Behavior and Modeling of Existing Reinforced Concrete Columns

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Questions?

- What is the **stiffness** of the column?
- What is the **strength** of the column?
- What **failure mode** is expected?
- What is the **drift capacity**...
  - at shear failure?
  - at axial failure?
- How can we account for **uncertainty** in the models?
- How can we **model** this behavior for the analysis of a structure?
- What is the influence of poor **lap splices**?
Column response

Effective Stiffness

Strength

Drift Capacities
Effective stiffness

\[ \frac{I_{eff}}{I_g} \]

- Column tests
- Moment curvature

FEMA 356
Effective stiffness

Flexural Deformations:

\[ \Delta_{flex} = \frac{L^2}{6} \phi_y \]

Does not account for end rotations due to bar slip!
Effective stiffness

**Slip Deformations:**

\[ \text{Slip Deformations:} \]

\[ L \]

\[ C \]

\[ T = A_s f_y \]

\[ u \]

\[ \delta_{\text{slip}} = \frac{1}{2} \varepsilon_y l \]

\[ \Delta_{\text{slip}} = \frac{L d_b f_y \phi_y}{8u} \]

\[ u \approx 6\sqrt{f_c'} - 12\sqrt{f_c'} \]
Effective stiffness

Modeling options:

Option 1 (with slip springs)

Stiffness based on moment-curvature analysis (or FEMA 356 recommendations)

Slip springs:

\[ k_{slip} = \frac{8u}{d_b f_y} \frac{M_y}{\phi_y} \]
Effective stiffness

Modeling options:

Option 2 (without slip springs):

Use effective stiffness determined from column tests. More flexible than FEMA recommendations for low axial load.
Effective stiffness

\[
\frac{I_{\text{eff}}}{I_g} \quad \text{ column tests} \quad \text{flexure and slip} \quad \text{Proposed}
\]

FEMA 356
Yield displacement

FEMA 356 Proposed stiffness
Column response

![Image of a damaged column]

- Effective Stiffness
- Strength
- Drift Capacities

Graph showing shear (kN) vs. drift ratio.
Shear Strength

Several models available to estimate shear strength:

- Aschheim and Moehle (1992)
- Priestley et al. (1994)
- Konwinski et al. (1995)
- FEMA 356 (Sezen and Moehle, 2004)
- ACI 318-05

All models (except ACI) degrade shear strength with increasing ductility demand.
Shear Strength

\[ V_n = V_c + V_s \]
\[ V_c = k \left( \frac{1}{a/d} \right) \left( 6\sqrt{f_c'} \sqrt{1 + \frac{P}{6\sqrt{f_c'A_g}}} \right) 0.8A_g \]
\[ V_s = k \frac{A_{sw}f_yd}{s} \]

- Based on principle tensile stress exceeding \( f_t = 6\sqrt{f_c'} \)

Sezen and Moehle, 2004
Shear Strength

\[ V_n = V_c + V_s \]

\[ V_c = k \left( \frac{1}{a/d} \right) \left( 6\sqrt{f'_c} \sqrt{1 + \frac{P}{6\sqrt{f'_c A_g}}} \right) 0.8A_g \]

\[ V_s = k \frac{A_{sw} f_y d}{s} \]

• Based on principle tensile stress exceeding \( f_t = 6\sqrt{f'_c} \)
• Accounts for degradation due to flexural and bond cracks

Sezen and Moehle, 2004
Shear Strength

\[ V_n = V_c + V_s \]

\[ V_c = k \left( \frac{1}{a/d} \right) \left( 6\sqrt{f_c'} \left[ 1 + \frac{P}{6\sqrt{f_c' A_g}} \right] \right)^{0.8A_g} \]

\[ V_s = k \frac{A_{sw} f_y d}{s} \]

- Based on principle tensile stress exceeding \( f_t = 6\sqrt{f_c'} \)
- Accounts for degradation due to flexural and bond cracks
- Degrades both \( V_s \) and \( V_c \) based on ductility

Sezen and Moehle, 2004
Shear Strength

Sezen and Moehle, 2004
Shear Strength

Priestley et al., 1994

FEMA 273

Sezen and Moehle, 2004
Column response

- Effective Stiffness
- Strength
- Drift Capacities

![Graph showing shear versus drift ratio with labels for effective stiffness, strength, and drift capacities.]
Shear Failure

