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Uncertainty Specification and Propagation for Loss Estimation Using FOSM Method

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ABSTRACT

The estimation of repair costs in future earthquakes is one component of loss estimation currently being developed for use in performance-based engineering. An important component of this calculation is the estimation of total uncertainty in the result, as a result of the many sources of uncertainty in the calculation. Monte Carlo simulation is a simple approach for estimation of this uncertainty, but it is computationally expensive. The procedure proposed in this report uses the first-order second-moment (FOSM) method to collapse several conditional random variables into a single conditional random variable, total repair cost given *IM* (where *IM* is a measure of the ground motion intensity). Numerical integration is then used to incorporate the ground motion hazard. The ground motion hazard is treated accurately because it is the dominant contributor to total uncertainty. Quantities that can be computed are expected annual loss, variance in annual loss, and the mean annual rate (or probability) of exceeding a given loss.

A general discussion of element-based loss estimation is presented, and a framework for loss estimation is outlined. The method works within the framework proposed by the Pacific Earthquake Engineering Research (PEER) Center

The report makes suggestions for the representation of correlation among the random variables, such as repair costs, where data and information are very limited. Guidelines for the estimation of uncertainty in peak interstory drift given *IM* are also presented. This includes using structural analysis to estimate aleatory uncertainty, and correlations for an example structure. Several studies attempting to characterize epistemic uncertainty are referenced as an aid.

A simple numerical calculation is presented to illustrate the mechanics of the procedure. The results of the example are also used to illustrate the effect of uncertainty on the rate of exceeding a given total cost. This illustrates that uncertainty in total repair cost given *IM* may or may not have a significant effect on the annual rate of exceeding a given cost.

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1 Introduction

The estimation of annual losses in a building due to earthquake damage is a quantity of interest to decision makers, and is a current topic of study in performance-based earthquake engineering. Among the quantities to be determined are the uncertainty in the annual losses and the contribution of each source of uncertainty to the total uncertainty in annual losses. Current efforts in this field consider the ground motion hazard, building response, damage to building elements, element repair costs, and total repair cost as individual random variables, and then propagate uncertainty through each step to find a final result.

One option for propagating this uncertainty is through Monte Carlo simulation (e.g. Porter and Kiremidjian 2001). Although straightforward, it can be very expensive computationally, especially when multiple runs are required to calculate sensitivities and/or when low probabilities are needed. In this paper, we present an alternative solution employing a hybrid of simple numerical integration supplemented by the first-order second-moment, or FOSM, method (e.g., Melchers 1999) applied to the higher dimensional response and loss variables.

Regardless of the propagation method used, it is necessary to represent and estimate what may be a complex variance and covariance structure. This report presents several models which may be adopted for these estimates, and the models are equally useful when using Monte Carlo simulation.

The report is broken into several sections, which may be viewed somewhat independently. They are as listed below:

1. Section 2 reviews a framework used in loss estimation, and describes the elements of this framework.
2. Section 3 presents a procedure for propagating uncertainty through the various random variables. Sections 3.1 through 3.5, 3.8 and 3.9 present the propagation procedure considering aleatory uncertainty only. Section 3.6 and 3.7 describe several

generalizations of the model that have been included for completeness, but are not needed for an understanding of the basic procedure. Sections 3.10 through 3.14 incorporate epistemic uncertainty and the model generalizations into the calculations. Consideration of epistemic uncertainty results in equations of increased length and complexity, but if implemented in a computer program, the computations should not be overwhelming. For a first-time reader, Sections 3.6, 3.7 and 3.10 through 3.14 can be skipped without losing the primary methods of the procedure.

3. Section 4 presents a numerical example to illustrate how the equations of Section 3 would be implemented. It is intended only as a simple illustration, and so the model generalizations are excluded from the example, as is consideration of epistemic uncertainty.
4. Section 5 uses the results from the example in Section 4 to examine the effect of variance in *TC* given *IM* on the final results.
5. During the course of this procedure, there are many times when variances and covariances of random variables need to be estimated. Although the quantification of these second moments is a relatively undeveloped field at present, several useful resources are described in Section 6.

