fault using reasonable parameters describing propagation in the crust?

**Transition Frequency between Deterministic and Stochastic Seismograms**

We expect that complexities in the rupture process and scattering by random heterogeneities in the crust between the source and receiver will cause incoherence in the summation of waveforms produced by different portions of the rupture surface. Many previous studies assumed that this transition occurred at about 1 Hz, partly because of computational limitations when 3D simulations were used for the long-period part of the calculation (e.g., Kamae et al., 1998; Graves and Pitarka, 2004). However, on the source side of the problem, we would expect that the frequency of transition between coherent and incoherent summation will vary with magnitude.

In the absence of scattering, we would expect to see coherent pulses at periods as short as a couple of tenths of a second for $M$ 5.5 earthquakes and several seconds for $M$ 7.5 earthquakes, based on their rise time of slip from modeling waveforms as summarized in Somerville et al. (1999). Figure 1 shows the velocity waveform for station TCU075 that recorded the $M$ 7.6 Chi-Chi, Taiwan, earthquake (data from Lee et al., 2001). This station is located updip from the hypocenter and samples forward directivity. This east–west component is oriented perpendicular to the fault strike and parallel to the slip on the fault. The pulse has about a 6 sec duration when considering the duration of the two sided velocity pulse. In Figure 2a, the Fourier spectrum of the east–west component of velocity shows

![Figure 1. Velocity pulses from station TCU075 (T75) from the M 7.6 Chi-Chi, Taiwan, earthquake and the Rinaldi station from the M 6.7 Northridge, California, earthquake. Both stations were updip from the hypocenter. The component of the seismograms is approximately parallel to the direction of slip on the fault and perpendicular to the strike of the fault. Note the longer period of the velocity pulse for the Chi-Chi earthquake compared to the Northridge earthquake.](image-url)
higher amplitudes at low frequencies (<0.8 Hz) than the north–south component, reflecting the coherent pulse on the east–west component that is not present on the north–south component. The frequency where the east–west and north–south spectra join, about 0.8 Hz, represents the frequency of transition between coherent summation that comprises the velocity pulse and incoherent summation.

Figure 1 also displays the velocity pulse for the $M_{6.7}$ Northridge earthquake at Rinaldi, which is updip from the hypocenter and also samples forward directivity. This component is oriented at 228° clockwise from north, approximately perpendicular to the fault strike and parallel to the slip direction. The velocity pulse width (two sided) is now only about 1.5 sec in duration. The velocity spectrum of the 228 component displayed in Figure 2b is peaked at about 1 Hz, inversely related to the duration of the velocity pulse. Spectra of the two horizontal components merge at about 2.5 Hz at this station, indicating that the transition frequency between the coherent pulse and incoherent behavior occurs at about 2.5 Hz. As these examples from the Chi-Chi and Northridge earthquakes illustrate, this frequency of transition is inversely related to the pulse duration, which is a function of the rise time of slip on the fault, the subevent dimension, and the directivity.

At sufficiently high frequencies and/or large distances, scattering will likely cause incoherence regardless of the coherence of the source pulse radiated from the rupture. Obviously, scattering will alter even the long-period portion of waveforms at a sufficient distance from the source. The transition frequency between coherent and incoherent summation will likely be affected by the distance. In this study, we are most interested in capturing the coherent directivity effects at close-in distances by using the deterministic synthetics and including the proper timing of the surface waves at all distances.

Methodology for Making Broadband Synthetics

In this study I considered ground motions from vertical strike-slip faults observed at firm-rock sites typical of rock sites in the western United States. Based on Boore and Joyner (1997), I used a $V_{s30}$ for these firm-rock sites of 620 m/sec for the high-frequency stochastic synthetics. I calculated ground motions for earthquakes with moment magnitudes $M$ of 5.5, 6.5, and 7.5 at source–receiver distances from 2 to 200 km.

Figure 3 shows a flow chart of the method used to produce broadband seismograms. For the long-period seismograms, we generate synthetic Green's functions for a plane-layered model using the frequency wavenumber integration program Greenbank written by Zhu and Rivera (2002). This method could also incorporate long-period synthetics from a 3D finite-difference program. Table 1 shows the velocity model used here for the long-period calculation. I started with the velocity model of Wald et al. (1996) but reduced the depth of the Moho to 30 km, which is a reasonable average for the western United States. I also modified the top 300 m of the model to correspond approximately to the shear-wave velocity ($V_S$) profile for generic rock sites determined by Boore and Joyner (1997). Here I used layer thicknesses of 0.1 km and shear-wave velocities corresponding to the layer thickness divided by the $S$-wave travel time determined for that depth range using the power law functions of $V_S$ given in Boore and Joyner (1997). Synthetic Green's functions were calculated for a range of distances and source depths. The fault plane was divided up into a set of cells or subfaults. For all three magnitudes, I used a
cell size of 0.31 by 0.31 km (see the Dimensions of Faults and Distribution of Stations section).