Shear capacity

Flexural capacity

Large variation in response

Drift or Ductility

Elwood and Moehle, 2005a
Shear Failure

High axial load
Low axial load

Drift or Ductility

Elwood and Moehle, 2005a
Drift at Shear Failure Model

Drift capacity depends on:
- amount of transverse reinforcement
- shear stress
- axial load

\[
\frac{\Delta_s}{L} = 4\rho'' - \frac{1}{500} \sqrt{f_c} - \frac{1}{40} \frac{P}{A_g f_c} + \frac{3}{100} \geq \frac{1}{100}
\]

Elwood and Moehle, 2005a
Drift at Axial Failure Model

**Simplifying Assumptions**

- $V$ assumed to be zero since shear failure has occurred
- Dowel action of longitudinal bars ($V_d$) ignored
- Compression capacity of longitudinal bars ($P_s$) ignored

\[
\sum F_y \to P = N \cos \theta + V_{sf} \sin \theta
\]

\[
\sum F_x \to N \sin \theta = V_{sf} \cos \theta + \frac{A_{st} f_y d_c \tan \theta}{s}
\]

*Classic Shear-Friction* $\to V_{sf} = N \mu$

\[
P = \frac{A_{st} f_y d_c \tan \theta \left( \frac{\cos \theta + \mu \sin \theta}{\sin \theta - \mu \cos \theta} \right)}{s}
\]

Elwood and Moehle, 2005b
Drift at Axial Failure Model

- Relationships combine to give a design chart for determining drift capacities.

\[ P = \frac{A_{st} f_y d_c}{s} \tan \theta \left( \frac{\cos \theta + \mu \sin \theta}{\sin \theta - \mu \cos \theta} \right) \]

\[ \theta \approx 65^o \]

\[ \mu \text{ vs. Drift at axial failure} \]
Drift at Axial Failure Model

\[
\left( \frac{\Delta}{L} \right)_{axial} = \frac{4}{100} \left( \frac{1 + \tan^2 65^\circ \tan 65^\circ + P \left( \frac{s}{A_{st}f_{yt} d_c \tan 65^\circ} \right)}{100} \right)
\]

Elwood and Moehle, 2005b
Drift models for flexural failures

- Flexural strength will degrade for columns with $V_p << V_o$ due to spalling, bar buckling, concrete crushing, etc.

- Several drift models have been developed:
  - Drift at onset of cover spalling:
    \[
    \frac{\Delta_{\text{spall}}}{L} = 1.6\left(1 - \frac{P}{A_g'f'_c}\right)(1 + \frac{L}{10D})
    \] (Berry and Eberhard, 2004)
  
  - Drift at bar-buckling:
    \[
    \frac{\Delta_{bb}}{L} = 3.25\left(1 + k_e\frac{\rho_{\text{vol}}f_{yt}d_b}{f'_cD}\right)(1 - \frac{P}{A_g'f'_c})(1 + \frac{L}{10D})
    \] (Berry and Eberhard, 2005)
  
  - Drift at 20% reduction in flexural capacity:
    \[
    \frac{\Delta_f}{L} = 0.049 + 0.716\rho_l + 0.120\frac{\rho''f_{yt}}{f'_c} - 0.042\frac{s}{d} - 0.070\frac{P}{A_g'f'_c}
    \] (Zhu, 2005)

Note: Values of \(50\) for rectangular columns, \(150\) for spiral-reinforced columns.

How to apply models...

First classify columns based on shear strength:

- $V_p/V_o > 1.0 \rightarrow$ shear failure
- $1.0 \geq V_p/V_o \geq 0.6 \rightarrow$ flexure-shear failure
- $V_p/V_o < 0.6 \rightarrow$ flexure failure

where

$$V_o = \frac{6\sqrt{f'_c}}{a/d} \sqrt{1 + \frac{P}{6\sqrt{f'_c} A_g} 0.8A_g + \frac{A_{st} f_{yt} d}{s}}$$

$$V_p = \frac{2M_p}{L}$$
How to apply models...

**Shear failure**
- Force-controlled
- Define drift at shear failure using effective stiffness and $V_o$.
  - May be conservative if $V_p \approx V_o$
- Use drift at axial failure model to estimate $\Delta_a/L$
  - Very little data available for drift at axial failure for this failure mode, but model provides conservative estimate in most cases.
- Do not use as primary component if $V > V_o$
How to apply models...

