Identification of Site Parameters that Improve Predictions of Site Amplification

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The opinions, findings, and conclusions or recommendations expressed in this publication are those of the author(s) and do not necessarily reflect the views of the study sponsor(s) or the Pacific Earthquake Engineering Research Center.
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ABSTRACT

The effects of the local soil conditions on earthquake shaking are often quantified via an amplification factor, which is defined as the ratio of the ground motion at the soil surface to the ground motion at a rock site at the same location. Site amplification models are empirical equations that predict site amplification based on the general characteristics of the site. Most of the current site amplification models predict amplification based on the average shear wave velocity in the top 30 m ($V_{S30}$). However, additional site parameters influence site amplification and should be included in site amplification models.

To identify site parameters beyond $V_{S30}$ that influence site amplification and to develop an empirical site amplification model that includes these parameters, site response analyses are performed for a large suite of shear wave velocity profiles. These analyses identified the parameter $V_{ratio}$, defined as the ratio of the average shear wave velocity from 20 m and 30 m to the average shear wave velocity in the top 10 m, as an important site parameter that influences site amplification. An empirical site amplification model is developed based on the site response results that predicts amplification as a function of $V_{S30}$, $V_{ratio}$, spectral acceleration on rock, and depth to rock.
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1 Study Overview

1.1 INTRODUCTION

When an earthquake occurs, seismic waves are released at the source, travel through the earth, and generate ground shaking at the ground surface. The characteristics of shaking at a site depend on the source characteristics, and these characteristics change as they travel through their path to get to the site. The wave amplitudes generally attenuate with distance and are modified by the local soil conditions at the site (i.e., site effects). The important characteristic of the local soil conditions that influence ground shaking is the shear wave velocity profile.

The effects of local soil conditions are often quantified via an amplification factor (AF), which is defined as the ratio of the ground motion at the soil surface to the ground motion at a rock site at the same location. Although AFs can be defined for any ground motion parameter, they are most commonly assessed for acceleration response spectral values (Sa) at different periods.

Empirical estimates of site amplification are often used to evaluate site effects. This approach uses an empirical equation to predict site amplification based on the input motion intensity and the general characteristics of a site. This approach is incorporated in empirical ground motion prediction equations (GMPEs), which are statistical models that predict an acceleration response spectrum at a site as a function of earthquake magnitude (M), site to source distance (R), local site conditions, and other parameters. These GMPEs are developed predominantly from recorded ground motions obtained from previous earthquakes. To account for local site conditions, GMPEs characterize sites simply by one or two parameters (e.g., the average shear wave velocity over the top 30 m, V₃₀), and the amplification at each period is related to these parameters. The amplification relationship included in a GMPE is often called a site response or site amplification model. While these models are relatively simple and ignore important details about the shear wave velocity profile and nonlinear properties at a site, they are important tools in estimating site amplification for a range of applications. Yet enhancements in these models can be made to improve their ability to predict site amplification.

The main objective of this research is to improve the site amplification models included in GMPEs. Important site details that control site amplification are identified and statistical models are developed that include these parameters. These models or their functional forms then can be implemented in GMPEs. To meet these objectives, first the important site parameters that
influence site amplification are identified. To identify these site parameters, hypothetical shear wave velocity profiles are generated manually and their seismic response computed using the equivalent linear approach. Various site parameters are computed from the hypothetical velocity profiles and the relationship between each of these parameters and the computed site amplification. After identifying appropriate site parameters for use in the empirical site amplification model, appropriate functional forms for the statistical model are developed. The developed functional forms are fit to the computed amplification data.

1.2 SITE AMPLIFICATION MODELS

Site amplification has been included in GMPEs for several decades. The initial site amplification models simply distinguished between rock and soil sites and incorporated site amplification by a scaling parameter or by defining different statistical models for soil and rock sites (e.g., Boore et al. [1993]; Campbell [1993]; Sadigh et al. [1997]). Current models explicitly use $V_{s30}$ and also include the effects of soil nonlinearity, where the stiffness of the soil decreases and the damping increases as larger shear strains are induced in the soil. As a result of soil nonlinearity, amplification is a nonlinear function of the input rock motion. Some models also include the effects of depth to rock, which is important for long-period amplification.

Site amplification models typically predict the natural log of the amplification factor and the effect can be separated into the linear elastic and nonlinear components:

$$\ln(\text{AF}) = \ln(\text{AF})_{\text{lin}} + \ln(\text{AF})_{\text{nl}}$$  \hspace{1cm} (1.1)

Boore et al. [1997] were the first to use $V_{s30}$ in their site amplification model. Their model only included the linear elastic component and is written as:

$$\ln(\text{AF}) = a \cdot \ln\left(\frac{V_{s30}}{V_{\text{ref}}}\right)$$  \hspace{1cm} (1.2)

where $a$ and $V_{\text{ref}}$ are coefficients estimated by regression. This model results in amplification decreasing log-linearly with an increase in $V_{s30}$ (i.e., the coefficient $a$ is negative).

Choi and Stewart [2005] expanded the Boore et al. [1997] site amplification model to include both linear and nonlinear site amplification effects. The general form of the model is given as:

$$\ln(\text{AF}) = a \cdot \ln\left(\frac{V_{s30}}{V_{\text{ref}}}\right) + b \cdot \ln\left(\frac{\text{PGA}_{\text{rock}}}{0.1\,g}\right)$$  \hspace{1cm} (1.3)

where $\text{PGA}_{\text{rock}}$ is the peak ground acceleration on rock in units of $g$ and $0.1\,g$ is the reference $\text{PGA}_{\text{rock}}$ level for nonlinear behavior. The coefficient $b$ is generally negative, indicating that amplification decreases with increasing $\text{PGA}_{\text{rock}}$. The Choi and Stewart [2005] model was
developed by considering recorded ground motions at sites with known $V_{S30}$ and computing the difference between the observed $\ln(Sa)$ and the $\ln(Sa)$ predicted by an empirical GMPE for rock conditions. This difference represents $\ln(AF)$ because the observed motion is $\ln(Sa_{soil})$ and the predicted motion on rock is $\ln(Sa_{rock})$. Using the observed $\ln(AF)$, Choi and Stewart [2005] found that $b$ generally decreases towards zero as $V_{S30}$ increases (Figure 1.1). This decrease in $b$ with increasing $V_{S30}$ indicates that nonlinearity becomes less significant as sites become stiffer.

Chiou and Youngs [2008] used a modified version of Choi and Stewart [2005] site amplification model in their GMPE. While Choi and Stewart [2005] normalized $PGA_{rock}$ by $0.1g$, Chiou and Youngs [2008] use the following form:

$$\ln(AF) = a \cdot \ln \left( \frac{V_{S30}}{V_{ref}} \right) + b \cdot \ln \left( \frac{Sa_{rock} + c}{c} \right)$$  \hspace{1cm} (1.4)

This functional form uses $Sa_{rock}$ instead of $PGA_{rock}$, and separates the linear and nonlinear components through the term $\ln \left( \frac{Sa_{rock} + c}{c} \right)$, which tends towards zero for small $Sa_{rock}$. Again, the coefficient $b$ is $V_{S30}$-dependent.

![Figure 1.1](image_url)  
*Figure 1.1 Derived values of coefficient $b$ as a function of $V_{S30}$ for periods of 0.3 and 1.0 sec [Choi and Stewart 2005].*
Other functional forms for site amplification models are available (e.g., Walling et al. [2008]), but all generally attempt to model the effect of $V_{S30}$ and soil nonlinearity. As an example of how these models predict amplification, Figure 1.2 plots amplification versus $V_{S30}$ for different input intensities as predicted by the Chiou and Youngs [2008] model. Amplification is shown for spectral periods of 0.01, 0.1, 0.3, and 1.0 sec. At small input intensities (i.e., 0.01 g) where the linear term dominates, amplification increases log-linearly as $V_{S30}$ decreases. This effect is larger at longer periods. At larger input intensities, the amplification at each $V_{S30}$ is reduced due to soil nonlinearity (i.e., soil stiffness reduction and increased damping). This effect is largest at small $V_{S30}$ and shorter periods.

Some site amplification models (e.g., Abrahamson and Silva [2008], Campbell and Bozorgnia [2008], and Chiou and Youngs [2008]) include a soil-depth term in addition to $V_{S30}$ when predicting site amplification at long periods. Because the natural period of a soil site is proportional to the soil depth (i.e., deeper sites have longer natural periods), deeper soil sites will experience more amplification at long periods than shallow soil sites. The scaling of site amplification with soil depth is commonly considered independent of input intensity (i.e., not influenced by soil nonlinearity).

The soil-depth term is defined based on the depth to a specific shear wave velocity horizon. Some models (e.g., Abrahamson and Silva [2008] and Chiu and Youngs [2008]) use the depth to $V_S$ equal to or greater than 1.0 km/sec (called $Z_{1.0}$), while others (i.e., Campbell and Bozorgnia [2008]) use the depth to $V_S$ equal to or greater than 2.5 km/sec (called $Z_{2.5}$).
Essentially, $Z_{1.0}$ represents the depth to “engineering” rock while $Z_{2.5}$ represents the depth to hard rock.

As noted previously, soil depth predominantly affects long-period amplification because soil depth affects the natural period of a site and the associated periods of amplification. Figure 1.3 shows the predicted acceleration response spectra for a soil site with $V_{S30} = 270$ m/sec and different values of $Z_{1.0}$ as predicted by the Abrahamson and Silva [2008] GMPE for a $M = 7.0$ earthquake at a distance of 30 km. At short periods (less than 0.4 sec) $Z_{1.0}$ does not influence the response spectrum, while at longer periods the response spectra are significantly affected by $Z_{1.0}$. For example, at a spectral period of 1.0 sec the spectral acceleration for $Z_{1.0} = 0.1$ km is 0.08g while the spectral acceleration for $Z_{1.0} = 1.1$ km is close to 0.25g. This represents an amplification of greater than 3.0. At longer periods the effect of $Z_{1.0}$ is even more pronounced. At a spectral period of 5.0 sec, the response spectra in Figure 1.3 indicate an amplification of greater than 4.0 between $Z_{1.0} = 0.1$ km and 1.1 km.

![Figure 1.3](image.png)

Figure 1.3  Effect of soil depth on ground shaking for a site with $V_{S30} = 270$ m/sec [Abrahamson and Silva 2008].
2 Identification of Site Parameters that Influence Site Amplification

2.1 INTRODUCTION

While the average shear wave velocity in top 30 m ($V_{S30}$) and depth to rock ($Z_{1.0}$ or $Z_{2.5}$) are considered important site parameters that influence site amplification, this research aims to identify additional site parameters to improve site amplification predictions in empirical GMPEs. First we performed wave propagation analysis (i.e., site response analysis) for sites with different velocity profiles and related the computed amplification factors to characteristics of the site profiles. This study focuses on parameters that could be determined from the shear wave velocity profile within the top 30 m of the ground surface because the shear wave velocity information below 30 m is not always available.

In this exploratory part of the research, 99 $V_s$ profiles were generated manually and analyzed by the equivalent-linear site response program Strata [Kottke and Rathje 2008]. The manually generated profiles allow for different velocity structures within the top 30 m while at the same time maintaining a constant $V_{S30}$. Amplification factors were then calculated for all the generated profiles at multiple input intensities and spectral periods. These data were used to identify parameters that strongly influence site amplification.

2.2 SITE PROFILES

Profiles with the same average shear wave velocity ($V_{S30}$) but different shear wave velocity structures within the top 30 m were generated. The profiles also had the same depth to engineering rock ($Z_{1.0} = 150$ m). The same $V_{S30}$ and $Z_{1.0}$ in the profiles facilitates investigation of other site parameters that influence the site response. Profiles of 150 m depth are developed for five different $V_{S30}$ values ($V_{S30} = 225, 280, 350, 450,$ and $550$ m/sec) using the baseline profiles shown in Figure 2.1. For each $V_{S30}$ value, the profiles were manually varied in the top 30 m (keeping $V_{S30}$ constant) with the profiles below 30 m kept at the baseline values. The half space below 150 m for all baseline profiles had a $V_s$ equal to 1100 m/sec. Eighteen to 24 profiles were generated for each $V_{S30}$ value, and 99 total profiles were analyzed.
The top 50 m of all of the generated profiles, along with the baseline profile, for each $V_{S30}$ value are shown in Figure 2.2. In all the generated profiles, the velocity increased with depth with no inversion in the shear wave velocity (i.e., an inversion is when a smaller $V_S$ is found below a larger $V_S$). The minimum shear wave velocity was limited to 100 m/sec in the generated profiles, and for each $V_{S30}$ the profiles all had the same maximum $V_S$ as controlled by the baseline velocity profile at 30 m.

In addition to the shear wave velocity profile, the unit weight and the shear modulus reduction and damping curves of the soil layers were required for site response analysis. The shear modulus reduction and damping curves describe the variation of the shear modulus and damping ratio with shear strain, and represent the nonlinear properties of the soil. For each of the profiles, the same unit weights, as well as shear modulus reduction and damping curves, were used. The Darendeli [2001] model was used to develop the modulus reduction and damping curves as a function of mean effective stress ($\sigma'_m$), Plasticity Index (PI), and over-consolidation ratio (OCR). In this study the PI and OCR are taken to be 10 and 1.0, respectively, for all layers. To model the stress dependence, the 150 m of soil was split into five layers, and the nonlinear property curves computed for the mean effective stress at the middle of each layer ($\sigma'_m = 0.6, 1.4, 2.7, 5.0, \text{ and } 8.0 \text{ atm}$).
2.3 INPUT MOTIONS

The random vibration theory (RVT) approach to equivalent-linear site response analysis was used. The RVT method allows equivalent linear site response to be calculated without the need to specify an input time series. Rather, the RVT method specifies the Fourier amplitude spectrum (FAS) of the input motion and propagates the FAS through the soil column using frequency domain transfer functions. The program *Strata* can generate input FAS from a specified input response spectrum or through seismological theory. For this study, the input motion was specified by seismological theory using the single-corner frequency, $\omega^2$ point source model.
(Brune [1970]). See Boore [2003] for additional discussion on this model and its use in RVT predictions of ground shaking. To specify the input motion, the earthquake magnitude, site-to-source distance, and source depth is provided by user. The other seismological parameters in the model are taken from Campbell [2003] and represent typical values for the western U.S. region.

To consider the nonlinear behavior of the soil, analyses were performed at multiple input intensities. Earthquake magnitude and site-source distance were varied to obtain different input intensities from the seismological method. The corresponding magnitude, distance, and depth combinations used to generate the different input intensities are given in Table 2.1 along with the resulting PGA<sub>rock</sub>. The range of magnitudes is between 6.5 and 7.8, while the range of distances, is 6 to 180 km. The resulting PGA<sub>rock</sub> values are from 0.01g to 1.5g and the resulting rock response spectra are shown in Figure 2.3.