Fractal distributions of fault slip and stress drop were produced for each earthquake scenario. For the fault slip I utilized a model where the spectral amplitude as a function of wavenumber $k$ was flat up to some corner wavenumber and then decayed as $k^{-2}$ above that. This is similar to the model advocated by Somerville et al. (1999) and this spectral fall-off is consistent with constant stress drop scaling with seismic moment (Frankel, 1991; Herrero and Bernard, 1994). I used different correlation distances (inversely proportional to the corner wavenumber) for the along strike and downdip directions on the fault plane, applying the magnitude dependent values of Mai and Beroza (2002). Figure 4 shows one realization of the slip. The mean level of the slip is adjusted so that the minimum slip value equals zero. For each source cell, the long-period Green’s functions are convolved with a source time function whose seismic moment equals the slip for that cell multiplied by the shear modulus and cell area, with a shape described subsequently. The seismic moments of the cells sum to the moment for the main shock.

I specify the average slip velocity for the entire fault plane, allowing some random variation between cells. This average slip velocity was set to be equal for the three moment magnitudes studied here: 5.5, 6.5, and 7.5. The rise time $T_i$ for each cell $i$ is equal to the final slip $S_i$ for that cell divided by the average slip velocity $S_v$, such that $T_i = S_i / S_v$. Because $S_i = M_{0i} / (\mu A)$, where $M_{0i}$ is the seismic moment for each cell, $\mu$ is the shear modulus, and $A$ is the area of each cell, then

$$T_i = M_{0i} / (\mu A S_v). \quad (1)$$

Thus, portions of the fault with larger slip (higher seismic moment) will have longer rise times, in keeping with results of inversions of strong-motion data, such as those reported for the $M$ 7.9 Denali fault, Alaska, earthquake by Frankel (2004). The average slip velocity was chosen so that the maximum rise time was approximately equal to that found from Somerville et al. (1999) for $M$ 7.5 (about 2.5 sec). This yielded an average slip velocity of 2.7 m/sec, which was used for all the simulations.

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<th>$V_s$ (km/sec)</th>
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Figure 3. Flow chart showing the procedure to make broadband synthetics.

Figure 4. (Top panel) Fractal distribution of slip on the fault plane used in one of the realizations for the $M$ 7.5 earthquake simulations. (Middle panel) Fractal distribution of stress drop on the fault plane used in the same simulation. This is derived from the same random number seed as the slip in the top panel, but the wavenumber spectral decay is less steep, causing more roughness than the slip. (Bottom panel) Rupture time derived from 2D finite-difference simulation where the local perturbation of the rupture velocity is proportional to the perturbation of slip relative to the average slip. A random component is then added to this rupture time.
other studies to make broadband synthetics (e.g., Graves and Pitarka, 2004; Liu et al., 2006). I found that such a pulse shape produced spectral peaks at high frequencies that were not consistent with a constant acceleration spectrum.

The timing of the rupture initiation at any point on the fault plane is critical to the summation of the synthetics. The local rupture velocity is assumed to be related to the slip at that location on the fault, in keeping with findings from dynamic rupture simulations (Guatteri et al., 2003). This is implemented by having the perturbation of the rupture velocity relative to the average rupture velocity proportional to the perturbation of the slip at that point relative to the average slip on the fault. I then adjust the amplitude of the rupture velocity perturbations so that they have a standard deviation equal to 20% of the average rupture velocity and limit the rupture velocity so that it cannot exceed ±40% of the average rupture velocity. The average rupture velocity is taken here to be 2.8 km/sec.

Given the variations in the local rupture velocity, I determined the timing of rupture across the fault plane using a 2D finite-difference program (Frankel and Clayton, 1986). This program is for propagation of SH waves through a medium with an arbitrary $V_S$ field. Here I am substituting the rupture velocity on the fault plane for the shear-wave velocity in the simulation. I determined the timing of rupture initiation along the fault plane by picking the times of the $SH$ pulse propagating through the medium. I found that the most stable results were achieved by picking the time of the peak amplitude of the pulse because the pulse shape did not change significantly as the pulse propagated. The simulation was initiated by imposing an initial displacement that was shaped as a Gaussian in space, centered on the hypocenter. A raytracing program for a complex 2D medium would have achieved a similar result. Figure 4 displays the timing of the rupture front derived from the finite-difference simulation for the slip distribution shown in the same figure. It is apparent how the contours of rupture time are more widely separated in areas with high slip, indicating higher local rupture velocity.

I also added a small random component of rupture timing for each cell, in order to apply some realistic small-scale variation in rupture initiation. This random component consists of a variation between each cell and a variation over blocks of cells, where each block consisted of 5 cells along strike by 5 cells along dip. I chose this timing variation through trial and error, by comparing the synthetics to data. For the magnitude 6.5 and 7.5 earthquakes uniform distributions were used with ±0.2 and ±0.4 sec for the intercell and interblock variations, respectively. For the $M$ 5.5 case, it was found that these variations caused too low spectral accelerations (SAs) at 1 Hz compared to NGA. Therefore, I used an intercell variation of ±0.1 sec and an interblock variation of ±0.2 sec for $M$ 5.5. A small random component of the focal mechanism was also applied by varying the rake, strike, and dip between blocks with a uniform distribution of ±20°.