- **Flexure-Shear failure**
  - Deformation-controlled
  - Max shear controlled by $2M_p/L$
  - Use drift at shear failure model to estimate $\Delta_s/L$
  - Use drift at axial failure model to estimate $\Delta_a/L$
  - Do not use as primary component if drift demand $> \Delta_s/L$

FEMA 356 limit for primary component failing in shear.
How to apply models...

**Flexure failure**
- Deformation-controlled
- Max shear controlled by $2M_p/L$
- Use model for drift at 20% reduction in flexural capacity to estimate $\Delta_f/L$
- Do not use as primary component if drift demand $> \Delta_f/L$
- For low axial loads, axial failure not expected prior to P-delta collapse.
- For axial loads above the balance point and light transverse reinforcement, column may collapse after spalling of cover.

\[
\frac{12EI_{eff}}{L^3}
\]

\[
\Delta_f/L
\]
Points to remember

- Models provide an estimate of the mean response.
- 50% of columns may fail at drifts less than predicted by the models.

<table>
<thead>
<tr>
<th>Drift Model</th>
<th>Mean $\Delta_{\text{meas}}/\Delta_{\text{calc}}$</th>
<th>CoV $\Delta_{\text{meas}}/\Delta_{\text{calc}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear Failure</td>
<td>0.97</td>
<td>0.34</td>
</tr>
<tr>
<td>Axial Failure</td>
<td>1.01</td>
<td>0.39</td>
</tr>
<tr>
<td>Flexural Failure</td>
<td>1.03</td>
<td>0.27</td>
</tr>
<tr>
<td>Spalling</td>
<td>0.97</td>
<td>0.43</td>
</tr>
<tr>
<td>Bar Buckling</td>
<td>1.00</td>
<td>0.26</td>
</tr>
</tbody>
</table>
Points to remember

- Shear and axial failure models based on database of columns experiencing:
  - flexure-shear failures
  - uni-directional lateral loads

- All models except bar buckling and spalling only based on database of rectangular columns.

- Use caution when applying outside the range of test data used to develop the models!

- Shear and axial failure models are not coupled.
  - If calculated drift at axial failure is less than the calculated drift at shear failure, assume axial failure occurs immediately after shear failure.
Application of Drift Models – Shake Table Tests

**Characteristics**
- Half-scale, three column planar frame
- Specimen 1:
  - Low axial load
    \[ P = 0.10f'_c A_g \]
- Specimen 2:
  - Moderate axial load
    \[ P = 0.24f'_c A_g \]

**Objective**
- To observe the process of dynamic shear and axial load failures when an alternative load path is provided for load redistribution
Specimen #1 – Low Axial Load

Top of Column - Total Displ.

Center Column - Relative Displ.

Center Column Hysteresis

Full Frame - Total Displ.
Specimen #2 – Moderate Axial Load

Top of Column - Total Displ.  Center Column Hysteresis

Center Column - Relative Displ.  Full Frame - Total Displ.
Axial Load Comparison

Low Axial Load (Spec 1)  Moderate Axial Load (Spec 2)

Center Column Axial Load Time History
Application of Drift Models – Shake Table Tests

$P = 0.10 f'_c A_g$

$P = 0.24 f'_c A_g$

Elwood and Moehle, 2003
Application of Drift Models – Van Nuys, Holiday Inn

- 7-story reinforced concrete frame building (1965)
- Damaged during San Fernando and Northridge Earthquakes

Did columns sustain axial load failures?
Application of Drift Models – Van Nuys, Holiday Inn

**Column C4 Properties:**

- $f_{y\,\text{long}} = 496$ MPa
- $f_{y\,\text{trans}} = 345$ MPa
- $f'_c = 28$ MPa
- $P = 0.13 A_g f'_c$
- $L = 2.08$ m

Direction of shear failure

Section A-A
Application of Drift Models – Van Nuys, Holiday Inn

Fourth story column: Max. interstory drift $\geq 0.018$

Interpolated from recorded motions on third and sixth floors → Lower Bound

Axial load failure expected to follow rapidly after shear failure.