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>Distance (km)</th>
<th>PGA&lt;sub&gt;rock&lt;/sub&gt; (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0</td>
<td>180</td>
<td>0.01g</td>
</tr>
<tr>
<td>7.0</td>
<td>68</td>
<td>0.05g</td>
</tr>
<tr>
<td>7.0</td>
<td>40</td>
<td>0.09g</td>
</tr>
<tr>
<td>6.5</td>
<td>20</td>
<td>0.16g</td>
</tr>
<tr>
<td>7.0</td>
<td>21</td>
<td>0.22g</td>
</tr>
<tr>
<td>7.0</td>
<td>16</td>
<td>0.3g</td>
</tr>
<tr>
<td>7.0</td>
<td>21</td>
<td>0.4g</td>
</tr>
<tr>
<td>7.0</td>
<td>10</td>
<td>0.5g</td>
</tr>
<tr>
<td>7.0</td>
<td>7</td>
<td>0.75g</td>
</tr>
<tr>
<td>7.6</td>
<td>9</td>
<td>0.9g</td>
</tr>
<tr>
<td>7.5</td>
<td>7</td>
<td>1.1g</td>
</tr>
<tr>
<td>7.8</td>
<td>6</td>
<td>1.5g</td>
</tr>
</tbody>
</table>
2.4 SITE CHARACTERISTICS

Identification of those spectral periods that are influenced by the seismic response of a site is critical. One simple parameter that can be used to consider the period range most affected by a site’s response is the site period, $T_S$, which is the period corresponding to first mode and represents the entire $V_S$ profile from the rock to the surface. The site period is estimated as:

$$T_S = \frac{4 \cdot H}{\bar{V}_s} \quad (2.1)$$

where $H$ is the soil thickness, and $\bar{V}_s$ is the average shear wave velocity of the soil. $\bar{V}_s$ is computed from the travel time for a shear wave traveling through the entire soil profile. The $V_S$ profiles developed for a given $V_{S30}$ category, each having same $V_{S30}$ and same $V_S$ profile below 30 m, all have the same $T_S$. The values of $T_S$ for the five $V_{S30}$ values considered are listed in Table 2.2. $T_S$ ranges from approximately 0.75 to 1.5 sec for the five $V_{S30}$ profiles considered.

The detailed velocity structure in the top 30 m affects site amplification as well. To estimate the period range affected by the top 30 m, another site period corresponding to the top 30 m is defined and called $T_{30}$, which is computed as:

$$T_{30} = \frac{4 \cdot (30 \text{ m})}{V_{S30}} \quad (2.2)$$
Table 2.2 presents $T_{30}$ values for the five $V_{S30}$ categories. $T_{30}$ ranges from approximately 0.2 to 0.55 sec for the five $V_{S30}$ profiles considered.

Because the period is inversely proportional to $V_S$, $T_S$ and $T_{30}$ decrease as $V_{S30}$ increases; therefore, stiffer sites have shorter site periods and shorter periods are affected most by site amplification. The $T_S$ for each category of $V_{S30}$ is greater than its corresponding $T_{30}$ because $T_S$ is associated with the entire depth of the profile and $T_{30}$ with only the top 30m.

To further investigate the period range in which the detailed velocity structure in the top 30 m affects the response, one-dimensional (1D) frequency domain transfer functions were computed for different profiles. A transfer function describes the ratio of the FAS of acceleration at any two points in the soil column. Figure 2.4 plots the acceleration transfer functions between the surface and the bedrock outcrop for three selected velocity profiles in the $V_{S30} = 225$ and 450 m/sec categories. In calculating these transfer functions, the soil properties were assumed to be linear elastic. The transfer functions are plotted versus period in Figure 2.4 and the corresponding periods for $T_{30}$ and $T_S$ are indicated. For periods near $T_S$, the transfer functions of different profiles in the same $V_{S30}$ category are very similar because the transfer function in this period range is controlled by the full $V_S$ profile. Starting at periods around $T_{30}$ and at periods shorter than $T_{30}$, the transfer functions vary significantly between the different profiles even though they have the same $V_{S30}$. This variability in the transfer function illustrates the influence of the details of the top 30 m $V_S$ profile in this period range. It can be concluded that the details of the top 30 m of a site are important at periods shorter than $T_{30}$. As a result, the period range influenced by the top 30 m depends on $V_{S30}$ (since $T_{30}$ is $V_{S30}$ dependent). Because the transfer functions in Figure 2.4 are for linear-elastic conditions, an additional consideration will be the influence of input intensity and soil nonlinearity. As the input intensity increases the soil becomes more nonlinear, and both $T_S$ and $T_{30}$ will shift to long periods. As a result, the period range affected by the top 30 m will increase to longer periods as input intensity increases.

<table>
<thead>
<tr>
<th>$V_{S30}$ (m/sec)</th>
<th>$T_S$ (sec)</th>
<th>$T_{30}$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>225</td>
<td>1.54</td>
<td>0.53</td>
</tr>
<tr>
<td>280</td>
<td>1.45</td>
<td>0.43</td>
</tr>
<tr>
<td>350</td>
<td>1.10</td>
<td>0.34</td>
</tr>
<tr>
<td>450</td>
<td>0.87</td>
<td>0.27</td>
</tr>
<tr>
<td>550</td>
<td>0.72</td>
<td>0.22</td>
</tr>
</tbody>
</table>
All the generated profiles in each category of $V_{S30}$ had the same value of $V_{S30}$ but a different $V_S$ structure in the top 30 m. Several parameters were identified from the velocity profiles as candidates that affect the computed site amplification. These parameters are $V_{\text{min}}$, $\text{th}V_{\text{min}}$, depth$V_{\text{min}}$, MAXIR, and $V_{\text{ratio}}$. These parameters are defined as:

- $V_{\text{min}}$ is the minimum shear wave velocity in the $V_S$ profile
- $\text{th}V_{\text{min}}$ is the thickness of the layer with the minimum shear wave velocity
- depth$V_{\text{min}}$ is the depth to the top of the layer with $V_{\text{min}}$
- MAXIR is the maximum impedance ratio within the $V_S$ profile as defined by the ratio of the $V_S$ of two adjacent layers ($V_{s,\text{upper}} / V_{s,\text{lower}}$)
- $V_{\text{ratio}}$ is the ratio of the average shear wave velocity ($\bar{V}_S$) between 20 m and 30 m to the average shear wave velocity in top 10 m. $V_{\text{ratio}}$ is defined as:

$$V_{\text{Ratio}} = \frac{V_{S(20-30)}}{V_{S10}}$$  \hspace{1cm} (2.3)

$$V_{S(20-30)} = \frac{10 \text{ m}}{\sum \left( \frac{h_i}{V_{S,i}} \right)} \text{ over depths 20 to 30 m}$$  \hspace{1cm} (2.4)

$$V_{S10} = \frac{10 \text{ m}}{\sum \left( \frac{h_i}{V_{S,i}} \right)} \text{ over depths 0 to 10 m}$$  \hspace{1cm} (2.5)
The concept of $V_{\text{ratio}}$ is similar to the impedance ratio for MAXIR, except that it represents a more global impedance ratio in the top 30 m. It also has the advantage of using information from a significant portion of the top 30 m of a profile, and it indicates how much the shear wave velocity increases in top 30 m. $V_{\text{ratio}}$ can also indicate if a large scale velocity inversion occurs in the top 30 m when it takes on values less than 1.0.

### 2.5 INFLUENCE OF SITE CHARACTERISTICS ON SITE AMPLIFICATION

#### 2.5.1 Low Input Intensities

If $V_{S30}$ and $Z_{1.0}$ are the dominant factors in explaining site amplification, sites with the same $V_{S30}$ and $Z_{1.0}$ should have the very similar AFs. Figure 2.5 plots AF versus period for all of the generated sites for each of the $V_{S30}$ categories subjected to the lowest input intensity ($\text{PGA}_{\text{rock}} = 0.01 g$). The amplification factors for a given $V_{S30}$ are not constant and in some instances show significant scatter. The amount of scatter (i.e., variability) varies with $V_{S30}$ and period. At smaller $V_{S30}$, the variability in the AFs is more significant. The period range over which the variability in AF is most significant also depends on $V_{S30}$. As $V_{S30}$ increases, this period range decreases. At periods greater than $T_{30}$, less variability is observed. The period at which the maximum variability in the AF occurs is also $V_{S30}$ dependent. For $V_{S30} = 225$ m/sec, the maximum variability is observed at a spectral period of 0.3 sec. Stiffer sites ($V_{S30} = 280$ m/sec and 350 m/sec) display the maximum variability at a spectral period of 0.2 sec, and the stiffest sites ($V_{S30} = 450$ m/sec and 550 m/sec) display the maximum variability at a spectral period of 0.1 sec.

To quantify the variability in AF, the standard deviation of ln(AF) at each period for each category of $V_{S30}$ was calculated. The standard deviation ($\sigma_{\ln \text{AF}}$) is calculated for the ln(AF) because ground motions are commonly assumed to be log-normally distributed and to be consistent with its use in GMPEs. Figure 2.6 shows the $\sigma_{\ln \text{AF}}$ values computed from the data in Figure 2.5. $\sigma_{\ln \text{AF}}$ smaller than about 0.05 is considered small enough such that the variability is minimal. The data in Figure 2.6 show that $\sigma_{\ln \text{AF}}$ is greater than 0.05 at periods less than about $T_{30}$ for each $V_{S30}$, which is consistent with the observations from the transfer functions. Additionally, the values of $\sigma_{\ln \text{AF}}$ are $V_{S30}$ dependent, with sites with smaller $V_{S30}$ producing larger values of $\sigma_{\ln \text{AF}}$.

The identification of the site parameters that explain the variability in AF is initiated by relating the data in Figure 2.5 to various site parameters. The data in Figure 2.5 were used to compute the difference between each ln(AF) and the average ln(AF) for a given period, input intensity, and $V_{S30}$ is considered. This difference represents the residual and is defined as:

$$\text{Residual} = \ln(\text{AF}) - \mu_{\ln \text{AF}}$$

(2.6)

where ln(AF) represents the AF for a single $V_S$ profile with a given $V_{S30}$, and $\mu_{\ln \text{AF}}$ is the average ln(AF) for all sites with the same $V_{S30}$. Considering the periods that have larger $\sigma_{\ln \text{AF}}$ values, only periods of 0.1 sec, 0.2 sec, and 0.3 sec will be considered here.
Figure 2.5 Amplification factor versus period for all generated profiles, $\text{PGA}_{\text{rock}} = 0.01g$. 
Figure 2.6 $\sigma_{\ln \text{AF}}$ versus period for all generated profiles, $\text{PGA}_{\text{rock}} = 0.01g$. 
The residual measures the difference between a specific value of AF and the average value of AF for all sites with the same $V_{S30}$ for a given period and input intensity. If a relationship is observed between the calculated residuals and a site parameter, then that parameter influences site amplification and potentially should be included in predictive models for AF to reduce its variability. As mentioned previously, the minimum velocity in the profile ($V_{min}$), the thickness of the layer with the minimum velocity ($thV_{min}$), the depth to the layer with the minimum velocity ($depthV_{min}$), the maximum impedance ratio (MAXIR), and $V_{ratio}$ are the site characteristics considered.

The first candidate parameter is $V_{min}$. While the absolute value of $V_{min}$ is important, its value relative to $V_{S30}$ provides information about the range of velocities within the top 30 m. To consider the relative effect of $V_{min}$, residuals were plotted versus $V_{S30}/V_{min}$ instead of $V_{min}$. The minimum value of $V_{S30}/V_{min}$ is 1.0, which represents a site with constant velocity equal to $V_{S30}$ in the top 30 m. Larger values of $V_{S30}/V_{min}$ indicate smaller values of $V_{min}$. Figure 2.7 shows the residuals versus $V_{S30}/V_{min}$ for all $V_{S30}$ categories at a spectral period of 0.2 sec and PGA$_{rock}$ = 0.01 g. For $V_{S30} \leq 350$ m/sec the residuals generally increase with increasing $V_{S30}/V_{min}$, while there is little influence of $V_{S30}/V_{min}$ on the residuals for $V_{S30} = 450$ and 550 m/sec. That said, as shown in Figure 2.6, there is little variability in AF for sites with $V_{S30} = 450$ and 550 m/sec at this period ($\sigma_{lnAF} \approx 0.05$).

Other parameters that may influence AF are $thV_{min}$, MAXIR, $depthV_{min}$, and $V_{ratio}$. In all the generated profiles in this study, the minimum velocity occurs at the ground surface, such that all profiles have $depthV_{min}$ equal to zero. Thus, this parameter cannot be considered with the present dataset. The residuals versus $thV_{min}$, MAXIR, and $V_{ratio}$ for a spectral period of 0.2 sec and PGA$_{rock}$ = 0.01 g are plotted in Figures 2.8, 2.9, and 2.10, respectively. The relationship between the residuals and $thV_{min}$ is quite weak (Figure 2.8). The relationship between the residuals and MAXIR (Figure 2.9) is stronger, particularly for $V_{S30} = 280$ m/sec and 350 m/sec, but the relationship is weak for $V_{S30} = 225$ m/sec. The relationship between the residuals and $V_{ratio}$ (Figure 2.10) is very strong for $V_{S30} = 280$ m/sec and 350 m/sec, and moderately strong for $V_{S30} = 225$ m/sec.

Evaluating the relationship between the residuals and the four parameters, $V_{ratio}$ best explains the variability in the AF at $T = 0.2 \text{ sec}$ as the relationship between that residual and $V_{ratio}$ is stronger than the three other parameters. Generally a linear relationship between the residual and $ln(V_{ratio})$ is observed. Consider $\sigma_{lnAF}$ in Figure 2.6: the variability in AF is significant for periods of 0.1 sec, 0.2 sec, and 0.3 sec for most of the $V_{S30}$ values. Residuals plotted for these periods also show a strong linear relationship between the residuals and $ln(V_{ratio})$; however, the intercept and slope of the linear fit is $V_{S30}$ and period dependent [Navidi 2012].
Figure 2.7  Residual versus $V_{S30}/V_{min}$ for spectral period of 0.2 sec and $PGA_{rock} = 0.01g$. 
Figure 2.8 Residual versus $thV_{\min}$ for spectral period of 0.2 sec and $PGA_{\text{rock}} = 0.01g$. 
Figure 2.9  Residual versus MAXIR for spectral period of 0.2 sec and PGA\textsubscript{rock} = 0.01g.
Figure 2.10  Residual versus $V_{\text{ratio}}$ for a spectral period of 0.2 sec and $\text{PGA}_{\text{rock}} = 0.01g$. 
2.5.2 Larger Input Intensities

Soil layers show nonlinear behavior at larger input intensities because larger strains are induced that soften the soil and increase the material damping. Therefore, amplification becomes a nonlinear function of input intensity at higher shaking levels. To investigate the variability in AF at moderate intensities, the results for PGA\textsubscript{rock} = 0.3g are presented.

In Figure 2.11, AF versus period is shown for all the generated sites at PGA\textsubscript{rock} = 0.3g. Comparing the AFs at each spectral period in Figure 2.11 with those in Figure 2.6 for PGA\textsubscript{rock} = 0.0g, it is clear that there is an increase in amplification variability. Figure 2.12 shows \(\sigma_{\ln(\text{AF})}\) versus period for each V\textsubscript{S30} category for the AF results shown in Figure 2.11. The largest values of \(\sigma_{\ln(\text{AF})}\) are observed at V\textsubscript{S30} = 225 m/sec. All the periods in this category of V\textsubscript{S30} have significant variation in AF (i.e., \(\sigma_{\ln(\text{AF})} > 0.05\)). \(\sigma_{\ln(\text{AF})}\) is as large as 0.4 at T = 0.66 sec for this value of V\textsubscript{S30}. For all sites with V\textsubscript{S30} \leq 350 m/sec, \(\sigma_{\ln(\text{AF})}\) is significant at almost all periods considered (\(\leq 2.0\) sec). The maximum value of \(\sigma_{\ln(\text{AF})}\) occurs at longer periods as V\textsubscript{S30} decreases. Comparing each V\textsubscript{S30} category subjected to PGA\textsubscript{rock} = 0.3g to their corresponding profiles subjected to PGA\textsubscript{rock} = 0.01g, the period range with \(\sigma_{\ln(\text{AF})}\) greater than 0.05 increases. The maximum \(\sigma_{\ln(\text{AF})}\) occurs generally at longer periods for PGA\textsubscript{rock} = 0.3g than for PGA\textsubscript{rock} = 0.01g. These observations indicate that the period range that is affected by the detailed velocity structure in the top 30 m increases as the shaking level increases.

