Slab – Column Frames

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Presentation Overview

- Current Practice
  - Modeling & analysis
  - Connection design
  - Progressive collapse
  - Deformation compatibility

- Existing Construction
  - Post-earthquake observations
  - Modeling and Model Assessment
  - Backbone curves/Rehabilitation

- Shear reinforcement
Current Practice

- Non-participating or “gravity” system
- Post-tensioned slab-column frame
- Span-to-depth ratios typically ~40+
- Use of shear reinforcement at slab-column connection to allow for thinner slabs or to eliminate drop panels

~1/3 scale shake table test specimen
Shear Reinforcement

Gravity Load Analysis & Design

- ACI 318 Chapter 11, 13, & 21 Materials
  - Slab moments: Use direct design, Equivalent frame, or computer program
  - Connection design – Chapter 11 & 13

\[ w_u = 1.2D + 1.6L \]

- Effective slab width

\[ EI_{column} \]

- \[ EI_{slab} = E_c(\alpha \beta I_2) \]
  Effective slab width
Gravity Load Analysis - Moments

Gravity Analysis: 1.2D + 1.6L

Slab Moments

Unbalanced Moment

Design slab-column connection to transfer unbalanced moment to column

FEMA 356 refers to ACI 318 provisions
Unbalanced Moment Transfer

Unbalanced moment at the slab-column connection is transferred by two mechanisms:

- Moment transfer (flexure) over a transfer width of $c + 3h$ centered on the column
- Eccentric shear on a critical section around the slab-column connection
- Code provisions are covered in Chapter 13 (13.5) and Chapter 11 (11.12) of ACI 318

\[
M_f = \gamma_f M_{unbalanced}
\]

where \[
\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1 / b_2}}
\]

\[b_1, b_2 = \text{widths of critical section defined in 11.12.1.2}\]

\[
M_v = (1 - \gamma_f)M_{unbalanced} = \gamma_v M_{unbalanced}
\]

If $b_1 = b_2$, then:

\[\gamma_f = 0.6 \quad \text{and} \quad \gamma_v = 0.4\]
Unbalanced Moment Transfer

Unbalanced Moment (Interior connection)

Flexural Transfer: \( c_2 + 3h \)

- \( \gamma_f M_{unb} \) where \( \gamma_f \) is typically \( \sim 0.6 \) for square columns
- Ratio of top to bottom reinforcement of 2:1 recommended in ACI 318 (R13.5.3.3)

\[ M_{unb} = M_L + M_R \]

FEMA 356 6.5.4.3(2) allows use of \( c_2 + 5h \)
Unbalanced Moment Transfer

- **Eccentric Shear transfer**
  - Critical section is defined d/2 from column face
  - Direct shear stress
    - \( b_0 = \text{perimeter of critical section} \)
    - \( v_{gravity} = \frac{V_{u(direct)}}{b_0d} \)
  - Eccentric shear stress due to \((1-\gamma_f)M_{unb} = \gamma_v M_{unb}\)
    - \( v_{unb} = \gamma_v \frac{M_{unb}Z}{J} \)
Unbalanced Moment Transfer

- Combined shear stresses
- Check punching failure per 318

\[
(\phi v_n = \phi v_c) \phi v_n \geq v_u \quad \text{where } \phi = 0.75
\]

\[
v_c = \text{Min} \left\{ \frac{\sqrt{f'_c} \left(2 + \frac{4}{\beta_c} \right)}{2 \sqrt{f'_c} \left(2 + \frac{\alpha_s + d}{b_0} \right)}, \frac{4\sqrt{f'_c}}{2 \sqrt{f'_c}} \right\}
\]

ACI Eq. 11-33, -34, -35

Direct shear stress + Eccentric shear stress = Total shear stress

\[
v_{u,\text{max}} \leq \phi v_n
\]
Laboratory Studies

Progressive collapse - continuous bottom steel (2 bars)
ACI 318-05 7.13.2.5 (13.3.8.5)

ACI Committee 352.1R89
Slab – Column Report

Recommendations for the design of slab-column connections in monolithic RC Structures, ACI-ASCE Committee 352, Report 352.1R-89 (reapproved 1997)

Fig. 5.4-Model of connection during punching failure

$$A_{sm} = 0.5 \frac{w_u l_1 l_2}{\phi f_y}$$

Bottom bar at angle of 30 degrees from horizontal
Deformation Compatibility

- Slab – column (gravity) frame assessment
  - Included in the model with the lateral system

\[ w_u = 1.2D + 0.5L \]

\[ EI_{\text{column}} \]

\[ EI_{\text{slab}} = E_c(\alpha \beta I_2) \]
Deformation Compatibility

- Determine if the connection can resist the $V_u$ & $M_{unb}$ without punching failure – Adequate strength. (ACI 318-05 21.11.5)
  - Flexural transfer, eccentric shear stress model
  - Limit analysis approach – for connections with a fuse
  - this does not consider the potential for shear strength degradation.