For the high-frequency seismograms, I summed point-source stochastic synthetics derived from the method of Boore (1983) using his program SMSIM (Boore, 1996). I applied the frequency-dependent site amplification determined for a generic rock site by Boore and Joyner (1997). The rupture timing along the fault is identical to that used for the long-period synthetics. The high-frequency synthetics for each cell are multiplied by a stress-drop factor derived from a fractal distribution of stress drop along the fault plane, rather than from the fractal slip used for the long-period synthetics. I chose this because the spectral amplitude of radiated energy at any given frequency above the corner frequency, divided by the rupture area, is related to the stress drop, not the slip, for a model where the acceleration spectrum is flat above the corner frequency (Joyner, 1984; Heaton and Hartzell, 1989; Frankel, 2004). Inversions of strong-motion data often indicate that areas of high slip on a fault are not necessarily areas that produce large amounts of high-frequency energy (e.g., Frankel, 2004). That said, I used the same random number seed for the stress drop and slip fields of each run. I found that, on average, the high-frequency SAs were not sensitive to the random number seed used for the fractal stress drop. Figure 4 shows an example of the fractal stress drop, which has a high-frequency decay of the wavenumber spectrum proportional to $k^{-1}$ and produces constant stress-drop scaling (Frankel, 1991). A stress drop of 100 bars was specified for the point-source synthetics. The amplitude of the fractal distribution of stress drop was adjusted so that its root mean square value over the fault plane was equal to 100 bars.

For the high-frequency synthetics, I used the frequency-dependent $Q = 180^{1/0.45}$ and geometrical spreading reported by Raoof et al. (1999) for southern California earthquakes. Here the geometrical spreading is $R^{-1}$ for distances up to 40 km and $R^{-0.5}$ for greater distances. For each point-source–receiver combination, the start of the short-period seismogram is set at the sum of the $S$-wave travel time calculated for the plane-layered model used for the long-period synthetics and the time to rupture initiation for that point source that is applied in the long-period calculation. A value of 0.035 was used for the value of site $\kappa$, based on the average value for a generic rock site given in Boore and Joyner (1997). The duration was specified as equal to $1/f_c + 0.05$ times the distance, based on an input file given in Boore (1996), where $f_c$ is the corner frequency of the subevent. The corner frequency of the subevent is determined from the stress drop, subevent moment, and shear-wave velocity using the relation given by Brune (1970).

After the point-source stochastic synthetics are summed over the fault plane, they are convolved with a source time function that ensures that the resulting acceleration spectrum is flat for frequencies lower than the corner frequency of the point-source seismograms. I used the source time function described in Frankel (1995) for this purpose.

Basically, this ensures that the acceleration spectrum (Fourier) is flat for frequencies less than the corner frequency used for the subevents and frequencies greater than the transition frequency with the long-period synthetics. The method of Frankel (1995) requires specification of a lower corner
frequency that determines the lower frequency limit of the flat acceleration spectrum. As long as this lower limit is much lower than the transition frequency, the acceleration Fourier spectrum of the broadband synthetics will be approximately flat between the corner frequency of the subevent and the transition frequency. I chose the lower corner frequency so that it would be much lower than the transition frequency. It should also be noted that the proper moment for the mainshock is ensured in the summation of the long-period synthetics from the subevents.

Finally, the long-period and short-period synthetics are summed after applying a matched filter. I used high- and low-pass phaseless Butterworth filters with identical corner frequencies and fall-offs (see Hartzell et al., 1999). A second-order low-pass filter is applied to the deterministic synthetics and a second-order high-pass filter is applied to the stochastic synthetics. The filters do not change the phase of the signals. The responses of the two filters sum to one at all frequencies.

Based on the spectra shown in Figure 2 for the $M_{7.6}$ Chi-Chi earthquake, the transition or crossover frequency for the $M_{7.5}$ synthetics was set at 0.8 Hz. The transition frequency (related to the inverse of the pulse duration) may be lower for a site that is not in the direction of rupture propagation, but I chose a transition frequency based on the forward directivity pulse from Chi-Chi, to ensure that the coherence of forward directivity pulses would be captured in the broadband synthetics. Then, assuming that the crossover frequency should be inversely proportional to the rise time on the fault (or moment to the one-third power), I used a crossover frequency of 2.4 Hz for $M_{6.5}$. Based on this scaling, the crossover frequency for $M_{5.5}$ should be about 7.5 Hz. Because scattering effects will be especially important at this high frequency, I chose to use a crossover frequency of 3.0 Hz for the $M_{5.5}$ runs. This was chosen so that the 5 Hz SA values are controlled by the stochastic portion of the calculation. 