Considering the periods of maximum \(\sigma_{\ln(\text{AF})}\) in Figure 2.12, the residuals are investigated at periods of 0.2 sec (period of maximum \(\sigma_{\ln(\text{AF})}\) for V\textsubscript{S30} = 280 and 450 m/sec) and a period of 0.66 sec (period of maximum \(\sigma_{\ln(\text{AF})}\) for V\textsubscript{S30} = 225 m/sec).

Figure 2.13 plots the residuals for the AF results for PGA\textsubscript{rock} = 0.3g at a spectral period of 0.2 sec versus V\textsubscript{ratio} in. Generally, a linear trend between the residuals and \(\ln(\text{V\textsubscript{ratio}})\) is observed, similar to the results for PGA\textsubscript{rock} = 0.01g; however, the relationship appears to break down at small V\textsubscript{S30} (i.e., 225 and 280 m/sec) and larger V\textsubscript{ratio} (i.e., 2 to 3). Figure 2.14 plots the velocity profiles over the top 30 m for four sites with V\textsubscript{S30} = 225 m/sec and with V\textsubscript{ratio} around 2.5 but very high residuals (+0.4) and very low residuals (−0.4). The profiles with very low residuals have a thick soft layer (i.e., layer with V\textsubscript{s} \leq 160 m/sec and thickness > 10 m) with a large impedance ratio (i.e., MAXIR) immediately below. The MAXIR is well above 2.0 for these profiles, while the profiles with large residuals have a MAXIR of between 1.5 and 1.7. The induced shear strains for the four profiles are also shown in Figure 2.14. The large MAXIR leads to significant shear strains—in excess of 2%—in the layers above the depth of MAXIR. The rapid increase in strain across the impedance contrast induces a rapid change in stiffness and damping that reduces the amplification at high frequencies. While the sites with large residuals also experience large strains (~ 1 to 1.5%), the increase in strain with depth is not as rapid, allowing for more wave motion to travel through the soil. The data in Figures 2.13 and 2.14 indicate that sites with very large MAXIR may experience very large strains at moderate input motion intensities, leading to smaller amplification.

The maximum value of \(\sigma_{\ln(\text{AF})}\) for V\textsubscript{S30} = 225 m/sec occurs at T = 0.66 sec, while the value of \(\sigma_{\ln(\text{AF})}\) is also significant at a spectral period of 0.66 sec for V\textsubscript{S30} = 280 m/sec (Figure 2.12). Figure 2.15 shows the residuals versus V\textsubscript{ratio} for all the generated sites subjected to PGA\textsubscript{rock} =
0.3g at spectral period of 0.66 sec. For $V_{S30} \geq 350$ m/sec, the residuals are almost zero because $\sigma_{\ln AF}$ is less than 0.05 (Figure 2.12). A linear trend is generally observed between the residuals and $\ln(V_{ratio})$ for the softer profiles ($V_{S30} = 225$ and 280 m/sec); however, the data is scattered for $V_{S30} = 225$ m/sec and $V_{ratio}$ greater than about 2.3. These are the same sites discussed in Figure 2.14, and the scatter is due to the large MAXIR and thick soft layers in the profiles. Generally at all periods where the variability in amplification is significant (i.e., $\sigma_{\ln AF} > 0.05$), the calculated residuals for these AF have a linear trend with $\ln(V_{ratio})$. That said, there are some profiles that break down this trend. These profiles tend to have a thick, very soft layer near the surface that may be unrealistic.
Figure 2.11 Amplification factor versus period for all generated profiles, $\text{PGA}_{\text{rock}} = 0.3g$. 
Figure 2.12  $\sigma_{lnAF}$ versus period for all generated profiles, PGA$_{rock} = 0.3g$. 

$V_{S30} = 225 \text{ m/s}$

PGA$_{rock} = 0.3g$

$\ln AF < 0.05$

$T_{30}$

$V_{S30} = 280 \text{ m/s}$

PGA$_{rock} = 0.3g$

$\ln AF < 0.05$

$T_{30}$

$V_{S30} = 350 \text{ m/s}$

PGA$_{rock} = 0.3g$

$\ln AF < 0.05$

$T_{30}$

$V_{S30} = 450 \text{ m/s}$

PGA$_{rock} = 0.3g$

$\ln AF < 0.05$

$T_{30}$

$V_{S30} = 550 \text{ m/s}$

PGA$_{rock} = 0.3g$

$\ln AF < 0.05$

$T_{30}$
Figure 2.13 Residual versus $V_{\text{ratio}}$ for spectral period of 0.2 s and $PGA_{\text{rock}} = 0.3g$. 
Figure 2.14  $V_s$ profiles and induced shear strains for sites with $V_{\text{ratio}} \sim 2.5$. 
2.6 SUMMARY

To investigate the site parameters beyond $V_{S30}$ that influence site amplification, 99 profiles were manually generated using five baseline profiles. The generated profiles from each baseline profile had the same average shear wave velocity in top 30 m, the same velocity structure at
depths greater than 30 m, and the same depth to bedrock. Equivalent-linear site response analyses were performed and the site AFs computed for the profiles were studied.

At multiple input intensities, sites with the same average shear wave velocity and depth to rock but a different structure in the \( V_S \) profile in the top 30 m display different AFs at some periods. These periods are correlated to \( T_{30} \). The variability in the AFs at these periods indicates that the detailed velocity structure in the top 30 m of a \( V_S \) profile influences the computed AF. As input intensity increased, the period range affected by the top 30 m increased.

The parameters \( V_{\text{min}} \), \( \text{th}V_{\text{min}} \), \( \text{depth}V_{\text{min}} \), MAXIR, and \( V_{\text{ratio}} \) were considered to explain the variability in AFs. The parameter \( V_{\text{ratio}} \) was identified as the parameter that most strongly influenced the computed amplification. A linear relationship was observed between residuals and \( \ln(V_{\text{ratio}}) \). Residuals versus \( \ln(V_{\text{ratio}}) \) plots for different \( V_{S30} \) and at different periods show that the effect of \( V_{\text{ratio}} \) on amplification is \( V_{S30} \) and period dependent.
3 Approach to Development of Site Amplification Model

3.1 INTRODUCTION

To develop a site amplification model that includes the effects $V_{ratio}$, the seismic response of sites with a wide range of velocity profiles is assessed and the computed AFs used in the statistical analysis. Hypothetical velocity profiles are developed using Monte Carlo simulations in which soil layer thickness, shear wave velocity, and depth to the bedrock are statistically varied. This chapter describes the generation of the velocity profiles and the statistical analyses to be performed, while the next chapters describe the development of the site amplification model.

3.2 STATISTICAL GENERATION OF VELOCITY PROFILES

A soil profile consists of discrete layers that describe the variation of soil properties with depth. Seismic site response analysis characterizes each soil layer by a thickness, mass density, shear wave velocity, and nonlinear properties (i.e., $G/G_{\max}$ versus shear strain, and $D$ versus shear strain).

The site response program Strata [Kottke and Rathje 2008] uses Monte Carlo simulations to develop different potential realizations of site properties. The goal of a Monte Carlo simulation is to estimate the statistical properties of the response of a complex system. To achieve this goal, each of the properties of the system is selected from defined statistical distributions, and the response of the system is computed. The calculated response from each realization is then used to estimate the statistical properties of the system’s response. Monte Carlo simulations require that each of the components in the system has a complete statistical description.

The Monte Carlo randomization feature in Strata can randomize layer thickness, shear wave velocity of a layer, nonlinear soil properties of a layer, and depth to bedrock. In this study layer thicknesses, shear wave velocity, and depth to bedrock were randomized; but nonlinear properties were not. Strata uses the statistical models developed by Toro [1995] to randomize the layer thicknesses and associated shear wave velocities. In this approach, layer thicknesses are first generated and then shear wave velocities are assigned to each layer.
Layering is modeled as a Poisson process, which is a stochastic process with the event occurring at a given rate ($\lambda$). For a homogeneous Poisson process this rate is constant, while for a non-homogeneous Poisson process the rate of occurrence varies. For the layering problem, the event is a layer interface, and its rate is defined in terms of the number of layer interfaces per meter. The rate of interface occurrence is depth dependent (i.e., thinner layers tend to occur near the surface, while thicker layers tend to occur at depth), thus a non-homogeneous Poisson process is used. Toro [1995] developed a depth-dependent layering rate model using the layer thicknesses measured at 557 sites, mostly from California. The resulting model of depth-dependent layering rate is shown in Figure 3.1. Note that the rate varies from 0.22 1/m at the ground surface (i.e., average layer thickness = $1/\lambda = 4.5$ m) to 0.05 1/m at a depth 50 m (i.e., average layer thickness = 20 m) to 0.02 1/m at a depth of 200 m (i.e., average layer thickness = 50 m).

After developing the layering profile (i.e., layer thicknesses) using the non-homogeneous Poisson process, a shear wave velocity is assigned to each layer. The Toro [1995] model describes the shear wave velocity at mid-depth of the layer by the log-normal distribution, and the use of this distribution is based on statistical investigation of shear wave velocity data from the same 557 sites used in developing the layering model. The log-normal distribution is described by the median shear wave velocity [i.e., the average of $\ln(V_s)$] at mid-depth of the layer and the standard deviation of the natural logarithm of the shear wave velocity ($\sigma_{\ln V_s}$). The median shear wave velocity for the layer is taken from a user-specified baseline velocity profile for the site, at the depth of the layer. The $\sigma_{\ln V_s}$ is assigned by the user to model a specific amount of variability. Given the baseline shear wave velocity for layer $i$ [$V_{s,0}(i)$, assumed to represent the mean in logarithmic space], the standard deviation of the natural logarithm of $V_s$ ($\sigma_{\ln V_s}$), and a random standard normal variable for layer $i$ ($Z_i$), the shear wave velocity of layer $i$ [$V_s(i)$] can be computed as [Toro 1995]:

![Figure 3.1 Variation of layer interface occurrence rate with depth [Toro 1995].](image)
\[ V_S(i) = \exp\left(\ln[V_{S,0}(i)] + Z_i \cdot \sigma_{\ln V_S}\right) \]  

(3.1)

The key issue is then the selection of the values of \( Z_i \) for each layer. Some researchers have assumed that \( Z_i \) values are perfectly correlated between layers [McGuire 1989; Toro et al. 1992], while others have assumed zero correlation [Costantino 1991]. Neither of these assumptions is consistent with velocity data, and they represent extreme conditions (i.e., perfect correlation versus statistical independence). Toro [1995] developed a model for the interlayer correlation of \( Z_i \) based on analysis of the same 557 shear wave velocity profiles previously discussed. In this model the standard normal variable for the surface layer (\( Z_1, i = 1 \)) is independent of all other layers and defined as:

\[ Z_1 = \varepsilon_1 \]  

(3.2)

where \( \varepsilon_1 \) is a random normal variable with zero mean and unit standard deviation. \( Z_i \) is correlated with the layer above it using [Toro 1995]:

\[ Z_i = \rho_{IL} \cdot Z_{i-1} + \varepsilon_i \cdot \sqrt{1 - \rho_{IL}^2} \]  

(3.3)

where \( Z_{i-1} \) is the standard normal variable of the previous layer, \( \varepsilon_i \) is a new normal random variable with zero mean and unit standard deviation, and \( \rho_{IL} \) is the interlayer correlation coefficient. Toro [1995] modeled the interlayer correlation as depth (\( d \)) and layer thickness (\( t \)) dependent:

\[ \rho_{IL}(d,t) = [1 - \rho_d(d)] \cdot \rho_t(t) + \rho_d(d) \]  

(3.4)

where \( \rho_d(d) \) is the depth-dependent component of the correlation coefficient and \( \rho_t(t) \) is the thickness-dependent component of the correlation coefficient. These correlation coefficients are defined as [Toro 1995]:

\[ \rho_d(d) = \begin{cases} 
\rho_{200} \cdot \left[ \frac{d + d_o}{200 + d_o} \right]^b, & d \leq 200 \text{ m} \\
\rho_{200}, & d > 200 \text{ m} 
\end{cases} \]  

(3.5)

\[ \rho_t(t) = \rho_0 \cdot \exp\left(\frac{-t}{\Delta}\right) \]  

(3.6)

where \( \rho_{200}, d_o, b, \rho_0, \) and \( \Delta \) are model parameters.

Toro [1995] developed median shear wave velocity profiles for different site classes (i.e., ranges in \( V_{S30} \) that are incorporated in the building code) for use in developing randomized velocity profiles for generic site conditions. Toro [1995] also developed estimates of \( \sigma_{\ln V_S} \) and interlayer correlation coefficient model parameters for site classes of \( V_{S30} > 760 \text{ m/sec}, 360–760 \text{ m/sec}, 180–360 \text{ m/sec}, \) and \(< 180 \text{ m/sec}. \) The depth to bedrock can be modeled using either a
uniform, normal, or log-normal distribution. The statistical properties of the distribution are entered by the user.

### 3.3 GENERATED VELOCITY PROFILES

The Monte Carlo simulation feature in the program *Strata* is used to generate generic site profiles for use in site response analysis. As discussed in the previous section, Monte Carlo simulations require a median shear wave velocity profile and $\sigma_{\ln V_s}$. The median shear wave velocity profiles are called baseline profiles in this study. Four baseline profiles with $V_{S30}$ equal to 200, 250, 400, and 550 m/sec are used to generate a total of four hundred soil profiles. Figure 3.2 shows the developed baseline shear wave velocity profiles. These baseline velocity profiles were developed based on the velocity profiles presented by Toro [1995] for different site classes, which were then modified in this study to achieve the desired $V_{S30}$ for each baseline profile. The minimum velocity at the surface varies between about 175 and 400 m/sec in the baseline profiles. The baseline profiles with $V_{S30}$ of 200 and 250 m/sec reach their maximum shear wave velocities of 750 m/sec and 890 m/sec, respectively, at a depth of 400 m. The stiffer baseline profiles reach $V_S$ equal to 1000 m/sec at shallower depths (150 m for $V_{S30}$ of 550 m/sec and 300 m for $V_{S30}$ of 400 m/sec). The site class dependent values of $\sigma_{\ln V_s}$ developed by Toro [1995] were used in generating the velocity profiles (see Table 3.1).

![Figure 3.2 Baseline shear wave velocity profiles.](image-url)
Generic layering was developed for the site profiles using the non-homogenous layering model of Toro [1995]. Depth to bedrock was varied using a uniform distribution. The minimum and maximum depths used for the uniform distribution were specified differently for each baseline velocity profile because each profile encounters rock-like velocities (~750 to 1000 m/sec) at different depths. The minimum and maximum depths used are listed in Table 3.1 for each baseline profile. The shear wave velocity of the half-space below the velocity profile was specified as 1000 m/sec. Velocities were assigned to each layer using Toro’s interlayer correlation model and depth and thickness-dependent model for $\rho_{IL}$. The predicted $\rho_{IL}$ from Toro [1995] is plotted versus depth for thicknesses of 5 m, 20 m, and 50 m in Figure 3.3 for site class $V_{S30} = 180–360$ m/sec. This is the site class that is associated with the $V_{S30}$ values of two of the baseline profiles. As shown in Figure 3.3, the interlayer correlation increases with depth for a given thickness; at the depth of 200 m approaches 1.0 for this site class. Note that thicker layers generally have a smaller interlayer correlation coefficient. In generating the velocity profiles, the shear wave velocity of layers is not allowed to exceed 1000 m/sec nor go below 100 m/sec.