The report is concluded in Section 7, which summarizes the findings and features of this report. Several Appendices are also included to give additional background into ideas presented in the body of the report. It is hoped that the modular format of this report will allow the reader to examine sections of interest, while skipping sections (e.g. Section 2) which may not be new material.

2 The Model Framework

The proposed procedure works within the framework proposed by the Pacific Earthquake Engineering Research (PEER) Center. Familiarity with this general framework is presumed (Cornell and Krawinkler 2000; Krawinkler 2002a). This framework is presented in the form of the following equation:

$$\lambda_{TC}(z) = \int_u \int_v \int_y \int_x G_{TC|\mathbf{DVE}}(z,\mathbf{u}) f_{\mathbf{DVE|DM}}(\mathbf{u},\mathbf{v}) f_{\mathbf{DM|EDP}}(\mathbf{v},\mathbf{y}) f_{\mathbf{EDP}|IM}(\mathbf{y},x) \left| d\lambda_{IM}(x) \right| \tag{2.1}$$

with terms defined as follows:

$\lambda_{TC}(z)$ is the annual rate of exceeding a total repair cost of z, where total repair cost, TC is the decision variable under study.

$G_{TC|\mathbf{DVE}}(z,\mathbf{u})$ is the Complimentary Cumulative Distribution Function (CCDF) of TC, conditioned on the vector of damage values of each element (DVE_j is the damage value of element j)

$f_{\mathbf{DVE|DM}}(\mathbf{u},\mathbf{v})$ is the PDF of the vector of damage values of each element, given the vector of damage states of each element (DM_j is the damage state of element j)

$f_{\mathbf{DM|EDP}}(\mathbf{v},\mathbf{y})$ is the PDF of the vector of (the typically discrete) damage states, given the vector of engineering demand parameters

$f_{\mathbf{EDP}|IM}(\mathbf{y},x)$ is the PDF of the vector of engineering demand parameters, given the intensity measure

$\left| d\lambda_{IM}(x) \right|$ is the absolute value of the derivative of the annual rate of exceeding a given value of the intensity measure (the seismic hazard curve). The absolute value is needed because the derivative is negative.

Elements of this framework are developed further in the following subsections.

2.1 TOTAL REPAIR COST

The assumption in this study is that the total cost of repair to the structure is the sum of the repair costs of all elements in the structure, but this can easily be generalized. A modification to this assumption accounting for collapse cases is presented in Section 3.6.

2.2 ELEMENT DAMAGE VALUES

Means and variances of repair costs for each possible damage state are needed for all element types under consideration. Mean repair costs can be estimated from sources such as R.S. Means Company's published materials on construction cost estimating (R.S. Means Co. 2002). Additional quantification of repair costs is a topic of current research.

2.3 ELEMENT DAMAGE MEASURES

In current research, damage measures are typically not continuous, but a discrete set of damage states (e.g., Porter and Kiremidjian 2001; Aslani and Miranda 2002). We shall adopt this format to facilitate such efforts. Discrete damage states are described by fragility functions, which return the probability of an element exceeding given damage states at a given Engineering Demand Parameter (*EDP)* level. One fragility function is needed for each potential damage state of the element. Typically, fragility functions with cumulative lognormal functional shapes are used. The development of fragility functions for structural and nonstructural components is a topic of current research.

2.4 BUILDING RESPONSE

A probabilistic model is needed for the distribution of *EDPs* for the structure (e.g., the interstory drifts and peak floor accelerations for each floor), conditioned on a level of *IM*. Conditional mean values and variances due to aleatory uncertainty can be determined from analysis techniques such as Incremental Dynamic Analysis (Vamvatsikos and Cornell 2002).

2.5 SITE SEISMIC HAZARD

It is necessary to determine the hazard curve for the predictor *IM* at the location of interest, through either PSHA or seismic hazard maps. This topic has been developed in detail elsewhere (e.g., Kramer 1995), and is not further covered in this study.