![Diagram showing deformation compatibility](image)
**Alternative - Deformation Compatibility**

- Verify that punching failures do not occur for gravity shear combined with imposed interstory displacement for $\Delta_M$ (new) or $\delta_{\text{target}}$ (Rehab). Adequate deformability. (ACI 318-05 21.11.5)

- RC interior and exterior (limited data) connections

![Graph showing relationship between drift ratio at punching and gravity shear ratio](image-url)
Deformation Compatibility

- PT Connections without shear reinforcement

Gravity Shear Ratio ($V_g / \phi V_c$), where $V_c = (0.29f'_c^{1/2} + 0.3f_{pc}) b_o d$
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- Shear reinforcement
Older Construction

- Gravity design, or relatively low lateral forces used for design

No continuous bottom reinforcement through column cage

Bent reinforcement sometimes used
Post-Earthquake Observations

Bullock’s Department Store - Northridge Fashion Mall
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❖ Shear reinforcement
Modeling Overview

- How to model...
  - Lateral stiffness?
  - Connection behavior?
- How good are our models?
  - Shake table studies
- FEMA 356 backbone curves
  - Basis of existing curves
  - New information?
Modeling Assumptions - Typical

\[ EI_{\text{eff}} = \text{effective column stiffness} \]

Rigid end zones at joints

\[ EI_{\text{eff}} = \alpha \beta EI_g \]

Model slab with “an effective beam”
Analytical Model - Column Stiffness

- $P_G = \text{axial from gravity}$ and $P_E = \text{axial from earthquake}$
- Anchorage slip – not likely as significant as noted for beam – column frames (see Elwood presentation)
Analytical Model – Slab Flexural Stiffness

Effective Beam Width Model

\[ \alpha : \text{Effective Beam Width Factor} \]
\[ \beta : \text{Coefficient accounting for Cracking} \]

Kang & Wallace (2005)

<table>
<thead>
<tr>
<th></th>
<th>RC</th>
<th>PT</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>0.75</td>
<td>0.65</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.33</td>
<td>0.5</td>
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Analytical Modeling - Connections

- Column (fiber element)
- Slab
- Connection (rigid plastic spring)
- Column strip spring

**Munbalanced @connection**
- Flexure $c_2+3h$ (5h)
- Eccentric shear
- $M_n = M_f/0.6$
- $M^-n / M^+_n @ column strip$

Punching before or after yielding


Limit State Model: Mean – 1σ

This model satisfies FEMA 356 6.5.4.2.2, which states that the connection must be modeled separately from slab and column elements.
Connection Modeling - Punching

Gravity Shear Ratio \( \left( \frac{V_g}{\phi V_c} \right) \), where \( V_c = \frac{(1/3)f'_c}{b_o d} \)

Drift Ratio at Punching

- Isolated RC "Interior" Connections
- Subassemblies
- Nine-panel Frame
- Isolated RC "Edge" Connections

ACI 318-05 Limit

Relationship for RC with stud-rails (Robertson et al.10)

Best-Fit Line for Interior Connections without Shear Reinforcement

Subassemblies6

- Nine-panel Frame18
- Isolated RC "Edge" Connections9
Shake Table Studies

- Two stories, $2 \times 2$ bays
- Approximately $1/3$ scale

RC Specimen

- Six 200 x 200 mm columns
- 90 mm thick slab
- 9.5 mm rebar $f_y = 414$ MPa
- $f'_c = 28$ MPa

Dimensions:
- 4.3 m
- 4.1 m
RC Specimen - Reinforcement

**Interior Connection**

**Shear Reinforcement**

Expected connection behavior: Flexural yielding, followed by punching
PT Specimen

- 5.7 m
- 5.7 m
- 8 mm 7-wire strand
- 6.35 mm deformed rebar
PT Specimen – Interior

ACI318-05 Requires only bottom (integrity) reinforcement
PT Video – Run 5
Model Assessment - NSP

Top Drift [%]

Base Shear [kN]

RC-RUN4-Exp
Push-over (2:1)
Push-over (1:2)

θ_u = 2.5%

Top displacement relative to footing [mm]
Model Assessment - PT

See Kang et al., 13WCEE, August 2004, paper 1119
Direct measurement of footing rotations
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Deformation – Backbone Curves

<table>
<thead>
<tr>
<th>Slabs Controlled by Flexure</th>
<th>Model Parameters, Radians</th>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{V_{\text{gravity}}}{V_0}$</td>
<td>Continuity Reinforcement</td>
</tr>
<tr>
<td>$\leq 0.2$</td>
<td>Yes</td>
</tr>
<tr>
<td>$&gt; 0.4$</td>
<td>Yes</td>
</tr>
<tr>
<td>$\leq 0.2$</td>
<td>No</td>
</tr>
<tr>
<td>$\geq 0.25$</td>
<td>No</td>
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Continuity reinforcement defined as at least one bottom bar or pt bar continuous through the column cage in each direction.