While the nonlinear properties are not varied in the Monte Carlo simulation, modulus reduction and damping curves are assigned to each layer. The Darendeli [2001] model was used to develop the modulus reduction and damping curves as a function of mean effective stress ($\sigma_m'$), PI, and OCR. In this study PI and OCR are taken to be 10 and 1.0, respectively, for all layers. To model the stress dependence, nonlinear property curves were generated for $\sigma_0'$ equal to 0.6 atm, 1.4, atm, 2.7 atm, 4.9 atm, 8.0 atm, 15.7 atm, and 33.5 atm. The appropriate curves were assigned to each layer of the baseline profile based on the depth and a computed mean effective stress at the middle of each layer.

Examples of generated profiles from each baseline profile are shown in Figures 3.4 and 3.5. Note that the $V_{S30}$ for each generated profile is different than the $V_{S30}$ for the baseline profile. Each profile generated from a single baseline profile has different layering and a different shear wave velocity profile. While the baseline velocity profiles (Figure 3.2) vary smoothly with depth, the generated profiles vary more irregularly (Figures 3.4 and 3.5). In some profiles, an inversion in the shear wave velocity occurs (i.e., the velocity decreases with depth). The irregular pattern is more representative of a real site, while the smooth baseline profiles represent the average over many different sites. The average velocity profile for the generated profiles for a given baseline profile varies smoothly and matches the baseline profile well.

For each generated velocity profile, site characteristics such as the minimum shear wave velocity, $V_{S30}$, and the depth to bedrock are different. Various site parameters, including the average shear wave velocity in top 30 m ($V_{S30}$), the depth to engineering rock ($Z_{1.0}$), and $V_{ratio}$ [i.e., $V_{ratio} = V_{S(20-30)} / V_{S10}$, where $V_{S(20-30)}$ is the average shear wave velocity between depths 20 m and 30 m and $V_{S10}$ is the average shear wave velocity in top 10 m] are calculated for each generated profile. Histograms showing the distribution of each of these site parameters are given in Figure 3.6. $V_{S30}$ in the generated profiles varies between 118 m/sec and 818 m/sec. The $V_{S30}$ values of the generated profiles are evenly distributed between 150 m/sec and 750 m/sec, with fewer values less than 150 m/sec or greater than 750 m/sec. This range indicates that the generated sites include a representative range of soft to stiff soil profiles. The $Z_{1.0}$ values of the generated profiles range from 16 m to 640 m. Because softer sites with small $V_{S30}$ tend to be
found in deeper alluvial valleys, there is a relationship between $V_{S30}$ and $Z_{1.0}$. As shown in Figure 3.7 for the generated profiles in this study, the softest sites are associated with the largest values of $Z_{1.0}$. The histograms in Figure 3.6 show that $V_{ratio}$ varies from 0.56 to 2.76 in the generated profiles. Profiles with $V_{ratio}$ less than 1.0 have $V_{S10}$ greater than $V_{S20-30}$, indicating that the shear wave velocity generally does not increase with depth in the top 30 m. In these cases an inversion in the velocity occurs. In 11% of the generated profiles $V_{ratio}$ is less than one. Large values of $V_{ratio}$ indicate a significant increase in shear wave velocity within the top 30 m.

Table 3.1 $\sigma_{lnVs}$ and minimum and maximum rock depths used in generating velocity profiles.

<table>
<thead>
<tr>
<th>$V_{S30}$ (m/sec)</th>
<th>$\sigma_{lnVs}$</th>
<th>Minimum Rock Depth (m)</th>
<th>Maximum Rock Depth (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.31</td>
<td>150</td>
<td>650</td>
</tr>
<tr>
<td>250</td>
<td>0.31</td>
<td>100</td>
<td>600</td>
</tr>
<tr>
<td>400</td>
<td>0.27</td>
<td>30</td>
<td>550</td>
</tr>
<tr>
<td>550</td>
<td>0.27</td>
<td>15</td>
<td>300</td>
</tr>
</tbody>
</table>

Figure 3.3 The predicted $\rho_{IL}$ from Toro [1995] as a function of depth for layer thicknesses of 5 m, 20 m, and 50 m and for site class $V_{S30} = 180–360$ m/sec.
Figure 3.4 Examples of generated velocity profiles for $V_{S30}$ = 200 and 250 m/sec baseline profiles.
Figure 3.5  Examples of generated velocity profiles for $V_{S30} = 400$ and 550 m/sec baseline profiles.
Figure 3.6 Distribution of $V_{S30}$, $Z_{1.0}$, and $V_{ratio}$ of the generated soil profiles.
3.4 MODEL DEVELOPMENT

An empirical site amplification model typically includes two components: a linear elastic component and a nonlinear component. The linear elastic component represents amplification under linear elastic soil conditions (i.e., low intensity shaking), while the nonlinear component includes the effects of soil nonlinearity at high intensity shaking. These AFs are generally multiplicative (additive in logarithmic space), which can be written as:

$$\ln(AF) = \ln(AF)_{\text{lin}} + \ln(AF)_{\text{nl}}$$

(3.7)

The linear elastic component is derived from the computed amplification for low intensity input motions. In this study AFs from input motions with a rock input PGA of 0.01g were used to develop the linear elastic AF model. Computed AFs from larger intensity input motions were used to develop the nonlinear component of the AF model. As discussed earlier, equivalent-linear analysis using the RVT approach was used to compute the seismic response of the generated profiles under a wide range of input intensities. In the RVT approach, the input motion is described by a response spectrum, this response spectrum is converted to an FAS, the FAS is propagated to the ground surface using the site frequency domain transfer function, and the surface FAS is converted to an acceleration response spectrum. The ratio of the surface response spectrum to the input response spectrum at each period defines the AF for each period. The input motions are defined using seismological source theory and are essentially the same as those described in Chapter 3.

After computing AFs for a range of sites subjected to a range of input intensities, the AF values are used to develop the empirical amplification model. A functional form for the model is developed and the model coefficients are determined through maximum likelihood regression.
4 Models for Linear Elastic Site Amplification

4.1 INTRODUCTION

This chapter presents the development of the linear elastic component of the site amplification model. The functional form is developed separately for shorter periods ($T < 1.0$ sec) and longer periods ($T \geq 1.0$ sec). At short periods the amplification model includes the effects of $V_{S30}$ and $V_{ratio}$, while at long periods the amplification model includes the effects of $V_{S30}$ and $Z_{1.0}$.

4.2 $V_{S30}$ COMPONENT OF LINEAR ELASTIC SITE AMPLIFICATION MODEL

Figures 4.1 and 4.2 show the computed AFs versus $V_{S30}$ for spectral periods shorter and longer than 1.0 sec, respectively. The data in Figures 4.1 and 4.2 generally show that a decrease in $V_{S30}$ corresponds with an increase in amplification, results that are consistent with previous studies. At shorter periods (i.e., $T \leq 0.5$), amplification increases over the full $V_{S30}$ range. At longer periods ($T = 5.0$ and $10.0$ sec in Figure 5.2), the amplification stays close to 1.0 for a range of larger $V_{S30}$ values before beginning to increase at smaller $V_{S30}$. The $V_{S30}$ below which amplification starts to increase is called $V_{ref}$, which generally decreases as the period increases.

Linear elastic AF models developed previously and used in most GMPEs incorporate a linear dependence between $\ln(AF)$ and $\ln(V_{S30}/V_{ref})$ using:

$$\ln(AF)_{\text{lin}} = \begin{cases} a_1 \cdot \ln \left( \frac{V_{S30}}{V_{ref}} \right) & \text{if } V_{S30} < V_{ref} \\ 0 & \text{if } V_{S30} \geq V_{ref} \end{cases} \quad (4.1)$$

A maximum likelihood regression is used to fit Equation (4.1) to the AF data at each spectral period. $V_{ref}$ is fixed, based on visual identification from the data (Table 5.1) because regressed values of $V_{ref}$ using the maximum likelihood method were not consistent with the data. Figures 4.3 and 4.4 show the AF data again, along with a linear fit from Equation (4.1). The model parameters ($a_1$ and $V_{ref}$) and the resulting standard deviation of the regression ($\sigma_{\ln(AF)}$) are shown in Table 4.1. As shown in Table 4.1, $V_{ref}$ is equal to 1000 m/sec at short periods and then decreases as spectral periods increase beyond 0.5 sec. Slope $a_1$ is almost the same for shorter periods ($\sim -0.45$ for $T \leq 0.3$ sec), increases in the middle periods ($\sim -0.5$ to -0.7 for $T = 0.5$ to 2.0 sec), and then becomes smaller at long periods greater than 2.0 sec. $\sigma_{\ln(AF)}$ ranges from 0.11 at
PGA, then increases to 0.2 to 0.25 for \( T = 0.2 \) to 1.0 sec, and then decreases at the longest periods. Comparing the model predictions to the data in Figures 4.3 and 4.4, it appears that a linear fit does not match the data appropriately, particularly at shorter periods and smaller \( V_{S30} \). In these cases, a second-order polynomial may fit the AF data better.

A second-order polynomial is considered for the relationship between \( \ln(AF) \) and \( \ln\left(\frac{V_{S30}}{V_{ref}}\right) \) and is described as:

\[
\ln(AF)_{\text{lin}} = \begin{cases} 
  a_1 \cdot \ln \left(\frac{V_{S30}}{V_{ref}}\right) + a_2 \cdot \left[ \ln \left(\frac{V_{S30}}{V_{ref}}\right) \right]^2 & \text{if } V_{S30} < V_{ref} \\
  0 & \text{if } V_{S30} \geq V_{ref}
\end{cases} 
\]  

(4.2)

A maximum likelihood regression is used to fit Equation (4.2) to the data at each period. The same \( V_{ref} \) values previously identified are used. The values of \( V_{ref} \), \( a_1 \), \( a_2 \), and \( \sigma_{\ln AF} \) for each period are listed in Table 4.2. The resulting second order polynomials are shown in Figures 4.5 and 4.6, along with the linear fit and the AF data. As compared with the linear fit, the second order polynomial better fits the data, particularly at smaller \( V_{S30} \). At periods longer than 1.0 sec, the parameter \( a_2 \) is close to zero, indicating that a linear fit best represents the data. Comparing the \( \sigma_{\ln AF} \) of polynomial and linear fits, the polynomial fit decreases \( \sigma_{\ln AF} \) by 15 to 25% at periods shorter than about 0.5 sec but does not significantly decrease it at longer periods. Therefore, the linear fit appears to be appropriate for periods greater than 0.5 sec.

Table 4.1

<table>
<thead>
<tr>
<th>( T ) (sec)</th>
<th>( a_1 )</th>
<th>( V_{ref} ) (m/sec)</th>
<th>( \sigma_{\ln AF} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGA</td>
<td>-0.49</td>
<td>1000</td>
<td>0.11</td>
</tr>
<tr>
<td>0.05</td>
<td>-0.47</td>
<td>1000</td>
<td>0.12</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.44</td>
<td>1000</td>
<td>0.15</td>
</tr>
<tr>
<td>0.2</td>
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<td>1000</td>
<td>0.24</td>
</tr>
<tr>
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<td>0.24</td>
</tr>
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<td>-0.52</td>
<td>1000</td>
<td>0.20</td>
</tr>
<tr>
<td>1.0</td>
<td>-0.62</td>
<td>850</td>
<td>0.20</td>
</tr>
<tr>
<td>2.0</td>
<td>-0.72</td>
<td>600</td>
<td>0.19</td>
</tr>
<tr>
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<td>-0.46</td>
<td>500</td>
<td>0.18</td>
</tr>
<tr>
<td>10.0</td>
<td>-0.21</td>
<td>500</td>
<td>0.06</td>
</tr>
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</table>
Figure 4.1  Amplification factor versus $V_{s30}$ for all generated profiles at short periods ($T \leq 0.5$ sec).
Figure 4.2 Amplification factor versus $V_{S30}$ for all generated profiles at long periods ($T \geq 1.0$ sec).
Figure 4.3  Linear fit to amplification factor versus $V_{S30}$ data for all generated profiles at short periods ($T \leq 0.5$ sec).
Figure 4.4  Linear fit to amplification factor versus $V_{S30}$ data for all generated profiles at long periods ($T \geq 1.0$ sec).
Figure 4.5  Linear and nonlinear fits to amplification factor versus $V_{S30}$ data for all generated profiles at short periods ($T \leq 0.5$ sec).
Figure 4.6 Linear and nonlinear fits to amplification factor versus $V_{S30}$ data for all generated profiles at long periods ($T \geq 1.0$ sec).
<table>
<thead>
<tr>
<th>T (sec)</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$V_{ref}$ (m/sec)</th>
<th>$\sigma_{\ln AF}$ [Eq. (4.2)]</th>
<th>$\sigma_{\ln AF}$ [Eq. (4.1)]</th>
<th>% Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGA</td>
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<td>-0.15</td>
<td>1000</td>
<td>0.09</td>
<td>0.11</td>
<td>18</td>
</tr>
<tr>
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<td>-0.17</td>
<td>1000</td>
<td>0.10</td>
<td>0.12</td>
<td>17</td>
</tr>
<tr>
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<td>-0.24</td>
<td>1000</td>
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<td>0.15</td>
<td>27</td>
</tr>
<tr>
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<td>-0.94</td>
<td>-0.36</td>
<td>1000</td>
<td>0.19</td>
<td>0.24</td>
<td>21</td>
</tr>
<tr>
<td>0.3</td>
<td>-0.93</td>
<td>-0.33</td>
<td>1000</td>
<td>0.20</td>
<td>0.24</td>
<td>17</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.70</td>
<td>-0.12</td>
<td>1000</td>
<td>0.19</td>
<td>0.20</td>
<td>5</td>
</tr>
<tr>
<td>1.0</td>
<td>-0.49</td>
<td>0.09</td>
<td>850</td>
<td>0.19</td>
<td>0.20</td>
<td>5</td>
</tr>
<tr>
<td>2.0</td>
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<td>0.09</td>
<td>600</td>
<td>0.19</td>
<td>0.19</td>
<td>1</td>
</tr>
<tr>
<td>5.0</td>
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<td>-11</td>
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<tr>
<td>10.0</td>
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<td>0.09</td>
<td>500</td>
<td>0.07</td>
<td>0.06</td>
<td>-17</td>
</tr>
</tbody>
</table>

### 4.3 $V_{\text{ratio}}$ COMPONENT OF LINEAR ELASTIC SITE AMPLIFICATION MODEL

To define the functional form describing the variation in AF with $V_{\text{ratio}}$, the residual of each data point relative to the predicted value from the model is computed and plotted versus $V_{\text{ratio}}$. The residual is calculated as:

$$\text{Residual} = [\ln(\text{AF})]_{\text{Data}} - [\ln(\text{AF})]_{\text{Predicted}}$$

(4.3)

where $[\ln(\text{AF})]_{\text{Predicted}}$ is calculated using Equation (4.2) for periods shorter than 1.0 sec (short periods) and Equation (4.1) for $T \geq 1.0$ sec (long periods). $[\ln(\text{AF})]_{\text{Data}}$ is the computed AF for the soil profile. The calculated residuals are plotted versus $V_{\text{ratio}}$ in Figures 4.7 and 4.8 for short and long periods, respectively. There is a strong positive trend between the residuals and $V_{\text{ratio}}$ at all six short periods. In these cases, sites with larger $V_{\text{ratio}}$ experience larger amplification. At long periods, the trend between the residuals and $V_{\text{ratio}}$ is not significant, indicating that the AF at long periods is not significantly influenced by $V_{\text{ratio}}$. 
Figure 4.7 Residuals versus $V_{ratio}$ at short periods ($T \leq 0.5$ sec).
As previously discussed, AF is $V_{S30}$ dependent, and thus it is likely that the influence of $V_{ratio}$ on site amplification is also $V_{S30}$ dependent. To identify the functional form that describes the dependence of AF on $V_{S30}$ and $V_{ratio}$, residuals were plotted versus $V_{ratio}$ for different bins of $V_{S30}$. Soil profiles were separated into 8 $V_{S30}$ bins as follows: $V_{S30} < 150$ m/sec, $150$ m/sec $< V_{S30} < 200$ m/sec, $200$ m/sec $< V_{S30} < 250$ m/sec, $250$ m/sec $< V_{S30} < 300$ m/sec, $300$ m/sec $< V_{S30} < 350$ m/sec, $350$ m/sec $< V_{S30} < 450$ m/sec, $450$ m/sec $< V_{S30} < 550$ m/sec, and $550$ m/sec $< V_{S30} < 750$ m/sec. To normalize the effect of $V_{ratio}$ on AF in a manner similar to $V_{ref}$, $V_{ratio}$ was normalized by a $V_{ratio}$ value of 1.4, which is the average $V_{ratio}$ of all four hundred generated profiles. Residuals were plotted versus $\ln \left( \frac{V_{ratio}}{1.4} \right)$ in Figures 4.9, 4.10, and 4.11 for periods PGA, 0.2 sec, and 0.5 sec, respectively. Because the other short periods show similar trend as these periods, only these three periods are shown.