Elwood and Moehle, 2004
Need for probabilistic model

\[
\frac{\Delta_s}{L} = \frac{3}{100} + 4\rho - \frac{1}{500} \sqrt{f_c'} - \frac{1}{40} \frac{P}{A_g f_c'}
\]

Probability of shear failure

\[f(\Delta_s/L)\]

\[\Delta_{dem}/L, \Delta_s/L\]

Ok?
Probabilistic Drift Capacity Models

*Median prediction of drift at shear failure:*

\[
\left( \frac{\Delta_s}{L} \right)_{\text{median}} = 2.02 \rho'' - 0.025 \frac{s}{d} + 0.013 \frac{a}{d} - 0.031 \frac{P}{A_g f_c'}
\]

*Median prediction of drift at flexural failure:*

\[
\left( \frac{\Delta_f}{L} \right)_{\text{median}} = 0.049 + 0.716 \rho_f + 0.120 \frac{\rho'' f_{yt}}{f_c'} - 0.042 \frac{s}{d} - 0.070 \frac{P}{A_g f_c'}
\]

*Median prediction of drift at axial failure:*

\[
\left( \frac{\Delta_a}{L} \right)_{\text{median}} = 0.184 \exp\left( -1.45 \mu \right)
\]

\[
\mu = \frac{P}{A_{st} f_y d_c / s} - 1 - \frac{1}{\frac{P}{A_{st} f_y d_c / s} \tan \theta + \tan \theta}
\]

Now have distributions on coefficients, capturing uncertainty in model!!

Zhu, 2005
Fragility curves – Shear Failure

Drift limit = 0.64 \times \left( \frac{\Delta_s}{L} \right)_{median}

Zhu, 2005
Fragility curves – Flexural Failure

Drift limit = $0.72 \times \left( \frac{\Delta f}{L} \right)_{\text{median}}$

Zhu, 2005
Fragility curves – Axial Failure

\[
\text{Drift limit} = 0.64 \times \left( \frac{\Delta_a}{L} \right)_{\text{median}}
\]

Median - \( \sigma \)

Zhu, 2005
Probabilistic model for drift at axial failure

Zhu, 2005
Application of Probabilistic Drift Capacity Model

- The probabilistic drift capacity model is used to develop a fragility estimate for column from Van Nuys, Holiday Inn.

Confidence bounds:
Based on model (epistemic) uncertainty

85% confident that \( P_f \leq 6.1\%

15% confidence bound

85% confidence bound

Zhu et al., 2006
Application of Probabilistic Drift Capacity Model

The relation between the fragility curves of shear failure and axial failure gives useful information regarding the column axial load capacity after shear failure.

Zhu, 2005
Assessment of FEMA 356

- Probabilistic models can be used to assess the probability of failure implied by drift limits in FEMA 356.
  - “Shear-controlled” response
  - $\Delta_s/L$ model compared with LS criteria for secondary components
  - $\Delta_a/L$ model compared with CP criteria for secondary components

$P_f = 2.6\%$ at LS limit

$P_f = 0.65\%$ at CP limit

Zhu, 2005
Level of safety provided is not consistent for all columns.

Is this level of safety appropriate?

Zhu, 2005
Analytical Model for Shear-Critical Columns

Zero-length Axial Spring

Zero-length Shear Spring

Beam-Column Element (including flexural and slip deformations)

\[ P \rightarrow V \]

Shear-failure model

Axial-failure model

\[ \delta_s \]

\[ \delta_a \]

Elwood, 2004
Analytical Model for Shear-Critical Columns

Elwood, 2004
Benchmark Shake Table Tests

NCREE, Taiwan, 2005

E-Defence, Japan, 2006

PEER, 2006
NCREE Shake Table Test 1
NCREE Shake Table Test 2
Analysis using OpenSees

Graphs showing drift ratio and vertical displacement for columns C and D with and without collapse.
Lap Splice Failures
Effect of poor lap splices

- bar yield
- bar pullout

Diagram showing:
- $P$ vs $\Delta_{\text{bar}}$
- $\Delta_{\text{slip}}$
- $M$
- $\theta$

$\Delta_{\text{bar}}$
Melek and Wallace (2004)

- **Six Full-Scale Specimens**
  - 18 in. square column
  - 8 – #8 longitudinal bars
  - #3 ties with 90° hooks
  - 20d_b lap splice

- **Tested with Lateral and Axial Load**
  - Lateral Load
    - Standard Loading History
    - Near Field Loading History
  - Axial Load
    - Constant
Test Matrix