$V_u = 1.2D + 0.5L$
Slab – Column Tests

Typical test setup
New Data – Test Results #1

- Slab reinforcing details

10ft x 10 ft x 4.5”

2 Continuous bottom bars

Figure 2-Slab Reinforcement for ND1C, ND4LL, and ND5XL.
Test Results #1 – Control Specimen

\[ \frac{V_y}{V_c} = 0.23 \]

\[ M_n = (12)(71 \text{ mm}^2)(414 \text{ MPa})(95\text{mm}) = 33,500 \text{ kN-mm} \]

\[ P_n = 33,500 \text{ kN-mm}/1524\text{mm} = 22 \text{ kN} \]

\[ \theta_e = 0.015; \quad \theta_p = 0.02(17/20) = 0.017 \]

Yield: 1.5% (assumed)
Test Results #1

\[ \frac{V_g}{V_c} = 0.28 \]

\[ M_n = (12)(71 \text{ mm}^2)(414 \text{ MPa})(95\text{ mm}) = 33,500 \text{ kN-mm} \]

\[ P_n = 33,500 \text{ kN-mm}/1524\text{ mm} = 22 \text{ kN} \]

\[ \theta_c = 0.015; \quad \theta_p = 0.02(12/20) = 0.012 \]
$\frac{V_g}{V_c} = 0.48$

$M_n = (12)(71 \text{ mm}^3)(414 \text{ MPa})(95 \text{ mm}) = 33,500 \text{ kN-mm}$

$P_n = 33,500 \text{ kN-mm}/1524 \text{ mm} = 22 \text{ kN}$

$\theta_c = 0.015; \theta_p = 0$
Test Results #1 - Summary

- FEMA 356 – Overall comparison
Test Results - #2

- Slab reinforcing details – less reinforcement

Figure 4-Slab Reinforcement for ND7LR.

10ft x 10 ft x 4.5”

2 Continuous bottom bars

Robertson & Johnson, 13WCEE, August 2004, Paper 143
Test Results

\[ \frac{V_g}{V_c} = 0.36 \]

\[ \begin{align*} M_n &= (7)(71 \text{ mm}^2)(414 \text{ MPa})(95\text{ mm}) = 19,500 \text{ kN-mm} \\
\theta_e &= 0.015; \quad \theta_p = 0.02(4/20) = 0.004 \end{align*} \]
Subassembly Test

- Test specimens ~1/2 scale
- 4 specimens
  - DNY_1, DNY_2, DNY_3, DNY_4 (spandrel beam)
  - Bent-up (1,2,4), Straight (3)
  - \( \frac{V_g}{V_c} = 0.2, 0.3, 0.24, 0.28 \)

Test Results


Spandrel beam
Test Results - Summary

Backbone relation:
\[ P = 10 \text{ kip (arbitrary)} \]
\[ V_g / V_c \approx 0.30 \]
\[ \theta_e \approx 0.01 \quad \theta_p = 0.02 \]

Straight bars vs Bent up bars
- no difference
- except for collapse

Fig. 5—Envelopes of load-drift hysteresis loops for subassemblies
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Recent Tests – ERICO Fortress Steel

0.6 Scale model tests (6” thick slab) – Smith Emery
Preliminary Results – ERICO Steel Fortress

- Fortress steel appears to be very effective in improving the punching resistance
- Appear as effective as stud-rails

\[
A_v = 1.44 \text{ in}^2 \quad f_y = 72.5 \text{ ksi} \quad A_v = 0.88 \text{ in}^2 \quad f_y = 60 \text{ ksi}
\]
Summary

Modeling
- Effective beam width model
- Connection behavior – Limit state model

Backbone curves - RC
- Conservative – In general
- Review allowable plastic rotation for low gravity stress ratios $< 0.2$, mean $- \sigma$
- Potential to increase plastic rotation for low reinforcement ratios
- Remove residual capacity for RC connections
Summary

Backbone curves - PT

- Conservative
- Increase plastic rotation from 0.02 (RC) to 0.03 at gravity shear ratio of 0.2
- Review higher gravity shear ratios – allowable plastic rotation of 0.01 at a gravity shear ratio of 0.5
- Allow residual capacity of 0.2 up to drifts of about 5% where one strand pass within the column cage in both directions.
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