For PGA (Figure 4.9), there is a linear relationship between the residual and $\ln \left( \frac{V_{ratio}}{1.4} \right)$ for all $V_{S30}$ bins. This relationship that can be described as:
\[ \text{Residual} = a_3 \cdot \ln \left( \frac{V_{\text{ratio}}}{1.4} \right) \]  

(4.4)

Equation (4.4) is fit to the residuals within each \( V_{S30} \) bin, and the slope of each fit (i.e., \( a_3 \)) is shown in Figure 4.9. The slope \( a_3 \) is different for each \( V_{S30} \) bin, with the largest values occurring at smaller \( V_{S30} \). The slope decreases with increasing \( V_{S30} \), and for \( V_{S30} \) greater than about 450 m/sec the slope is essentially zero.

For periods of 0.2 sec (Figure 4.10), a linear trend between the residual and \( \ln \left( \frac{V_{\text{ratio}}}{1.4} \right) \) is also observed, but the scatter is more significant. Again, Equation (4.4) is fit to the data within each \( V_{S30} \) bin. The slope of the linear fit \( (a_3) \) varies across \( V_{S30} \) bins with larger \( V_{S30} \) values, displaying smaller slopes. For \( T = 0.5 \) sec (Figure 4.11), a similar trend is observed with the slope decreasing with increasing \( V_{S30} \); for this period the slope even becomes negative for larger \( V_{S30} \) (although significant scatter in the data exists). Figure 4.12 plots the derived values of \( a_3 \) versus the median \( V_{S30} \) of each bin for periods of PGA, 0.2 sec, and 0.5 sec. The data for PGA clearly shows \( a_3 \) as a constant at smaller \( V_{S30} \), and then \( a_3 \) decreases as \( V_{S30} \) increases. At \( V_{S30} \) of about 500 m/sec, \( a_3 \) becomes zero. The other periods show similar trends, although not as clearly.

Using the trends shown in Figure 4.12, an expression that describes the slope \( a_3 \) as a function of \( V_{S30} \) is developed. This expression models \( a_3 \) as decreasing linearly between \( V_{S30} \) values of \( V_a \) and \( V_b \), and remaining constant outside of these values. The \( V_{S30} \) below which \( a_3 \) stays constant is \( V_a \), and the \( V_{S30} \) value above which \( a_3 \) is zero is \( V_b \). \( a_0 \) is the value of the slope for \( V_{S30} \) less than \( V_a \). The resulting expression is given by:

\[
a_3 = \begin{cases} 
a_0, & V_{S30} \leq V_a \\
\frac{a_0}{(V_b - V_a)} \cdot (V_{S30} - V_a), & V_a < V_{S30} \leq V_b \\
0, & V_{S30} > V_b
\end{cases}
\]  

(4.5)

where \( V_a, V_b, \) and \( a_0 \) are period dependent.
Figure 4.9  Residuals versus ln(V\text{ratio}/1.4) for different V_{S30} bins at PGA.
Figure 4.10 Residuals versus ln(V\text{ratio}/1.4) for different V_{S30} bins at T = 0.2 sec.
Figure 4.11  Residuals versus $\ln(\text{V ratio}/1.4)$ for different $V_{s30}$ bins at $T = 0.5$ sec.
Figure 4.12   Computed $a_3$ for each $V_{s30}$ bin versus the median $V_{s30}$ of each bin for PGA, $T = 0.2$ sec and 0.5 sec.
Considering the influence of $V_{\text{ratio}}$ identified above and the functional forms that model this influence, the following model is proposed for linear elastic amplification for spectral accelerations at $T \leq 0.5$ sec:

$$\ln(\text{AF})_{\text{lin}} = \begin{cases} a_1 \cdot \ln \left( \frac{V_{S30}}{V_{\text{ref}}} \right) + a_2 \cdot \left[ \ln \left( \frac{V_{S30}}{V_{\text{ref}}} \right) \right]^2 + a_3 \cdot \ln \left( \frac{V_{\text{ratio}}}{1.4} \right) & \text{if } V_{S30} < V_{\text{ref}} \\ 0 & \text{if } V_{S30} \geq V_{\text{ref}} \end{cases}$$

(4.6)

In Equation (4.6) $a_3$ is defined by Equation (4.5). The parameters $a_1$, $a_2$, $a_0$, $V_a$, and $V_b$ are estimated by a maximum likelihood regression on the complete AF dataset for an input PGA$_{\text{rock}}$ of 0.01 g. Table 4.3 lists the model parameters for all six short periods.

To evaluate the effect of adding $V_{\text{ratio}}$ to the linear elastic amplification model, the standard deviation of the data relative to different models is computed. The standard deviation ($\sigma_{\ln(\text{AF})}$) is computed as the standard deviation of the ln residuals (i.e., $\ln(\text{AF})_{\text{Data}} - \ln(\text{AF})_{\text{predicted}}$), and it is computed for both the model that does not incorporate $V_{\text{ratio}}$ [Equation (4.2)] and the model that includes $V_{\text{ratio}}$ [Equation (4.6)]. The computed values of $\sigma_{\ln(\text{AF})}$ are listed in Table 4.4 for each period considered. At shorter periods, the inclusion of $V_{\text{ratio}}$ decreases the standard deviation by more than 30% while the decrease is about 10% at $T = 0.5$ sec.

The variation of amplification with $V_{S30}$ for different values of $V_{\text{ratio}}$ for the developed model [i.e., Equation (4.6)] is shown in Figure 4.13 for the six periods considered. The curves demonstrate that amplification is larger for larger values of $V_{\text{ratio}}$, and this effect is larger at smaller values of $V_{S30}$. 
Table 4.3  Regression coefficients of the linear elastic amplification model [Equations (4.5) and (4.6)].

<table>
<thead>
<tr>
<th>T (sec)</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_0$</th>
<th>$V_a$ (m/sec)</th>
<th>$V_b$ (m/sec)</th>
<th>$V_{ref}$ (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGA</td>
<td>-0.69</td>
<td>-0.13</td>
<td>0.34</td>
<td>176</td>
<td>481</td>
<td>1000</td>
</tr>
<tr>
<td>0.05</td>
<td>-0.70</td>
<td>-0.15</td>
<td>0.37</td>
<td>147</td>
<td>512</td>
<td>1000</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.76</td>
<td>-0.21</td>
<td>0.38</td>
<td>110</td>
<td>737</td>
<td>1000</td>
</tr>
<tr>
<td>0.2</td>
<td>-0.90</td>
<td>-0.32</td>
<td>0.44</td>
<td>414</td>
<td>726</td>
<td>1000</td>
</tr>
<tr>
<td>0.3</td>
<td>-0.89</td>
<td>-0.28</td>
<td>0.57</td>
<td>100</td>
<td>750</td>
<td>1000</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.67</td>
<td>-0.1</td>
<td>0.39</td>
<td>100</td>
<td>750</td>
<td>1000</td>
</tr>
</tbody>
</table>

Table 4.4  $\sigma_{\text{lnAF}}$ of linear elastic amplification models with and without $V_{\text{ratio}}$.

<table>
<thead>
<tr>
<th>T (sec)</th>
<th>$\sigma_{\text{lnAF}}$ (without $V_{\text{ratio}}$)</th>
<th>$\sigma_{\text{lnAF}}$ (with $V_{\text{ratio}}$)</th>
<th>% Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGA</td>
<td>0.09</td>
<td>0.06</td>
<td>33</td>
</tr>
<tr>
<td>0.05</td>
<td>0.10</td>
<td>0.06</td>
<td>40</td>
</tr>
<tr>
<td>0.1</td>
<td>0.11</td>
<td>0.08</td>
<td>27</td>
</tr>
<tr>
<td>0.2</td>
<td>0.19</td>
<td>0.14</td>
<td>26</td>
</tr>
<tr>
<td>0.3</td>
<td>0.20</td>
<td>0.16</td>
<td>20</td>
</tr>
<tr>
<td>0.5</td>
<td>0.19</td>
<td>0.17</td>
<td>10</td>
</tr>
</tbody>
</table>
Figure 4.13 Model predictions for different values of $V_{\text{ratio}}$. 
4.4 $Z_{1.0}$ COMPONENT OF LINEAR ELASTIC SITE AMPLIFICATION MODEL

As discussed above, a linear trend between $\ln(AF)$ and $\ln(V_{S30}/V_{ref})$ best fits the amplification data for periods greater than or equal to 1.0 sec. As shown, $AF$ is not strongly dependent on $V_{ratio}$ for these periods. Amplification at long periods is controlled predominantly by the depth of soil, such that $Z_{1.0}$ is an important parameter to include in the amplification model. To consider the appropriate functional form for the amplification model that includes $Z_{1.0}$, the amplification data are separated into 4 bins of $Z_{1.0}$, and $AF$ is plotted versus $V_{S30}/V_{ref}$ for each bin. Figure 4.14 plots $AF$ versus $V_{S30}/V_{ref}$ for a spectral period of 2.0 sec. The slope of the linear fit of $AF$ versus $V_{S30}$ varies for different bins of $Z_{1.0}$ and generally increases with increasing $Z_{1.0}$, indicating that the slope of the linear fit is $Z_{1.0}$ dependent. The $AF$ data for the other long periods show a similar relationship with $Z_{1.0}$ [Navidi 2012].

A model for this relationship has been proposed by Kottke [2011] and is adopted in this study. This model is described as:

$$\ln(AF)_{lin} = \begin{cases} a_1 \cdot \alpha \cdot \ln \left( \frac{V_{S30}}{V_{ref}} \right) & \text{if } V_{S30} < V_{ref} \\ 0 & \text{if } V_{S30} \geq V_{ref} \end{cases} \quad (4.7)$$

$$\alpha = \left( \frac{\min(Z^*, Z_{1.0}) + 1}{Z^* + 1} \right)^b \quad (4.8)$$

$Z^*$ is the depth (in meters) above which $Z_{1.0}$ no longer influences the amplification, and $a_1$ is the slope of the $\ln(AF)$ versus $\ln(V_{S30}/V_{ref})$ relationship. In the model proposed by Kottke [2011], $Z^*$ is defined as a function of frequency, and $b$ is constant at all periods. In this study, $Z^*$ and $b$ are estimated using the maximum likelihood regression method for each period.

The parameters $a_1$, $Z^*$, and $b$ computed via the maximum likelihood regression for the four long periods are listed in Table 4.5 along with the standard deviation of the model with and without considering the $Z_{1.0}$ effect. Small values of $Z^*$ at periods of 1.0 and 2.0 sec indicate that a smaller range of depth to bedrock influences amplification at these periods. Larger values of $Z_{1.0}$ do not influence amplification at $T = 1.0$ sec and 2.0 sec because these larger values of $Z_{1.0}$ generate amplification at much longer periods. The larger $Z^*$ values for periods of 5.0 and 10.0 sec indicates that all values of $Z_{1.0}$ influence amplification at these periods. Figure 4.15 shows the variation of $\alpha$ with $Z_{1.0}$ for each long period. At $T = 1.0$ sec, $\alpha$ approaches 1.0 for $Z_{1.0}$ greater than 100 m. While for $T = 10.0$ sec, $\alpha$ does not reach 1.0 until $Z_{1.0}$ is equal to 1000 m.

The variation of amplification with $V_{S30}$ for different values of $Z_{1.0}$ for the developed model in Equations (4.7) and (4.8) is shown in Figure 4.16 for the four periods considered. Relationships are shown for four values of $Z_{1.0}$ that represent the range in the data. The influence of $Z_{1.0}$ at $T = 1.0$ sec is not significantly different because the derived value of $Z^*$ is relatively small. The influence of different values of $Z_{1.0}$ is more readily apparent in the data for periods of
5.0 and 10.0 sec. Nonetheless, the reduction in $\sigma_{\ln AF}$ when $Z_{1.0}$ is included is relatively modest (Table 4.5).

<table>
<thead>
<tr>
<th>$T$ (sec)</th>
<th>$a_1$</th>
<th>$Z^*$ (m)</th>
<th>$b$</th>
<th>$\sigma_{\ln AF}$ (without $Z_{1.0}$)</th>
<th>$\sigma_{\ln AF}$ (with $Z_{1.0}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>-0.63</td>
<td>121</td>
<td>0.70</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>2.0</td>
<td>-0.75</td>
<td>292</td>
<td>0.71</td>
<td>0.19</td>
<td>0.17</td>
</tr>
<tr>
<td>5.0</td>
<td>-0.63</td>
<td>490</td>
<td>1.16</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td>10.0</td>
<td>-0.44</td>
<td>1000</td>
<td>0.76</td>
<td>0.06</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Figure 4.14 Amplification factor versus $V_{S30}/V_{ref}$ at $T = 2.0$ sec for four different bins of $Z_{1.0}$. 
Figure 4.14  Amplification factor versus $V_{S30}/V_{ref}$ at $T = 2.0$ sec for four different bins of $Z_{1.0}$. 
Figure 4.15 Variation of $\alpha$ with $Z_{1.0}$ for different periods.
Figure 4.16 Model predictions for different values of $Z_{1.0}$. 
5 Models for Nonlinear Site Amplification

5.1 INTRODUCTION

Next, we develop the nonlinear component of the site amplification models. A functional form for shorter periods that includes the effects of $V_{S30}$, $V_{ratio}$, and $S_{a,rock}$ will be presented, and a functional form for longer periods will be presented that includes $V_{S30}$ and $S_{a,rock}$.

5.2 $V_{S30}$ COMPONENT OF THE NONLINEAR SITE AMPLIFICATION MODEL

The nonlinear component of the soil amplification model includes the effects of soil nonlinearity at high intensity shaking such that the amplification changes with increasing input shaking intensity. The nonlinear effect can be modeled using a functional form represented by:

$$\ln(AF)_{nl} = b_1 \cdot \ln\left(\frac{S_{a,rock} + c}{c}\right)$$

(5.1)

where $S_{a,rock}$ is the spectral acceleration for rock conditions at the spectral period of interest, and $b_1$ and $c$ are regression coefficients. This functional form was used by Chiou and Youngs [2008] in their GMPE.