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Splice</th>
<th>Axial Load</th>
<th>Shear</th>
<th>Load History</th>
<th>Header Spacing</th>
<th>Column Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>S10MB</td>
<td>YES</td>
<td>10</td>
<td>1.57</td>
<td>SOD</td>
<td>12</td>
<td>F' - F&quot;</td>
</tr>
<tr>
<td>S20MB</td>
<td>YES</td>
<td>20</td>
<td>1.70</td>
<td>SOD</td>
<td>12</td>
<td>F' - F&quot;</td>
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<tr>
<td>S30MB</td>
<td>YES</td>
<td>30</td>
<td>1.68</td>
<td>SOD</td>
<td>12</td>
<td>F' - F&quot;</td>
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<tr>
<td>S20HB</td>
<td>YES</td>
<td>10</td>
<td>1.52</td>
<td>SOD</td>
<td>12</td>
<td>F' - F&quot;</td>
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<tr>
<td>S20HUN</td>
<td>YES</td>
<td>20</td>
<td>1.61</td>
<td>NEAR</td>
<td>12</td>
<td>F' - F&quot;</td>
</tr>
<tr>
<td>S30X1</td>
<td>YES</td>
<td>30</td>
<td>1.13</td>
<td>SOD</td>
<td>12</td>
<td>F' - F&quot;</td>
</tr>
</tbody>
</table>

(12-1) ACI-318-99

Melek and Wallace (2004)
## Experimental Results

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Maximum Lateral Load (kips)</th>
<th>Lateral Strength Degradation at</th>
<th>Type of Failure</th>
<th>Applied Axial Load (kips)</th>
<th>Axial Capacity Lost?</th>
</tr>
</thead>
<tbody>
<tr>
<td>S10MI</td>
<td>45.56</td>
<td>1.50% Drift</td>
<td>Bond Det.</td>
<td>120</td>
<td>No</td>
</tr>
<tr>
<td>S20MI</td>
<td>52.49</td>
<td>1.28% Drift</td>
<td>Bond Det.</td>
<td>240</td>
<td>Yes @ 7% Drift</td>
</tr>
<tr>
<td>S30MI</td>
<td>64.14</td>
<td>1.45% Drift</td>
<td>Bond Det.</td>
<td>360</td>
<td>Yes @ 5% Drift</td>
</tr>
<tr>
<td>S20HI</td>
<td>55.53*</td>
<td>1.33% Drift</td>
<td>Bond Det.</td>
<td>240</td>
<td>Yes @ 7% Drift</td>
</tr>
<tr>
<td>S20HIN</td>
<td>55.10*</td>
<td>1.00% Drift</td>
<td>Bond Det.</td>
<td>240</td>
<td>No</td>
</tr>
<tr>
<td>S30XI</td>
<td>63.82*</td>
<td>1.50% Drift</td>
<td>Bond Det.</td>
<td>360</td>
<td>Yes @ 5% Drift</td>
</tr>
</tbody>
</table>

*normalized

Melek and Wallace (2004)
Observed Damage

Specimen: S20MI

1.5% Drift
Splice Deterioration
\(F_{\text{ult}} = 53 \text{ kips}\)

3% Drift

5% Drift

7% Drift
Axial Load
Capacity Lost

Melek and Wallace (2004)
S10MI – S20MI – S30MI

Melek and Wallace (2004)
S20MI – S20HI – S20HIN

Melek and Wallace (2004)
Axial Load Capacity

**S20MI, S20HI, S30MI, S30XI**
- Axial load capacity lost due to buckling of longitudinal bars
- 90° ties at 4” and 16” above pedestal opened up
- Concrete cover lost

**S20HIN – Axial Load Capacity Maintained**
- Near Fault Displacement History (Less cycles)
- Concrete cover intact

Melek and Wallace (2004)
Axial load capacity

Axial capacity model assumes the column has sustained a shear failure.
• Not developed for splice failures and flexurally dominated columns.
FEMA 356 lap splice provisions

- Adjust yield stress based on lap splice length:

- Cho and Pinchiera (2005) evaluated provisions using detailed bar analysis calibrated to test data.
Spring model for anchored bar

- Stress vs. Strain
- Bond Stress
- Slip

$E_{sh}$

$\varepsilon_y$

$f_y$

$f_s$

$\ell_b$

(Cho and Pincheira, 2005)

Harajli (2002)
FEMA 356 lap splice provisions

FEMA 356 under-predicts steel stress.

(Cho and Pincheira, 2005)
Modified Equation
(Cho and Pincheira, 2005)
Modified Equation
(Cho and Pincheira, 2005)
Modified Steel Stress Equation
(Cho and Pincheira, 2005)
References