2.6 ASSUMPTIONS

Several assumptions are made in this framework, and are described below. They are believed to be consistent with the most advanced current seismic loss estimation efforts. Most can be reduced without formal difficulty.

Markovian dependence is assumed for all distributions in the framework. For example, it is assumed that the distribution of the *DM* vector can be conditioned solely on the *EDP* vector, and that knowledge of the *IM* provides no additional information. In this way, previous conditioning information does not need to be carried forward through all future distributions, reducing complexity. A conditioning variable that contains all necessary conditional information is deemed a "sufficient" descriptor (Luco 2002).

All damage is assumed to occur on an element level. The total cost of damage to the structure is then the sum of the damage cost of each element in the structure. This technique is explained in more detail by Porter and Kiremidjian (2001). This assumption can easily be generalized, if desired. In this paper, the exception to this assumption is when collapse occurs, and repair costs will be a function of the collapse rather than individual element responses. The collapse case is set aside in the initial presentation of the procedure, and then accounted for in a later section.

All relations in the framework are assumed to be scalar functions. For example, the conditional distribution of the Damage Measure of element j is a function of only the ith Engineering Demand Parameter. Or alternatively, $f_{DM_j|\mathbf{EDP}}(v_j, \mathbf{y}) = f_{DM_j|EDP_i}(v_j, y_i)$. Note also that the function is not conditioned on variables from any previous steps because of the Markovian process assumption.

To calculate total uncertainty in our decision variable, it will be necessary to incorporate both epistemic and aleatory uncertainty. These two sources of uncertainty are uncorrelated, allowing their contributions to be calculated separately for simplicity. It is recommended that the variance of *TC* given *IM* be calculated once for the variance due to epistemic uncertainty, and once for the variance due to aleatory uncertainty.

3 Procedure

The procedure outlined makes use of FOSM approximations to calculate the mean and variance of *TC* given *IM*. A distribution $F_{TC|IM}(z,x)$ is then fit to these moments, and integrated numerically or analytically over the hazard curve, i.e., $|d\lambda_{IM}(x)|$ to generate the mean annual rate of exceeding a given repair cost:

$$\lambda_{TC}(z) = \int_{IM} 1 - F_{TC|IM}(z,x)\left|d\lambda_{IM}(x)\right| \tag{3.1}$$

The FOSM approximations used to obtain moments of *TC*|*IM* from **EDP**, **DM** and **DVE** are justified by the assumption that the uncertainty in the *IM* hazard curve is the most significant contributor to variance of the total loss. Therefore, we are retaining the full distribution for *IM* itself, but using the FOSM approximations for all (first and second) moments conditioned on *IM*. In addition, we likely do not have information about the full distributions of some variables (e.g., repair costs), and so using only the first two moments of these distributions does not result in a significant loss of available information.

Note that we are working with natural logarithms of the variables described previously. This allows us to work with sums of terms, rather than products. We revert to a non-log form for the final result.

3.1 SPECIFY $\ln$ EDP | *IM*

The proposed model[1] in this study is $EDP_i \mid IM = H_i(IM)\varepsilon_i(IM)$, where $H_i(IM)$ is the (deterministic) mean value of EDP_i given *IM*, and $\varepsilon_i(IM)$ is a random variable with mean of one, and conditional variance adjusted to model the variance in EDP_i. Then when we use the log form of EDP_i, we have a random variable of the form $\ln\left(EDP_i \mid IM\right) = \ln\left(H_i(IM)\right) + \ln\left(\varepsilon_i(IM)\right)$.

[1] We introduce the random variable notation *X*|*Y*, to denote that the model of *X* is conditioned on *Y*.