It is helpful to consider how parameters $b_1$ and $c$ control the variation of AF with shaking intensity (i.e., $S_{a,rock}$). Coefficient $c$ essentially represents the $S_{a,rock}$ level in the middle of transition from linear behavior (i.e., where AF does not vary with $S_{a,rock}$) to nonlinear behavior (i.e., where AF does vary with $S_{a,rock}$). Coefficient $b_1$ represents the degree of nonlinearity in terms of the logarithmic change in AF with a logarithmic increase in the level of shaking. Generally, $b_1$ is negative such that an increase in $S_{a,rock}$ results in a decrease in AF. A more negative value of $b_1$ indicates a stronger reduction in AF with $S_{a,rock}$ (i.e., more nonlinearity), and as $b_1$ approaches zero the site amplification approaches the linear elastic condition. The degree of nonlinearity is a function of the stiffness of the site; with softer sites experiencing more nonlinearity. Therefore, $b_1$ is a function of $V_{S30}$.

Figure 5.1 shows AF versus $S_{a,rock}$ as predicted by the Chiou and Youngs [2008] model at periods of PGA, 0.2 sec, and 0.5 sec, and for three different $V_{S30}$ values (150 m/sec, 300 m/sec and 500 m/sec). Also shown are the locations of coefficient $c$ for each period ($c$ is not taken $V_{S30}$ dependent in the Chiou and Youngs [2008] model) and a representative slope ($b_1$) at larger $S_{a,rock}$. 
At each of the spectral periods, nonlinearity is stronger at smaller values of $V_{S30}$. At large values of $V_{S30}$ (e.g., $V_{S30} = 500$ m/sec in Figure 5.1), the reduction of AF with increasing $S_{a,rock}$ is insignificant, indicating more linear behavior for stiff sites. The coefficient $c$, which is the shaking level where there is a transition from linear to nonlinear behavior, is close to 0.1g at PGA and 0.5 sec, while it is higher (0.25g) at a spectral period of 0.2 sec.

To investigate the dependency of AF on $S_{a,rock}$ and the factors that influence this relationship, the computed AF values for the randomized site profiles subjected to 10 different input intensities were considered. Amplifications of the soil profiles at short and long periods are considered separately in the next sections.

![Figure 5.1](image)

**Figure 5.1** Amplification versus $S_{a,rock}$ as predicted by the Chiou and Youngs [2008] GMPE.
Building upon previous work, the nonlinear AF model was developed from Equation (5.1). The dependence of the slope $b_1$ on $V_{S30}$ was investigated by separating the AF data into the same eight $V_{S30}$ bins used in Chapter 4 ($V_{S30}<150$ m/sec, 150 m/sec $< V_{S30} < 200$ m/sec, 200 m/sec $< V_{S30} < 250$ m/sec, 250 m/sec $< V_{S30} < 300$ m/sec, 300 m/sec $< V_{S30} < 350$ m/sec, 350 m/sec $< V_{S30} < 450$ m/sec, 450 m/sec $< V_{S30} < 550$ m/sec, and 550 m/sec $< V_{S30} < 750$ m/sec). Figure 4.2 shows plots of AF versus $S_{a,rock}$ for PGA for the eight bins of $V_{S30}$. At the smaller $V_{S30}$ values, amplification almost immediately starts to decrease as $S_{a,rock}$ increases from 0.01 g. At larger $V_{S30}$ (i.e., greater than about 300 m/sec) AF remains relatively constant at $S_{a,rock}$ levels less than about 0.1 g but then decreases as $S_{a,rock}$ increases beyond 0.1 g. At the largest $V_{S30}$ values, AF does not vary significantly over the $S_{a,rock}$ values analyzed.

To identify the variation of $b_1$ with $V_{S30}$, a linear relationship is fit between the logarithm of AF in each $V_{S30}$ bin and the logarithm of $S_{a,rock}$, using only AF data from $S_{a,rock} > 0.1$ g in each bin. Only data with $S_{a,rock}$ larger than 0.1 g is considered because $b_1$ represents the slope at larger input intensities. The slope of the linear relationship ($b_1$) is shown in Figure 5.2 and varies across the $V_{S30}$ bins, with larger $V_{S30}$ values displaying smaller slopes. For $T = 0.2$ sec and 0.5 sec (Figures 4.3 and 4.4, respectively) a similar trend is observed with the slope decreasing with increasing $V_{S30}$. Figure 4.5 plots the derived values of $b_1$ versus the median $V_{S30}$ of each bin for periods of PGA, 0.2, and 0.5 sec. The data for all three periods show $b_1$ as approximately constant at smaller $V_{S30}$ and then decreasing towards zero as $V_{S30}$ increases. Above some value of $V_{S30}$, $b_1$ tends to remains constant.
Figure 5.2  Amplification factor for PGA versus S\text{a,rock} for different V_{S30} bins.
Figure 5.3 Amplification factor for $T = 0.2$ sec versus $S_{a,\text{rock}}$ for different $V_{S30}$ bins.
Figure 5.4 Amplification factor for $T = 0.5$ sec versus $S_{a,rock}$ for different $V_{S30}$ bins.
Figure 5.5  Computed $b_1$ for each $V_{S30}$ bin versus median $V_{S30}$ of each bin (PGA, $T = 0.2$ sec and 0.5 sec).
Using the trends in Figure 5.5, an expression is developed that describes the slope $b_1$ as a function of $V_{S30}$. This expression models $b_1$ as decreasing log-linearly between $V_{S30}$ values of $V_1$ and $V_2$, remaining constant outside of these values. The $V_{S30}$ below which $b_1$ stays constant is $V_1$, and the value above which $b_1$ stays constant is $V_2$. $b_{o1}$ and $b_{o2}$ are the values of the slope for $V_{S30}$ less than $V_1$ and greater than $V_2$, respectively. The resulting expression is given by:

$$b_1 = \begin{cases} 
    b_{o1}, & V_{S30} \leq V_1 \\
    b_{o2} - b_{o1} \cdot \ln \left( \frac{V_{S30}}{V_2} \right), & V_1 < V_{S30} \leq V_2 \\
    b_{o2}, & V_{S30} > V_2
\end{cases} \quad (5.2)$$

$V_1$, $V_2$, $b_{o1}$, and $b_{o2}$ are regression coefficients that are period dependent. This expression is incorporated into the final form of the nonlinear amplification model.

### 5.3 $V_{ratio}$ COMPONENT OF THE NONLINEAR SITE AMPLIFICATION MODEL

Chapter 4 demonstrated that $V_{ratio}$ influences the linear-elastic amplification of soil sites at shorter periods. Including $V_{ratio}$ into the linear-elastic model reduced the standard deviation by 10 to 30%. The $V_{ratio}$ effect modeled in the linear elastic amplification model will be present at larger intensities, but $V_{ratio}$ may also influence the nonlinear amplification. The AF data in Figures 5.2 through 5.4 clearly show that the scatter in the AF data increases as input intensity increases. Based on the observations at small input intensities, it is likely that $V_{ratio}$ influences the computed values of AF at large input intensities.

To investigate the influence of $V_{ratio}$ on the nonlinear component of the site amplification model, AF is plotted versus the normalized $V_{ratio}$ ($V_{ratio} / 1.4$) for each bin of $V_{S30}$ and input intensity. A linear relationship is fit to the $\ln(AF)$ versus $\ln(V_{ratio} / 1.4)$ for each $S_{a,rock}$ to identify if the slope of this relationship (i.e., regression parameter $a_3$ from Chapter 4) changes with $S_{a,rock}$. Figures 5.6 through 5.8 show plots of PGA amplification versus $V_{ratio} / 1.4$ for $V_{S30}$ bins of $\leq$150 m/sec, 250–350 m/sec and 550–750 m/sec, respectively. The slope of the log-linear fit for each of the $V_{S30}$ bins generally increases with $S_{a,rock}$. The slope, which indicates how strongly $V_{ratio}$ affects AF for a given $V_{S30}$, can increase by more than a factor of two as $S_{a,rock}$ increases from 0.01g to 0.9g. For the larger $V_{S30}$ values (Figure 5.8), the slope increases from 0.0 (i.e., no $V_{ratio}$ effect) at $S_{a,rock} = 0.01g$ to 0.4 at $S_{a,rock} = 0.9g$.

The slope of the linear fit between $\ln(AF)$ and $\ln(V_{ratio} / 1.4)$, which represents parameter $a_3$ in the linear-elastic model of Chapter 4, is plotted versus normalized $-S_{a,rock}$ (i.e., $S_{a,rock}/S_{a,rock-min}$, where $S_{a,rock-min}$ is the smallest input rock intensity considered) in Figure 5.9 for all eight $V_{S30}$ bins. Since $a_3$ is $V_{S30}$ dependent; Figure 5.9 allows us to investigate whether the increase in $a_3$ with $S_{a,rock}$ is also $V_{S30}$ dependent. The data in Figure 5.9 show that $a_3$ increases linearly with $\ln(S_{a,rock}/S_{a,rock-min})$ for all $V_{S30}$ bins at PGA. The linear trend between $a_3$ and $\ln(S_{a,rock}/S_{a,rock-min})$ is quite strong, and the slope of that relationship varies between 0.05 and 0.15, with most of the values between 0.07 and 0.09. The computed slopes are plotted versus $V_{S30}$ in Figure 5.10.
variation of the slope with $V_{S30}$ is not systematic; therefore, the slope is considered to be independent of $V_{S30}$. 
Figure 5.6 Amplification factor for PGA versus normalized $V_{\text{ratio}}$ for $V_{S30} < 150$ m/sec.
Figure 5.7 Amplification factor for PGA versus normalized $V_{\text{ratio}}$ for $250 < V_{S30} < 300 \text{ m/sec}$. 
Figure 5.8 Amplification factor for PGA versus normalized $V_{ratio}$ for $550 < V_{S30} < 750$ m/sec.
Figure 5.9  The slope of linear fit between $\ln(AF)$ and $\ln(V_{\text{ratio}}/1.4)$ versus $\ln(S_{\text{rock}}/S_{\text{rock-min}})$ for PGA.
Generally, the trends shown for PGA are similar for other short periods (i.e., \( \leq 0.5 \) sec), Figure 5.11 plots the slope of the \( a_3 \) versus \( \ln \left( \frac{S_{a,rock}}{S_{a,rock-min}} \right) \) relationship versus \( V_{S30} \) for these other periods. Again, the slope varies with \( V_{S30} \) but not in a systematic manner; therefore, the slope is modeled as \( V_{S30} \) independent for all of the short periods.

The data presented indicate that the \( V_{\text{ratio}} \) effect, previously modeled in the linear elastic case as \( a_3 \cdot \ln(V_{\text{ratio}}/1.4) \) with \( a_3 \) being \( V_{S30} \) dependent, is intensity dependent. The full \( V_{\text{ratio}} \) effect can be written as:

\[
\ln(AF)_{v_{\text{ratio}}} = \left[ a_3 + b_2 \cdot \ln \left( \frac{S_{a,rock}}{S_{a,rock-min}} \right) \right] \cdot \ln \left( \frac{V_{\text{ratio}}}{1.4} \right) \tag{5.3}
\]

Where \( a_3 \) represents the \( V_{\text{ratio}} \) effect under linear elastic conditions, as presented in Chapter 4, and \( b_2 \cdot \ln \left( \frac{S_{a,rock}}{S_{a,rock-min}} \right) \) models the effect of input intensity on the \( V_{\text{ratio}} \) effect. The parameter \( b_2 \) represents the slopes shown in Figures 5.9; this parameter does not vary with \( V_{S30} \) based on the data shown in Figures 5.10 and 5.11. To maintain the separation of AF under linear elastic conditions from the AF under nonlinear conditions, the second component in Equation (5.3) is added to the nonlinear amplification model. Additionally, to ensure that the nonlinear amplification model disappears at small input intensities, the smallest \( S_{a,rock} \) that can be used in Equation (5.3) is \( S_{a,rock-min} \). Based on the above considerations, the nonlinear component of the site amplification model can be written as:

\[
\ln(AF)_{n1} = b_1 \cdot \ln \left( \frac{S_{a,rock} + c}{c} \right) + b_2 \cdot \ln \left[ \max \left( \frac{S_{a,rock}}{S_{a,rock-min}} \right) \right] \cdot \ln \left( \frac{V_{\text{ratio}}}{1.4} \right) \tag{5.4}
\]
where $b_1$ is $V_{S30}$ dependent, as described by Equation (5.2), and the coefficient $b_2$ is $V_{S30}$ independent.
Figure 5.11  The variation of the slope of the linear fit between $a_3$ and $\ln(S_{a,\text{rock}}/S_{a,\text{rock-min}})$ with $V_{S30}$ at $T = 0.05, 0.1, 0.2, 0.3,$ and $0.5$ sec.
5.4 NONLINEAR SITE AMPLIFICATION AT LONG PERIODS

To investigate the dependency of amplification on input intensity at long periods, AF is plotted versus $S_{a,\text{rock}}$ for bins of $V_{S30}$ at the four long periods considered. Figures 5.12 and 5.13 show AF versus $S_{a,\text{rock}}$ for two select representative periods: $T = 1.0$ sec and 5.0 sec, respectively. At $T = 1.0$ sec, AF decreases with increasing input intensity ($S_{a,\text{rock}}$) for smaller $V_{S30}$ but the AF remains constant or slightly increases for larger $V_{S30}$ (i.e., greater than about 350 m/sec in Figure 5.12). At $T = 5.0$ sec (Figure 5.13), amplification generally increases as $S_{a,\text{rock}}$ increases for smaller $V_{S30}$, indicating that the $b_1$ slope may be positive at long periods. At larger values of $V_{S30}$, the amplification becomes insensitive to input intensity (slope ~ 0.0) similar to the results for $T = 1.0$ sec. A positive value of $b_1$ is technically justified for $T = 5.0$ sec because most sites have natural periods shorter than 5.0 sec, and the soil nonlinearity induced by large input intensities will cause period lengthening towards 5.0 sec. This lengthening will cause an increase in amplification with increasing input intensity at this period. Sites with larger $V_{S30}$ tend to have natural periods much shorter than 5.0 sec and they strain less as $S_{a,\text{rock}}$ increases, making the effect of period lengthening minimal.

The slope of the $\ln(\text{AF})-\ln(S_{a,\text{rock}})$ relationship at larger values of $S_{a,\text{rock}}$ (which represents $b_1$) are computed for each $V_{S30}$ bin and are shown in Figures 5.12 and 5.13. Figure 5.14 plots the derived values of $b_1$ versus the median $V_{S30}$ of each bin for periods of 1.0 sec and 5.0 sec. Figure 5.14 shows that $b_1$ is constant at smaller $V_{S30}$; it varies linearly with increasing $V_{S30}$, and then becomes constant again at larger $V_{S30}$. For $T = 1.0$ sec, $b_1$ is equal to –0.8 at smaller $V_{S30}$ and approaches 0.0 at $V_{S30}$ greater than about 400 m/sec. For $T = 5.0$ sec, $b_1$ is about 0.2 for smaller $V_{S30}$ and approaches 0.0 at $V_{S30}$ greater than about 500 m/sec. A positive value of $b_1$ indicates that AF increases with increasing $S_{a,\text{rock}}$, and positive values are observed for periods greater than and equal to 2.0 sec.