Table 2 lists some of the key parameters used in the simulations.

### Dimensions of Faults and Distribution of Stations

The fault dimensions are important to the ground-motion calculations. It is especially important to determine the proper dimensions for the subevents, after picking a magnitude for them. In order to do this, I considered the scaling of earthquakes with approximately equal dimensions in length and width, so that standard scaling relations with constant static stress drop could be used. The dimensions of the mainshocks should also be consistent with empirical studies of fault area versus magnitude. I started with the largest rupture that has equal length and width: a vertical rupture that extends from 3 km depth to 15 km depth, with a 12 km length. I determined the moment magnitude for this earthquake from Wells and Coppersmith (1994). Because I assumed the faulting reaches the surface, the area to use in the Wells and Coppersmith formula is 15 by 12 km, yielding a magnitude of 6.3.

In all the simulations I used a minimum depth of rupture of 3 km, assuming that depths above 3 km did not radiate significant seismic radiation in the frequency range we are considering here, following the reasoning of Campbell (1997). The minimum depth of seismogenic rupture is a controversial topic. It likely varies with the frequency of ground motion. Graves and Pitarka (2004) lowered the rupture velocity and lengthened rise time for the portion of the fault above 5 km.

Based on constant stress-drop scaling for circular ruptures, the area of the subevent $A_{sub}$ can be determined from

$$A_{sub} = A_{main}(M_{sub}/M_{0main})^{2/3},$$

where the area of the mainshock $A_{main}$ is 12 by 12 km. Choosing a subevent magnitude of 3.1, yields a dimension of 0.31 by 0.31 km for the subevent (here log $M_0 = 1.5M_{0} + 9.05$ in mks units), given a mainshock magnitude of 6.3 and area of 144 km$^2$. This subevent or cell size of 0.31 by 0.31 km is used in all the simulations. I found that the magnitude of the subevent did not make a significant difference in the SAs calculated for the mainshock. I chose a magnitude (3.1) small enough so that rupture directivity would be properly captured. I used this subevent size (0.31 by 0.31 km) for the $M_{5.5}$, $M_{6.5}$, and $M_{7.5}$ events, filling the fault plane with the subevent rupture zones in each case.

The rupture dimensions for the $M_{6.5}$ events were based on the area from Wells and Coppersmith (1994), resulting in 18 km length by 15 km width, with the top 3 km assumed to be nonseismogenic. For the $M_{7.5}$ rupture I used a 150 km length, approximately averaging the results of Hanks and Bakun (2002), the Ellsworth B relation given in Working Group on California Earthquake Probabilities (2003), Somerville (2006), and Wells and Coppersmith (1994) for that magnitude. This 150 km length is consistent with the rupture

<table>
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<th>Fault Width (km)</th>
<th>Depth Extent of Rupture (km)</th>
<th>Hypocentral Depth (km)</th>
<th>Average Slip Velocity (m/sec)</th>
<th>Approximate Maximum Rise time (sec)</th>
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length found from modeling the strong-motion records of the M 7.5 Izmit (Kocaeli), Turkey, earthquake by Bouchon et al. (2002). The maximum depth for the M 7.5 earthquakes was taken to be 18 km and that of the M 6.5 earthquakes taken to be 15 km (see Table 2). The dimensions of the M 5.5 earthquake were determined using equation (2), resulting in a 4.9 by 4.9 km rupture. I assumed the rupture zone of the M 5.5 extended from 3 to 7.9 km depth.

Figure 5 shows the station distribution applied for the M 7.5 simulations. The distribution contains stations at a range of azimuths from the ends of the fault and stations arranged parallel to the strike of the fault. For any given value of the closest distance to the fault, the stations at that distance are distributed so that they are approximately equal distance from each other. For close-in distances, the stations densely sample the ground motions along the length of the fault. This was done to ensure a sampling of areas with high and low slip along the fault surface. A similar distribution of stations was used for the M 6.5 and M 5.5 simulations. 243 stations were used for the M 7.5 simulations, 114 stations for the M 6.5 runs, and 105 stations for M 5.5 simulations.

An example of the summation of the deterministic and stochastic synthetics is depicted in Figure 6. These seismograms are for a station 3 km south of the southern end of the fault used in one of the M 7.5 simulations (hypocenter midway along the fault length). The deterministic synthetics are from the fault-normal (east–west) component and show a strong pulse because of forward rupture directivity. The deterministic synthetic is low-pass filtered at 0.8 Hz, and the stochastic synthetic is high-pass filtered at 0.8 Hz. Both filters are second-order phaseless Butterworth filters. The broadband synthetic is the sum of the two filtered synthetics.

In this initial study, I have not attempted to vary all the parameters to determine estimates of the variability of ground motions for any given magnitude and distance. For example, I have chosen one crustal model and one average stress drop. I have assessed the variability of spectral response values from differences in receiver location, slip distribution, and hypocenter location.