Note that the expected value of $\ln(EDP_i \mid IM)$ is $\ln(H_i(IM))$, and that the variance of $\ln(EDP_i \mid IM)$ is equal to the variance of $\ln(\varepsilon_i(IM))$. Both $\ln(H_i(IM))$ and $Var[\ln(\varepsilon_i(IM))]$, as well as the correlations between *EDP*s, can be determined from Incremental Dynamic Analysis. We will need the following information for our calculations[2]:

$E[\ln EDP_i \mid IM]$, denoted $h_i(IM)$ for all EDP_i (3.2)

$Var[\ln EDP_i \mid IM]$, denoted $h^*_i(IM)$ for all EDP_i (3.3)

$\rho(\ln EDP_i, \ln EDP_j \mid IM)$, denoted $\hat{h}_{ij}(IM)$ for all $\{EDP_i, EDP_j\}$ (3.4)

As an example, using nonlinear time history analysis of the Van Nuys building (Lowes 2002), the results needed for these equations are presented in Section 6.2 below. For illustration, plots of the logs of floor interstory drift ratios are shown in Figure 3.1. Note that these results have been conditioned on no collapse—a procedure that will be explained in Section 3.6. They are also presented as the exponential of the estimated natural logs of the results (denoted $\exp(\overline{\ln IDR_i})$), because we want the expected natural log of the drifts.

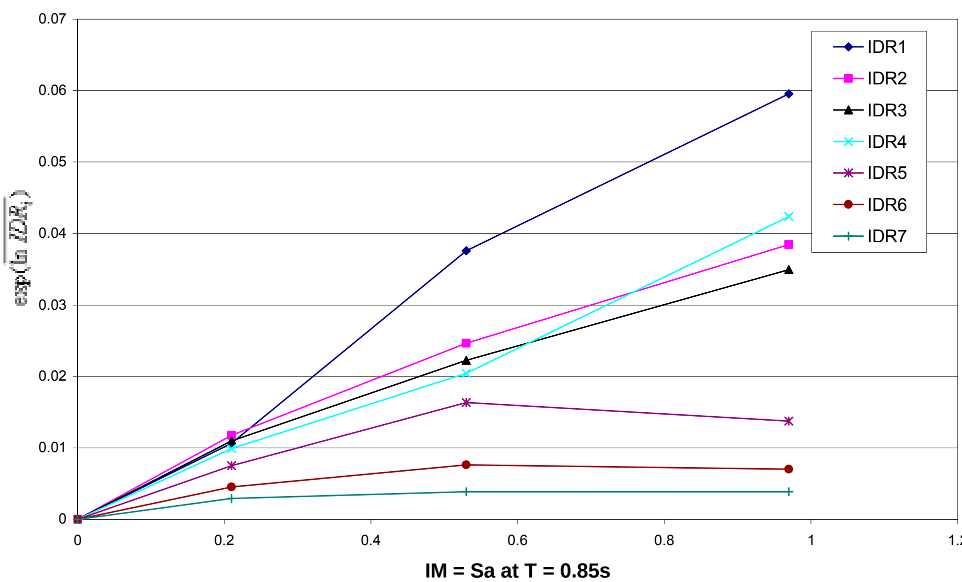

Fig. 3.1: Estimated median of IDR$_i$ ($\exp(\overline{\ln IDR_i})$), conditioned on no collapse and Sa

The data from this figure could be used in Equation 3.2 above.

[2] Note that our model is limited to the first and second moments. The full distribution model is not needed in what follows.

3.2 SPECIFY $\mathbf{DM} \mid \ln \mathbf{EDP}$ AND $\ln \mathbf{DVE} \mid \mathbf{DM}$, AND COLLAPSE TO $\ln \mathbf{DVE} \mid \ln \mathbf{EDP}$

The discrete states of the Damage Measure variable found in current loss estimation (Porter and Kiremidjian, 2001) are not compatible with the FOSM approach, which requires continuous functions for the moments. To deal with the discrete states, we take advantage of the fact that we can always "collapse" the two distributions $\mathbf{DM} \mid \ln \mathbf{EDP}$ and $\ln \mathbf{DVE} \mid \mathbf{DM}$ into one continuous distribution $\ln \mathbf{DVE} \mid \ln \mathbf{EDP}$ by integrating over the appropriate variable. This is illustrated in Figure 3.2 below.