Based on the data presented in Figure 5.14, the same functional form that is used at short periods to describe the variation $b_1$ with $V_{S30}$ can be used at longer periods (Equation 5.2). This relationship defines regions of constant $b_1$ at smaller and larger $V_{S30}$ values, and a linear relationship with $\ln(V_{S30})$ at $V_{S30}$ values in between. The parameters to be determined by regression are the $V_{S30}$ values above and below which $b_1$ is a constant ($V_2$ and $V_1$), and the $b_1$ values above and below these values ($b_{02}$ and $b_{01}$).

Chapter 4 demonstrated that the site amplification at long periods is not influenced by $V_{\text{ratio}}$, but that it is influenced by $Z_{1.0}$. The effect of $Z_{1.0}$ on soil amplification is not affected by soil nonlinearity; therefore the model for the effect of $Z_{1.0}$ does not include input intensity (i.e., $S_{a,\text{rock}}$). Additionally, the NGA models that include $Z_{1.0}$ [Abrahamson and Silva 2008; Chiou and Youngs 2008] also found the influence of $Z_{1.0}$ to be intensity independent.
Figure 5.12 Amplification for $T = 1.0$ sec versus $S_{a,\text{rock}}$ for different $V_{S30}$ bins.
Figure 5.13 Amplification for $T = 5.0$ sec versus $S_{a,\text{rock}}$ for different $V_{S30}$ bins.
Figure 5.14  
$b_1$ versus median $V_{S30}$ of each bin at $T = 1.0$ sec and 5.0 sec.
6 Final Site Amplification Model

6.1 INTRODUCTION

The linear and nonlinear components of the site amplification model presented in Chapters 4 and 5 are combined and summarized in this chapter. Separate functional forms are fitted to the amplification data for short and long periods. To demonstrate the fit of the developed model, the amplification data are plotted along with predictions from the developed models. The calculated residuals from the developed functional forms are plotted versus the independent variables in the site amplification model. Finally, the influence of $V_{\text{ratio}}$ on predicted acceleration response spectra is demonstrated.

6.2 SITE AMPLIFICATION MODEL FOR SHORT PERIODS

Combining the linear-elastic and nonlinear components of the site amplification model for short periods, the following model is proposed for the amplification factor for spectral accelerations at short periods ($T \leq 0.5$ sec):

If $V_{S30} < V_{\text{ref}}$:

$$\ln(AF) = a_1 \cdot \ln\left(\frac{V_{S30}}{V_{\text{ref}}}\right) + a_2 \cdot \left[\ln\left(\frac{V_{S30}}{V_{\text{ref}}}\right)\right]^2 + a_3 \cdot \ln\left(\frac{V_{\text{ratio}}}{1.4}\right) + b_1 \cdot \ln\left(\frac{S_{a,\text{rock}+c}}{c}\right) + b_2 \cdot \ln\left\{\max\left(\frac{S_{a,\text{rock} \cdot S_{a,\text{rock} - \text{min}}}}{S_{a,\text{rock} - \text{min}}}\right)\right\} \cdot \ln\left(\frac{V_{\text{ratio}}}{1.4}\right)$$

(6.1a)

If $V_{S30} \geq V_{\text{ref}}$:

$$\ln(AF) = b_1 \cdot \ln\left(\frac{S_{a,\text{rock} + c}}{c}\right) + b_2 \cdot \ln\left\{\max\left(\frac{S_{a,\text{rock} \cdot S_{a,\text{rock} - \text{min}}}}{S_{a,\text{rock} - \text{min}}}\right)\right\} \cdot \ln\left(\frac{V_{\text{ratio}}}{1.4}\right)$$

(6.1b)
This model is fit independently to each short period. The parameters $a_3$ and $b_1$ are $V_{S30}$ dependent, and the $V_{S30}$ dependencies are described by:

$$a_3 = \begin{cases} 
    a_0 - \frac{a_0}{(V_b - V_a)} \cdot (V_{S30} - V_a), & V_a < V_{S30} \leq V_b \\
    0, & V_{S30} > V_b 
\end{cases} \quad (6.2)$$

$$b_1 = \begin{cases} 
    b_{o1}, & V_{S30} \leq V_1 \\
    b_{o2} - b_{o1} \cdot \ln \left( \frac{V_{S30}}{V_1} \right), & V_1 < V_{S30} \leq V_2 \\
    b_{o2}, & V_{S30} > V_2 
\end{cases} \quad (6.3)$$

The regression process is executed in three steps. Each step uses the maximum likelihood method for regression. In the first step, the coefficients for the linear-elastic component are estimated (i.e., $a_1$, $a_2$, $a_0$, $V_a$, $V_b$) using the AF data from the lowest intensity input motions (i.e., PGA$_{rock}$ = 0.01g). These parameters are kept fixed in subsequent steps. In the second step, the parameters for the nonlinear model ($b_{o1}$, $b_{o2}$, $V_1$, $V_2$, and $c$) are estimated excluding the effect of $V_{ratio}$ (i.e., $b_2$) using the AF data for all input intensities. To ensure that the nonlinear component of the model does not influence the computed amplification at small $S_a_{rock}$, a minimum value of $c$ was set to 0.1g. If $c = 0.1g$, then $\ln \left( \frac{S_{a,rock}+c}{c} \right)$ term is equal to 0.095 for $S_{a,rock} = 0.01g$. Thus, the nonlinear effect on the computed amplification is minimal. In the last and final step, all previously parameters are fixed and $b_2$ (i.e., the $V_{ratio}$ effect in the nonlinear model) is computed. The final estimated values of all coefficients for the six short periods studied here are listed in Table 6.1.

Figure 6.1 shows the residuals [i.e., ln(data)-ln(predicted)] plotted versus $V_{S30}$, $S_{a,rock}$, and $V_{ratio}$ for the amplification factors for PGA. The residuals indicate overprediction in amplification (negative residuals) for smaller $V_{S30}$ and a slight overprediction over all $V_{S30}$ (overall average residual equal to –0.07). There is no systematic trend between the residuals and the various parameters. However the variability in the residuals increases with the increase of $S_{a,rock}$. For all the other considered short periods, the residuals also do not show a systematic trend with any of independent variables in the model; however, the average residual across these other periods range from about -0.07 to +0.08.

It is difficult to fully evaluate the model looking at the overall residuals, therefore AF predictions are plotted versus $S_{a,rock}$ along with the AF data in the next several figures. To demonstrate the effects of $V_{S30}$ and $V_{ratio}$ on the AF predictions, the predicted AF values are plotted versus $S_{a,rock}$ for select $V_{S30}$ bins and for a range of $V_{ratio}$ values. As discussed previously, the range of $V_{ratio}$ values is slightly different among the $V_{S30}$ bins, but most of the values fall between 0.8 and 2.5. Considering these ranges, AF predictions are shown for $V_{ratio} = 0.8, 1.4, and 2.5$ in Figures 6.2 to 6.7 for the six periods considered.
Table 6.1 Regression coefficients of the site amplification model for short periods.

<table>
<thead>
<tr>
<th></th>
<th>PGA</th>
<th>0.05 sec</th>
<th>0.1 sec</th>
<th>0.2 sec</th>
<th>0.3 sec</th>
<th>0.5 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>-0.69</td>
<td>-0.70</td>
<td>-0.76</td>
<td>-0.9</td>
<td>-0.89</td>
<td>-0.67</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-0.13</td>
<td>-0.15</td>
<td>-0.21</td>
<td>-0.32</td>
<td>-0.28</td>
<td>-0.10</td>
</tr>
<tr>
<td>$a_0$</td>
<td>0.34</td>
<td>0.38</td>
<td>0.38</td>
<td>0.44</td>
<td>0.57</td>
<td>0.39</td>
</tr>
<tr>
<td>$V_a$ (m/sec)</td>
<td>176</td>
<td>130</td>
<td>110</td>
<td>414</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>$V_b$ (m/sec)</td>
<td>481</td>
<td>513</td>
<td>737</td>
<td>726</td>
<td>750</td>
<td>750</td>
</tr>
<tr>
<td>$b_{o1}$</td>
<td>-0.91</td>
<td>-1.26</td>
<td>-0.98</td>
<td>-1.21</td>
<td>-1.93</td>
<td>-2.60</td>
</tr>
<tr>
<td>$b_{o2}$</td>
<td>-0.24</td>
<td>-0.21</td>
<td>-0.19</td>
<td>-0.16</td>
<td>-0.14</td>
<td>-0.11</td>
</tr>
<tr>
<td>$V_1$ (m/sec)</td>
<td>184</td>
<td>118</td>
<td>192</td>
<td>188</td>
<td>133</td>
<td>103</td>
</tr>
<tr>
<td>$V_2$ (m/sec)</td>
<td>454</td>
<td>581</td>
<td>583</td>
<td>557</td>
<td>530</td>
<td>447</td>
</tr>
<tr>
<td>$c$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.21</td>
<td>0.37</td>
<td>0.4</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.09</td>
<td>0.08</td>
<td>0.08</td>
<td>0.07</td>
<td>0.08</td>
<td>0.06</td>
</tr>
<tr>
<td>$V_{ref}$ (m/sec)</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>$S_{a,rock-min}$ (g)</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
</tr>
</tbody>
</table>
Figure 6.1 Model residuals for PGA versus $V_{S30}$, $S_{a,rock}$, and $V_{ratio}$. 
The predictions in Figures 6.2 through 6.7 are in general agreement with the data. Additionally, sites with larger values of \( V_{\text{ratio}} \) have larger amplification and sites with smaller \( V_{\text{ratio}} \) have smaller amplification. This effect helps explain some of the scatter shown in the AF data. The effect of \( V_{\text{ratio}} \) at low input intensity (i.e., smallest \( S_{a,\text{rock}} \)) is strongest for the smaller values of \( V_{S30} \) and essentially non-existent for larger \( V_{S30} \) (greater than about 350 m/sec). However, the input intensity effect modeled with \( V_{\text{ratio}} \) results in \( V_{\text{ratio}} \) becoming important for larger \( V_{S30} \) at larger input intensities (note the predictions for \( V_{S30} = 450–550 \) m/sec in Figures 6.2 to 6.7).

While there is general agreement between the model predictions and data shown in Figures 6.2 through 6.7, there are some areas of deviation. Note that at the smallest input intensity and the smallest \( V_{S30} \) values, the AF predictions tend to be smaller than observed for \( T \leq 0.2 \) sec. This result is due to the parameter \( c \) being small (~ 0.1g) relative to the smallest \( S_{a,\text{rock}} \) used in this study (0.01g) and the large, negative values of \( b_1 \) for small \( V_{S30} \) sites. As a result, the nonlinear component of the amplification model contributes to the AF prediction even for \( S_{a,\text{rock}} \sim 0.01g \). This issue is not apparent at periods of 0.3 and 0.5 sec because the parameter \( c \) is larger (Table 6.1). Another challenging area is amplification for the larger \( V_{S30} \) values (450–550 m/sec) at periods of 0.1 and 0.2 sec. The model tends to underpredict the amplification at moderate \( S_{a,\text{rock}} \) and overpredicts amplification at larger AF. This issue again appears to be due to \( c \) being relatively small and the model for the \( V_{S30} \)-dependence for \( b_1 \).
Figure 6.2 Amplification factor for PGA versus $S_{a,\text{rock}}$ and model predictions for different values of $V_{\text{ratio}}$. 
Figure 6.3 Amplification factor for $T = 0.05$ sec versus $S_{a,\text{rock}}$ and model predictions for different values of $V_{\text{ratio}}$. 
Figure 6.4 Amplification factor for $T = 0.1$ sec versus $S_{a, \text{rock}}$ and model predictions for different values of $V_{\text{ratio}}$. 
Figure 6.5 Amplification factor for $T = 0.2$ sec versus $S_{a,\text{rock}}$ and model predictions for different values of $V_{\text{ratio}}$. 

- $T = 0.2$ s, $150 < V_{S30} < 200$ m/s
- $T = 0.2$ s, $250 < V_{S30} < 300$ m/s
- $T = 0.2$ s, $350 < V_{S30} < 450$ m/s
- $T = 0.2$ s, $450 < V_{S30} < 550$ m/s
Figure 6.6 Amplification factor for $T = 0.3$ sec versus $S_{a,\text{rock}}$ and model predictions for different values of $V_{\text{ratio}}$. 
To evaluate the effect of adding $V_{ratio}$ to the full amplification model, the standard deviation of the residuals [i.e., $\ln(\text{data})-\ln(\text{predicted})$] for models with and without including $V_{ratio}$ is computed. The standard deviations (i.e., $\sigma_{\ln(AF)}$) are listed in Table 6.2 for each period. The standard deviation of the full model decreases when including $V_{ratio}$ (in most cases from about 8 to 15%), however the level of reduction is not as significant as it was for the linear elastic model.
(Chapter 4). The modest reduction in $\sigma_{\ln AF}$ may be influenced by the strength of $\ln(AF) - \ln(V_{ratio})$ relationship at larger input intensities. Considering the data from Chapter 5 shown in Figures 5.6 through 5.8, the strength of the $AF - V_{ratio}$ relationship decreases with increasing input intensity.

In addition to the standard deviation of the residuals, the models with and without $V_{ratio}$ can be compared through the parameter $R^2_{M1-M2}$, defined as:

$$R^2_{M1-M2} = \frac{\sum (\text{Residual}_{M1})^2 - \sum (\text{Residual}_{M2})^2}{\sum (\text{Residual}_{M1})^2}$$

(6.4)

where $\sum (\text{Residual}_{M1})^2$ is the sum of the squared residuals from Model 1 (i.e., the model that does not include $V_{ratio}$) and $\sum (\text{Residual}_{M2})^2$ is the sum of the squared residuals from Model 2 (i.e., the model that includes $V_{ratio}$). The calculated $R^2_{M1-M2}$ values for each period are also listed in Table 6.2. The calculated $R^2_{M1-M2}$ values range from 0.08 to 0.33, indicating that $V_{ratio}$ model explains 8 to 33% of the variation that exists in the non-$V_{ratio}$ model. At a spectral period of 0.2 sec, the calculated $R^2_{M1-M2}$ reaches its maximum value of 33%.