**Results and Comparison with NGA Relations**

Broadband synthetic seismograms (acceleration and velocity) for three stations at 3 km closest distance from one of the M 7.5 simulations are depicted in Figure 7. For the ver-

![Figure 5. Map of stations (circles) used for the M 7.5 simulations. Fault strikes north–south and is shown as solid line. The stations at any given value of the closest distance to the fault (Rjb) are arranged so that they have approximately equal distances from each other.](image)

![Figure 6. Construction of broadband seismogram. The top trace is the east–west (fault normal) component from the deterministic (plane-layered model) synthetics for a station 3 km from the south end of the fault from one of the M 7.5 simulations, low-pass filtered at 0.8 Hz. The middle trace is from the summation of the stochastic seismograms, high-pass filtered at 0.8 Hz. The bottom trace is the broadband seismogram derived from summing the deterministic and stochastic seismograms.](image)
tical strike-slip, surface-rupturing faults used here for \( M_{7.5} \) and \( M_{6.5} \), the closest distance to the fault is equal to the Joyner–Boore distance \( R_{jb} \) (Joyner and Boore, 1981). The hypocenter where the rupture starts is closer to the southern end of the fault. On the acceleration traces, the long-period forward directivity pulse is most apparent in the station off of the north end of the fault. The peak acceleration for the station to the south is significantly smaller. It is clear how rupture directivity causes large variations in ground motions for stations at different azimuths but identical distances from the closest portion of the fault. The accelerogram for the station near the middle of the fault has a more extended duration than those off the ends of the faults, again an expression of the differences caused by rupture directivity.

The velocity traces demonstrate the large pulses on the fault-normal component for these three receivers. The peak velocity is largest on the station to the north, even though it is farther from the hypocenter than the other stations. This pulse is especially dominant on the fault-normal velocity synthetics for these receivers.

Figure 7. Acceleration and velocity synthetics for three sites at 3 km \( R_{jb} \) from fault for the \( M_{7.5} \) simulation where the hypocenter is located 38 km from the southern end of a 150 km long fault. The top trace in each panel is the east–west component (perpendicular to fault strike, i.e., fault normal). The bottom trace in each panel is the north–south component (parallel to fault strike). (a) The station located 3 km from the north end of the fault. (b) The station located 3 km from the middle of the fault. (c) The station located 3 km from the south end of the fault. Seismograms are displaced vertically on each plot for clarity. Note strong pulses on the fault-normal component for all the stations. This pulse is especially dominant on the fault-normal velocity synthetics for these receivers.
in peak acceleration and the change in the envelope of the ground motions with distance are apparent. The seismogram at 200 km distance exhibits long-period surface waves that follow the shorter-period $S$-wave energy. Some synthetics for $M_{7.5}$, $M_{6.5}$, and $M_{5.5}$ earthquakes are shown in Figure 9. Here the station is 10 km east of the center of the fault. The peak accelerations are similar at this station regardless of the magnitude. The duration of shaking is much larger for the $M_{7.5}$ earthquake compared to the $M_{6.5}$ event, which has longer duration than the $M_{5.5}$ shock.

In Figures 10, 11, 12, and 13, I compare the spectral response amplitudes from the broadband synthetics with the values predicted using four of the NGA relations as a function of closest distance to the rupture. Nine simulations were considered for the $M_{6.5}$ and $M_{7.5}$ cases. Three hypocenters were specified, at $1/4$, $1/2$, and $3/4$ along the fault length, all at the base of rupture. For each hypocenter, three different random distributions of slip and stress drop were applied.

The geometrical mean of the SAs from the two horizontal components (north–south and east–west) of the synthetic seismograms at each receiver was calculated for each simulation. The NGA relations use a value of the geometrical mean of the SA from the two horizontal components of the data that is independent of sensor orientation and is called GMrotI50 (Boore et al., 2006). The average difference between the geometric mean and GMrotI50 in the NGA database is only a few percent (Beyer and Bommer, 2006; Boore et al., 2006; Campbell and Bozorgnia, 2008). Because our high-frequency synthetics are stochastic and set to be identical for both horizontal components, it is inappropriate to rotate the broadband synthetic seismograms to other sensor orientations. The SAs are for 5% of critical damping. When an NGA relation required a depth to top of rupture, I used 0 km for the $M_{6.5}$ and $M_{7.5}$ cases and 3 km for the $M_{5.5}$ case (see the following discussion). I assumed that the $M_{6.5}$ and $M_{7.5}$ cases will produce surface rupture, although the top of the seismogenic rupture applied here is 3 km. When finding the values from the NGA relations, a $V_{330}$ of 620 m/sec was specified, as well as strike-slip faulting and a $90^\circ$ dip.
Figure 10. SAs (open circles) from the broadband synthetics of the nine simulations of $M_{7.5}$ earthquakes, for 0.2 (top panel), 1.0 (middle panel), and 3.0 sec periods (bottom panel). Solid lines are predictions of the median values from the NGA relations.

Figure 11. Median values of the SAs (solid circles with error bars for $\pm 1$ standard deviation, based on the variation between the values at each distance) at each distance for 0.2 (top panel), 1.0 (middle panel), and 3.0 sec periods (bottom panel) from the broadband synthetics for $M_{7.5}$ compared to the predictions from the NGA relations.
Figure 12. Median values of the SAs (solid circles with error bars for ±1 standard deviation) at each distance for 0.2 (top panel), 1.0 (middle panel), and 3.0 sec periods (bottom panel) from the broadband synthetics for $M_{6.5}$ compared to the predictions from the NGA relations.

Figure 13. Median values of the SAs (solid circles with error bars for ±1 standard deviation) at each distance for 0.2 (top panel), 1.0 (middle panel), and 3.0 sec periods (bottom panel) from the broadband synthetics for $M_{5.5}$ compared to the predictions from the NGA relations.
Figure 10 shows the spectral response values for the nine $M_{7.5}$ scenarios plotted against the closest distance of the rupture to that station. Note the scatter in SA values at any given distance. The scatter is larger for the 3 sec values than for 1.0 and 0.2 sec. Because the crossover frequency between the deterministic and stochastic seismograms is at 0.8 Hz, the 0.2 sec values are controlled by the stochastic portion and the 1.0 sec values are a mix of the deterministic and stochastic aspects of the calculation. The 3.0 sec values are controlled by the deterministic procedure and show the most scatter because they are more sensitive to the directivity, radiation pattern, and slip distribution. The short-period synthetics do not include a specific focal mechanism, and the directivity effect is limited to altering the total duration of shaking and the resulting change in amplitudes. In general, the SA values in Figure 10 from the synthetics bracket the predictions of the median values from the NGA relations.

Figure 11 depicts the median values of the SAs displayed in Figure 10 for the $M_{7.5}$ synthetics for each distance along with error bars signifying ±1 standard deviation in the SAs at each distance. These error bars express the scatter of the values at each distance. In general, the variability is larger for longer periods, as is generally seen in strong-motion data. Again, this is due to the increased effect of rupture directivity and radiation pattern in the deterministic, long-period synthetics. The 0.2 sec SA values from the synthetics have very similar amplitude and distance decay rate from 2 to 100 km distance as all of the NGA relations. However, from about 100 to 200 km distance the synthetic values decay more steeply with distance than predicted by Abrahamson and Silva (2008) and Campbell and Bozorgnia (2008). The Chiou and Youngs (2008) and Boore and Atkinson (2008) relations are fairly similar to the synthetic values out to 200 km. The over prediction of the synthetics at 100–200 km by Abrahamson and Silva (2008) and Campbell and Bozorgnia (2008) is a consequence of those relations not having a $Q$ term where the log of the amplitude is negatively proportional to distance to supplement the term they do have where log of the amplitude is proportional to log of distance.

At a 1.0 sec period, the SA values from the $M_{7.5}$ synthetics generally agree well with the NGA predictions from 2 to 200 km (Fig. 10), with the exception of an over prediction of the synthetic values at 150–200 km by Abrahamson and Silva (2008). The median of the 3 sec SA values from the synthetics are systematically higher than the NGA values, although the values are within one standard deviation of three of the NGA relations. In general, the median is about a factor of 1.5 higher than three of the NGA relations. At distances between 30 and 150 km, the median values from the synthetics are about a factor of 2 higher than the predicted median values from Chiou and Youngs (2008). The significance and possible causes of this difference will be discussed later in the article.

An important question is how the deterministic and stochastic ground motions compare around the frequency of transition (D. Boore, personal comm., 2008), which is 0.8 Hz for the $M_{7.5}$ simulations. Fourier spectral amplitudes were calculated between 0.6 and 1.0 Hz from the deterministic and stochastic seismograms, before they were filtered. The geometrical average of the Fourier spectral amplitudes between 0.6 and 1.0 Hz was calculated for each synthetic seismogram. The geometrical average of the spectral amplitudes was taken for the two components from the deterministic case. Figure 14 shows the Fourier spectral amplitudes based on seismograms from 243 stations for one of the $M_{7.5}$ runs with the hypocenter midway along the fault length. The spectral amplitudes are similar between the deterministic and stochastic synthetics, except at distances of 100–150 km, where the deterministic synthetics have somewhat larger spectral amplitudes than the stochastic ones. This is probably caused by the presence of Moho reflections in the deterministic seismograms that cause larger amplitudes than the $R^{-0.5}$ geometrical spreading used in making the stochastic seismograms, for distances past 40 km. However, the overall similarity in the spectral amplitudes around the transition frequency for the deterministic and stochastic seismograms shows that there will not be a large jump in the spectrum in the vicinity of the transition frequency.