<table>
<thead>
<tr>
<th>T (sec)</th>
<th>$\sigma_{\ln AF}$ (\text{model without } V_{ratio})</th>
<th>$\sigma_{\ln AF}$ (\text{model with } V_{ratio})</th>
<th>% Reduction</th>
<th>$R^2_{M1-M2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGA</td>
<td>0.28</td>
<td>0.25</td>
<td>11</td>
<td>0.08</td>
</tr>
<tr>
<td>0.05</td>
<td>0.30</td>
<td>0.25</td>
<td>17</td>
<td>0.19</td>
</tr>
<tr>
<td>0.1</td>
<td>0.36</td>
<td>0.32</td>
<td>11</td>
<td>0.19</td>
</tr>
<tr>
<td>0.2</td>
<td>0.45</td>
<td>0.37</td>
<td>18</td>
<td>0.33</td>
</tr>
<tr>
<td>0.3</td>
<td>0.41</td>
<td>0.36</td>
<td>12</td>
<td>0.19</td>
</tr>
<tr>
<td>0.5</td>
<td>0.39</td>
<td>0.36</td>
<td>8</td>
<td>0.11</td>
</tr>
</tbody>
</table>

6.3 SITE AMPLIFICATION MODEL FOR LONG PERIODS

Combining the linear-elastic and nonlinear components of the amplification model for long periods, the following model is proposed for the amplification factor for spectral accelerations at long periods ($T \geq 1.0$ sec):
If $V_{S30} < V_{ref}$:

$$\ln(AF) = a_1 \cdot \alpha \cdot \ln \left( \frac{V_{S30}}{V_{ref}} \right) + b_1 \cdot \ln \left( \frac{S_{a,rock} + c}{c} \right)$$

(6.5a)

If $V_{S30} \geq V_{ref}$:

$$\ln(AF) = b_1 \cdot \ln \left( \frac{S_{a,rock} + c}{c} \right)$$

(6.5b)

with $\alpha$ and $b_1$ defined as:

$$\alpha = \left[ \frac{\min(Z^*, Z_{1.0}) + 1}{Z^* + 1} \right]$$

(6.6)

$$b_1 = \begin{cases} 
  b_{o1}, & V_{S30} \leq V_1 \\
  b_{o2} - b_{o1} \cdot \ln \left( \frac{V_{S30}}{V_1} \right), & V_1 < V_{S30} \leq V_2 \\
  b_{o2}, & V_{S30} > V_2 
\end{cases}$$

(6.7)

All other parameters are estimated in the maximum likelihood regression. The regression process is executed in two steps. In the first step, the coefficients of the linear-elastic component of the model (i.e., $a_1$, $Z^*$, and $b$) are estimated. These coefficients are kept fixed in the second step, which estimates the parameters in the nonlinear component of the model ($b_{o1}$, $b_{o2}$, $V_1$, $V_2$, and $c$). The derived values of all coefficients for the four long periods studied here are listed in Table 6.3. Note that positive values of $b_1$ (i.e., $b_{o1}$ and $b_{o2}$) are derived for periods of 5.0 and 10.0 sec, indicating that amplification increases with increases $S_{a,rock}$ at these periods. At $T = 1.0$ and 2.0 sec, $b_1$ becomes slightly positive at large $V_{S30}$ values (i.e., $b_{o2}$ become positive). The standard deviation for each model ($\sigma_{lnAF}$) is also listed in Table 6.3 and ranges from 0.32 to 0.14.

Figure 6.8 shows the residuals [i.e., ln(data)-ln(predicted)] plotted versus $V_{S30}$, $S_{a,rock}$, and $Z_{1.0}$ for spectral period of 1.0 sec. The residuals do not show any systematic trend with respect to $V_{S30}$, $S_{a,rock}$, and $Z_{1.0}$. For all the other long periods considered in this study, the residuals do not show any systematic trend with any of the independent variables in the model.

To investigate how the proposed functional form at long periods ($T \geq 1.0$ sec) fits the data, predictions of AF are plotted versus $S_{a,rock}$ for three different values of $Z_{1.0}$ in Figures 6.9 to 6.12. The AF data are also shown in these plots. As noted previously, there is a relationship between $V_{S30}$ of a site and its depth to bedrock (i.e., $Z_{1.0}$) with stiffer sites reaching bedrock at shallower depth. Based on the data shown in Chapter 3 for $V_{S30} < 300$ m/sec, $Z_{1.0}$ ranges from about 100 to 600 m. Therefore, the predicted AF values are plotted for $Z_{1.0}$ of 100, 300, and 600 m for $V_{S30} < 300$ m/sec. For $V_{S30} > 300$ m/sec, $Z_{1.0}$ ranges from 20 to 400 m. For $V_{S30} > 300$ m/sec, AF predictions are shown for $Z_{1.0}$ of 20, 200, and 400 m. As discussed in the previous chapter, the $\alpha$ parameter incorporates the effect of depth in the model and $\alpha$ is equal to 1.0 for sites with $Z_{1.0}$ greater than $Z^*$. Therefore, amplification for sites with $Z_{1.0} > Z^*$ is not influenced by $Z_{1.0}$. 

99
At a spectral period of 1.0 sec (Figure 6.9), $Z^*$ is equal to 121 m such that most of the curves shown do not show any $Z_{1.0}$ effect. The curve with $Z_{1.0}$ equal to 100 m for $V_{S30} < 300$ m/sec shows a very small effect; the curve for $Z_{1.0}$ equal to 20 m for $V_{S30} > 300$ m/sec shows a larger effect. However, few of the data have $Z_{1.0}$ equal to 20 m. Similar trends are observed at $T = 2.0$ sec, but with $Z^*$ equal to 292 m the $Z_{1.0}$ effect is more apparent. At longer periods (i.e., 5.0 and 10.0 sec), $Z^*$ is much larger (490 and 1000 m, respectively), such that a clear increase in amplification as the depth to the bedrock ($Z_{1.0}$) increases is observed for $V_{S30} < 300$ m/sec. For larger $V_{S30}$, the $V_{S30}$ approaches $V_{ref}$ (i.e., $V_{ref} = 500$ m/sec for these periods and $AF = 1.0$ for $V_{S30} > V_{ref}$) such that the $Z_{1.0}$ effect is not significant.

<table>
<thead>
<tr>
<th>Table 6.3</th>
<th>Regression coefficients of the site amplification model for long periods.</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (sec)</td>
<td>1.0</td>
</tr>
<tr>
<td>$a_1$</td>
<td>-0.62</td>
</tr>
<tr>
<td>$Z^*$ (m)</td>
<td>121</td>
</tr>
<tr>
<td>$b$</td>
<td>0.70</td>
</tr>
<tr>
<td>$b_{o1}$</td>
<td>-1.6</td>
</tr>
<tr>
<td>$b_{o2}$</td>
<td>0.06</td>
</tr>
<tr>
<td>$V_1$ (m/sec)</td>
<td>114</td>
</tr>
<tr>
<td>$V_2$ (m/sec)</td>
<td>387</td>
</tr>
<tr>
<td>$c$</td>
<td>0.2</td>
</tr>
<tr>
<td>$V_{ref}$ (m/sec)</td>
<td>850</td>
</tr>
<tr>
<td>$σ_{lnAF}$</td>
<td>0.32</td>
</tr>
</tbody>
</table>
Figure 6.8  Model residuals for $T = 1.0 \text{ sec}$ versus $V_{S30}$, $S_{a,\text{rock}}$, and $Z_{1.0}$. 
Figure 6.9 Amplification factor for $T = 1.0$ sec versus $S_{a,rock}$ and model predictions for different values of $Z_{1.0}$. 
Figure 6.10 Amplification factor for $T = 2.0$ sec versus $S_{a,\text{rock}}$ and model predictions for different values of $Z_{1.0}$. 
Figure 6.11 Amplification factor for $T = 5.0$ sec versus $S_{a,\text{rock}}$ and model predictions for different values of $Z_{1.0}$. 
Figure 6.12 Amplification factor versus $S_{a,\text{rock}}$ and model predictions for different values of $Z_{1.0}$ for $T = 10.0$ sec.
6.4 MODEL PREDICTIONS

Two sites with different $V_{S30}$ values (250 m/sec and 350 m/sec) and three input motion intensities were considered to demonstrate the site amplification value predicted by the developed model and the resulting surface response spectra. To show the influence of $V_{ratio}$ on the prediction of amplification, the amplification for each site was predicted for three values of $V_{ratio}$ (i.e., $V_{ratio} = 0.85$, 1.4, and 2.3), and their amplifications were compared. Both $V_{S30}$ values represent medium stiff soil sites and the $Z_{1.0}$ values assigned to these profiles are consistent with the $V_{S30}$ ($Z_{1.0} = 300$ m for $V_{S30} = 250$ m/sec, $Z_{1.0} = 100$ m for $V_{S30} = 350$ m/sec).

The response of the soil profiles were predicted for three different levels of shaking; low input intensity (i.e., $PGA_{rock} = 0.06g$), medium input intensity (i.e., $PGA_{rock} = 0.13g$), and high input intensity (i.e., $PGA_{rock} = 0.3g$). These input motions were derived using the average of the predictions from the Next Generation Attenuation (NGA) GMPEs for $M_w = 7.0$, $V_{S30} = 1000$ m/sec, and distances of 5, 20, and 50 km, respectively. The resulting input rock response spectra are shown in Figure 6.13.

The AF values predicted by the developed site amplification model for the selected scenarios and for $V_{ratio}$ equal to 0.85, 1.4, and 2.3 are shown in Figure 6.14. The different values of $V_{ratio}$ influence amplification at periods less than 1.0 sec; this difference can be significant at high input intensities. Here, an increase in $V_{ratio}$ from 0.85 to 2.3 can increase the amplification by as much as a factor of 2.0. The resulting soil surface response spectra obtained from applying the AFs from Figure 6.14 to the rock response spectra from Figure 6.13 are shown in Figure 6.15. In all cases, the response of the sites shifts the spectrum to longer periods with the peaks occurring at 0.3 to 0.4 sec rather than at 0.2 sec. The influence of different values of $V_{ratio}$ is readily apparent, with the PGA and maximum spectral acceleration significantly larger for larger $V_{ratio}$.

![Figure 6.13](image_url)
Figure 6.14 Model predictions of amplification factor for different values of $V_{ratio}$ and $PGA_{rock}$
Figure 6.15  Model predictions of surface response spectra for different values of $V_{\text{ratio}}$. 
6.5 MODEL LIMITATIONS

The site AFs computed in this study were all based on equivalent linear analysis. The equivalent linear assumption employed in the analysis is often considered invalid at shear strains greater than about 1.0%. At high levels of shaking, softer soil profiles may experience strains significantly larger than 1.0%. This issue introduces some limitations to the model when applying it to softer sites at larger input intensities.

The level of generated shear strain in a soil profile depends on the stiffness of the site and the level of shaking. Softer soil profiles experience larger strains than stiffer soil profiles subjected to the same level of input intensity. Figure 6.16 plots shear strain profiles induced in the softest site considered in this study ($V_{S30} = 118$ m/sec) for four different input intensities. At $PGA_{rock} = 0.1g$, the maximum generated strain in the site is about 0.4%. The maximum induced strain increases as $PGA_{rock}$ increases and exceeds 1% for $PGA_{rock} = 0.22g$ and larger. This result indicates that the AF values from this site are not realistic for $PGA_{rock}$ greater than 0.22g.

The $PGA_{rock}$ that first induces maximum shear strains greater than 1% is called $PGA_{rock-1%}$. This value is compiled for the profiles analyzed and used to identify the limitations of the developed model. Figure 6.17 plots $PGA_{rock-1%}$ versus $V_{S30}$ for the soil profiles analyzed in this study. For soil profiles with $V_{S30}$ less than 200 m/sec, the maximum shear strain exceeds 1% at $PGA_{rock} = 0.22g$. The maximum shear strain exceeds 1% at larger input intensities for stiffer sites (e.g., $PGA_{rock-1%} \sim 0.4g$ for $V_{S30} = 300$ m/sec, $PGA_{rock-1%} \sim 1.0g$ for $V_{S30} = 400$ m/sec). For soil profiles with $V_{S30}$ greater than 500 m/sec, the maximum induced shear strain does not exceed 1% for the sites and input motions analyzed in this study. Figure 6.17 shows the ranges of $V_{S30}$ and $PGA_{rock}$ for which the developed model is appropriate. For stiff sites with $V_{S30} > 500$ m/sec, the developed model can be used at all levels of shaking.

**Figure 6.16** Induced shear strain profile in a soft site ($V_{S30} = 118$ m/sec) for different levels of shaking.
The developed model is NOT applicable

The developed model is applicable

Figure 6.17  Relationship between $\text{PGA}_{\text{rock-1\%}}$ and $V_{S30}$ for the analyses performed.
7 Summary and Conclusions

The study presented herein developed an improved site amplification model that considers the effect of multiple site parameters that affect site amplification. This model includes parameters previously considered in GMPEs (e.g., $V_{S30}$ and $Z_{1.0}$), but also identifies an additional parameter that influences site amplification ($V_{ratio}$).

To identify the appropriate site parameters to be included in the site amplification model, 99 soil profiles were generated manually using five baseline velocity profiles. The top 30 m of each baseline shear wave velocity profile was modified to maintain the same $V_{S30}$ but to simulate a different $V_S$ structure. The seismic responses of the generated profiles were analyzed using the equivalent linear approach as implemented in the site response program Strata [Kottke and Rathje 2008]. Different site parameters such as $V_{min}$, $\theta V_{min}$, MAXIR, and $V_{ratio}$ were considered to explain the variability in AF across sites with a common $V_{S30}$. These analyses identified $V_{ratio}$ as the site parameter that influences site amplification most significantly, helping to explain the variability in amplification across sites with the same $V_{S30}$.

To generalize the findings from the analyses in which only the top 30 m of the velocity profile were varied, a suite of fully randomized velocity profiles were generated and their responses analyzed using Strata for a range of input motion intensities. The results of the site response analyses conducted on these 400 fully randomized velocity profiles confirmed the influence of $V_{ratio}$ on site amplification. The computed AFs were used to develop a functional form that incorporates its influence on both the linear-elastic and nonlinear components of the site amplification model. The computed site AFs under low input intensity ($PGA_{rock} = 0.01 \text{g}$) were used to develop the linear-elastic component of the model, and the AFs from higher input intensities were used to develop the nonlinear component.

The effect of $V_{ratio}$ on site amplification is significant only at shorter periods ($T < 1.0 \text{ sec}$). Therefore, different models were developed for shorter ($T < 1.0 \text{ sec}$) and longer ($T \geq 1.0 \text{ sec}$) periods. At short periods, the linear-elastic component of the model uses a second-order polynomial functional form for modeling the effect of $\ln(V_{S30})$ on $\ln(AF)$ rather than the linear relationship used in other site amplification models. The second-order polynomial better fits the AF data at low intensities, particularly at smaller values of $V_{S30}$. The linear-elastic model incorporates the effects of $V_{ratio}$ through a linear relationship between $\ln(V_{ratio})$ and $\ln(AF)$. The slope of this relationship is $V_{S30}$ dependent. At longer periods, the model uses a linear relationship between $\ln(V_{S30})$ and $\ln(AF)$ rather than a second-order polynomial, and no $V_{ratio}$
effect is modeled. Additionally, at long periods site amplification is affected by the depth to bedrock (i.e., \(Z_{1.0}\)) in addition to \(V_{S30}\). The effect of \(Z_{1.0}\) is considered in the linear component of the proposed model because it is not influenced by soil nonlinearity.

The nonlinear component of the model incorporates the effect of input intensity on the \(V_{S30}\) and \(V_{\text{ratio}}\) scaling. The effect of input intensity is coupled with the \(V_{S30}\) scaling by making \(\ln(AF)\) a function of \(S_{a,\text{rock}}\) and \(V_{S30}\). The effect of input intensity on the \(V_{\text{ratio}}\) scaling is incorporated by making the relationship between \(\ln(AF)\) and \(\ln(V_{\text{ratio}})\) a function of \(S_{a,\text{rock}}\).

The resulting amplification model accounts for the influence of \(V_{S30}\), \(V_{\text{ratio}}\), \(S_{a,\text{rock}}\), and \(Z_{1.0}\). Generally, larger amplification is predicted for sites with smaller \(V_{S30}\), larger \(V_{\text{ratio}}\), and larger \(Z_{1.0}\). For sites with the same average shear wave velocity in top 30 m (\(V_{S30}\)), a larger value of \(V_{\text{ratio}}\) indicates a larger change in \(V_S\) over the top 30 m; these sites will experience larger site amplification.

A limitation of the developed model is that it is based on AFs computed from equivalent-linear site response analysis. Soft soil profiles under high levels of shaking may experience strains significantly large enough (i.e., greater than about 1%) to make the equivalent-linear assumption invalid. The shaking level above which the maximum generated shear strain in a profile exceeds 1% is \(V_{S30}\) dependent. The PGA_{rock} at which shear strains exceeded 1% in the analyses was identified and used to define the limitations on PGA_{rock} for the developed model as a function of \(V_{S30}\).
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