The results for the $M_{6.5}$ simulations (Fig. 12) show overall agreement between the SA values of the synthetics at 1.0 and 3.0 sec and the respective predictions of the NGA relations, although the Boore and Atkinson (2008) 3.0 sec values from 30 to 200 km are larger than the synthetic values. At 0.2 sec, the Chiou and Youngs (2008) models fit the synthetic values the best. The other three relations predict larger
values than was found from the synthetics for distances greater than 100 km and, for Campbell and Bozorgnia (2008) and Boore and Atkinson (2008), for distances greater than about 30 km. At the 1.0 sec period, the synthetic values level off between 80 and 120 km because of reflection by the Moho; whereas the NGA values continue to decline with increasing distance. It should be stressed that the NGA relations are based on a worldwide data set mixing various crustal structures, as opposed to the one crustal model used here to make the synthetics. Thus, we would expect that Moho reflections may be averaged out of the NGA relations. All of these comparisons should be tempered by the fact that the synthetics do not include variations from different crustal models and parameters such as average stress drop.

For \( M \leq 5.5 \), I considered three different random slip distributions (with their corresponding stress-drop distributions). The hypocenter was taken to be the center of a 5 by 5 km fault that extends from 3 to 8 km depth. Because the rupture zone was so small, I did not use a rupture velocity that varied with the local slip on the fault. Figure 13 displays the comparison between the SA values of the synthetics with the NGA predictions. At 0.2 sec the Abrahamson and Silva (2008) and Chiu and Youngs (2008) models match the synthetic values fairly well. Note that the Abrahamson and Silva (2008) relations specify a steeper fall off past 100 km for earthquakes less than \( M \geq 6.5 \), causing the better fit to the 0.2 sec SA values from the synthetics at those distances. The Campbell and Bozorgnia (2008) and the Boore and Atkinson (2008) values are higher than the synthetics for distances greater than about 40 km, and the Boore and Atkinson (2008) value is lower than the synthetics for distances of less than 10 km. At 1.0 sec, the synthetic values agree with the predictions of Chiu and Youngs (2008) and Abrahamson and Silva (2008) and are somewhat lower than those of Campbell and Bozorgnia (2008) and Boore and Atkinson (2008) for distances greater than 60 km. At a 2 km distance, the 3.0 sec SA values from the synthetics are lower than those predicted from all the NGA relations. There are few long-period data from close-in stations for \( M = 5.5 \) earthquakes, so this discrepancy may not be significant. The synthetics also predict a relatively low value of 3.0 sec SA at 20 km distance that is not predicted by the NGA relations. For distances from 30 to 200 km the 3.0 sec SA values are similar between the synthetics and the NGA relations.

As a test of the finite-fault procedure, I compared the 0.2 sec SAs derived from the \( M = 5.5 \) finite-fault simulations with those from a point source, using the stochastic SMSIM program (Boore, 1996). I specified a stress drop of 100 bars for the point source, identical to the stress drop used in the high-frequency portion of the finite-fault procedure. At a 0.2 sec period, the broadband, finite-source synthetics are controlled by the stochastic portion of the summation. Figure 15 shows the results for 20 different point-source simulations using 20 random number seeds and the finite-source results also contained in Figure 13. The finite-source and point-source values of 0.2 sec SA are very similar for distances larger than 10 km, proving that the finite-source summation is equivalent to the point-source result at high frequencies for large distances compared to the fault length. As expected, at short distances of 10 km and less, the finite-source results are somewhat lower than the point-source results (Fig. 15).

Figure 16 contains the response spectra from the synthetics and NGA relations for specific distances. The error bars signify \( \pm 1 \) standard deviation for that period and distance. For the \( M = 7.5 \) case at 20 km \( R_{jb} \) distance, the NGA relations are similar to those from the synthetics for a 0.1 sec period to about a 1.5 sec period. For 2 sec and longer periods, the median values from the synthetics are higher than the NGA relations, although there is substantial scatter in the synthetic values. The difference between the synthetic values and NGA is greatest at the 5 sec period, such that two of the NGA relations are below the median minus one standard deviation value (Fig. 16). The discrepancy between the NGA relations and the synthetics for 5 sec (and other periods above 2 sec) deserves additional study. There are likely to be variations in travel time caused by lateral heterogeneities in seismic velocity that could decrease the amplitude at such long periods, especially for arrivals from distant portions of long ruptures (see the Discussion and Conclusion section). These lateral heterogeneities in seismic velocity are not contained in the 1D crustal model used here for the long-period synthetics. At a 0.1 sec period, Abrahamson and Silva (2008) predict somewhat lower values than the synthetics (and the other three NGA relations). For \( M = 6.5 \), the synthetic response spectrum at 20 km is similar to those from the NGA relations (Fig. 16).
There is an interesting difference in the shape of the \( M_{5.5} \) response spectrum at 30 km from the synthetics compared to those for the NGA relations (Fig. 16). There is a sag in the synthetic response spectrum from about 0.3 to 0.7 sec that is not apparent in the NGA relations. This occurs at periods between the rise time of slip on the fault (about 0.2 sec maximum) and the overall duration of the earthquake (about 1 sec based on the rupture dimension and the rupture velocity). This sag becomes less pronounced at larger distances. It will be interesting to see if this sag is apparent in the response spectra of actual \( M \) 5.5 earthquakes in the western United States, once site response has been accounted for.

To assess the effects of the transition or crossover frequency \( f_{\text{cross}} \) between the deterministic and stochastic synthetics on the SAs from the synthetics, I compared the 1 sec SA of the \( M \) 7.5 and \( M \) 6.5 synthetics using different values of \( f_{\text{cross}} \). Here I chose a hypocenter midway along the length of the fault and a single slip distribution. Figure 17 illustrates that the median 1 sec SA for the \( M \) 7.5 synthetics increased by almost a factor of 2 for close-in sites when \( f_{\text{cross}} \) was increased from 0.8 to 2.4 Hz. The 1 sec SAs for an \( f_{\text{cross}} \) of 2.4 Hz were substantially higher than the NGA predictions. This implies that assuming coherent summation above 0.8 Hz for \( M \) 7.5 earthquakes would overestimate their 1 sec SAs, at least using the procedure developed here with a rise time proportional to slip on the fault and a Brune pulse shape. For \( M \) 6.5 (Fig. 17), changing \( f_{\text{cross}} \) from 2.4 to 0.8 Hz resulted in relatively little change in the 1 sec SA values, with a small decrease for sites 10 km or closer. Using either value of \( f_{\text{cross}} \) yielded 1 sec SA values consistent with the NGA relations. However, if one applied a 1 Hz crossover frequency to make synthetics for the Northridge earthquake, this would cut off some of the frequency content of the observed forward directivity pulse at Rinaldi, which extends up to at least 2 Hz (Fig. 2).

**Discussion and Conclusions**

The general agreement between the SAs from the synthetics and the NGA relations provides some validation to the methodology used to make the broadband synthetics. The NGA relations support the constant stress-drop model. In particular, it is noteworthy that the 0.2 sec SAs predicted by the NGA relations for \( M \) 5.5–7.5 and distances less than 100 km are matched by the synthetics with a stress drop of 100 bars. While the differences at some periods and distances need to be studied further, the overall conclusion is that the NGA relations can be approximately matched by a physically plausible model of the earthquake source and seismic-wave propagation.

More work needs to be done to address the discrepancy in the SAs at a 5.0 sec period (and to some extent for \( \geq 2.0 \) sec) for \( M \) 7.5 between the synthetics and the NGA relations (Fig. 12). The larger values for the synthetics compared to the NGA predictions may be the result of neglecting small-scale lateral variations in seismic velocity in the simulations that could reduce constructive interference of 5.0 sec period energy, especially over the long fault length for a \( M \) 7.5 earthquake. We also need to evaluate the effects on long-period ground motions of including realistic curva-
ture of long faults, rather than using the straight fault in the simulations.

Another possibility is that the NGA relations under predict the SAs for large western U.S. earthquakes at periods of 3.0 sec and longer. Campbell and Bozorgnia (2008) note the under prediction by their NGA relation of the observed 3.0 sec SAs for five California earthquakes with $M > 6.7$. This may be the result of using a global data set in the NGA relations that may not be appropriate, in some cases, to the western United States.

Originally, I considered that the over prediction of the synthetics relative to NGA could be a result of assuming a continuous, plane-layered velocity model in the shallow crust. Such a continuous plane-layered model may not be realistic in many areas. However, the synthetics for this firm-rock velocity model predict the same distance decay (out to 200 km) of 3.0 and 5.0 sec SA for $M_{6.5}$ as the NGA relations (see Fig. 12). This suggests that the distance decay for a spatially limited source is properly modeled using the synthetics for this plane-layered model. It is likely, though, that a model with a continuous soil layer would not be realistic over large distances.

This study cannot resolve differences between the NGA relations and the previous generation of attenuation relations, which can be up to about 30%–40% for 1.0 sec SA (see, e.g., Boore and Atkinson, 2008; Campbell and Bozorgnia, 2008; Chiou and Youngs, 2008). The synthetics do support the steeper fall off with distance employed by Campbell and Bozorgnia (2008) for $M_{7.5}$ earthquakes compared to their previous study (Campbell and Bozorgnia, 2003).

I have not yet systematically investigated the effects of source depth, focal mechanism type, and site condition on ground motions. Somerville and Pitarka (2006) reported that earthquakes with deeper average rupture and/or buried rupture produce higher ground motions than shallow, surface-rupturing earthquakes. This could certainly be evaluated by imposing a depth-dependent stress drop and slip velocity. Three of the NGA studies find that sites on the hanging wall of a thrust or normal fault exhibit higher ground motions than those on the footwall, given the same closest distance to the fault. This will be addressed in future studies using the procedure described here to construct broadband synthetics.

Data and Resources

The acceleration records for the Northridge earthquake are available from the COSMOS Virtual Data Center at http://db.cosmos-eq.org (last accessed May 2008).

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