$$\lambda_{TC}(z) = \int_{\mathbf{u}}\int_{\mathbf{v}}\int_{\mathbf{y}}\int_{x} f_{TC|\mathbf{DVE}}(z,\mathbf{u}) f_{\mathbf{DVE|DM}}(\mathbf{u},\mathbf{v}) f_{\mathbf{DM|EDP}}(\mathbf{v},\mathbf{y}) f_{\mathbf{EDP}|IM}(\mathbf{y},x) \left|d\lambda_{IM}(x)\right| \tag{3.5}$$

$$\lambda_{TC}(z) = \int_{\mathbf{u}}\int_{\mathbf{y}}\int_{x} f_{TC|\mathbf{DVE}}(z,\mathbf{u}) f_{\mathbf{DVE|EDP}}(\mathbf{u},\mathbf{y}) f_{\mathbf{EDP}|IM}(\mathbf{y},x) d\lambda_{IM}(x) \tag{3.6}$$

because $f_{\mathbf{DVE|EDP}}(\mathbf{u},\mathbf{y}) = \int_{\mathbf{v}} f_{\mathbf{DVE|DM}}(\mathbf{u},\mathbf{v}) f_{\mathbf{DM|EDP}}(\mathbf{v},\mathbf{y})$ (3.7)

Fig. 3.2: Collapsing out *DM*

For a given element with n possible damage states, we use a set of element fragility functions F_1, F_2 … F_n, such that $F_i(y) = P(DM > d_i \mid EDP = y)$ (Fig. 3.3). These functions will have a corresponding set of distributions c_1, c_2 … c_n of element repair costs such that $c_i(v)$ is a probability distribution of DVE, given that the damage state equals d_i (Fig. 3.4). With this information, we can determine the first two moments of the collapsed distributions. Miranda et al. (2002) documents the development of one set of these functions.

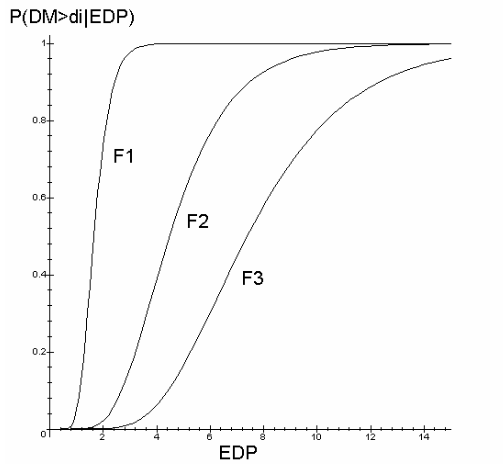

Fig. 3.3: Element Fragility Functions

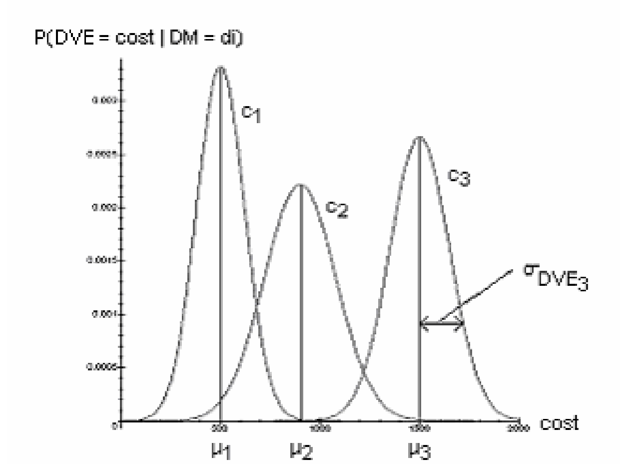

Fig. 3.4: Element Repair Costs

From the total probability theorem, we know that in this case, Equation 3.7 can be written in scalar form for each DVE as $f_{DVE|EDP} = \sum_{\text{Damage States}} f_{DVE|DM=d_i} P_{DM=d_i|EDP}$ (recall by assumption that each DVE is dependent on a single EDP). For our FOSM purposes, furthermore, it is sufficient to find simply the conditional means, variances, and covariances of the DVEs given the EDPs. Thus, taking the mean of this PDF, we have the result:

$$E[DVE \mid EDP] = \sum_{\substack{Damage\\States}} E[DVE \mid DM = d_i] P(DM = d_i) \tag{3.8}$$

$$= \sum_{\substack{Damage\\States}} \mu_i \left(F_i(EDP) - F_{i+1}(EDP) \right) \equiv \bar{\mu} \tag{3.9}$$

Applying the same thinking to $E[DVE^2 \mid EDP]$, and recognizing that $Var[X] = E[X^2] - \mu_X{}^2$, we have the following result (see Appendix B for derivation):

$$\sigma_{DVE|EDP}{}^2 = E_{DM}[Var_{DVE}[DVE \mid DM]] + Var_{DM}[E_{DVE}[DVE \mid DM]] \tag{3.10}$$

$$\begin{aligned} &= \sum_{\substack{Damage\\States}} \sigma^2_{DVE_i} ((F_i(EDP) - F_{i+1}(EDP)) \\ &\quad + \sum_{\substack{Damage\\States}} (\mu_i - \bar{\mu})^2 (F_i(EDP) - F_{i+1}(EDP)) \end{aligned} \tag{3.11}$$

Figure 3.5 shows an example of the mean and mean plus or minus one sigma, as generated from the example distributions shown in Fig. 3.3 and 3.4.

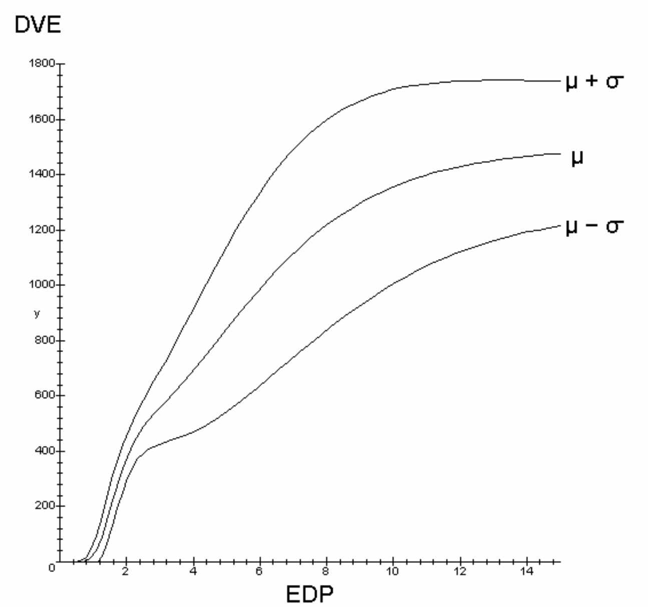

Fig. 3.5: Collapsed distribution *DVE* | *EDP*

Finally, to complete this collapsing exercise, we need to determine correlations among the *DVE*s of all elements in the structure. Note that like the mean and variance, these correlations are conditioned on the *EDP*s. While these calculations are straightforward, estimation of the necessary correlation inputs is a difficult task due to a lack of data. In the absence of additional information, it may be helpful to use the following characterization scheme. Let us assume for this purpose a model of the form: $\ln DVE_k \mid \ln EDP_i = g_k(\ln EDP_i) + \ln \varepsilon_{Struc} + \ln \varepsilon_{ElClass_m} + \ln \varepsilon_{El_k}$, where ε_{Struc} represents uncertainty common to the entire structure, $\varepsilon_{ElClass_m}$ represents uncertainty common only to elements of class "*m*" (e.g., drywall partitions, moment connections, etc.), and ε_{El_k} represents uncertainty unique to element *k*. All of these *ε*'s are assumed to be mutually uncorrelated. We then define $Var[\ln \varepsilon_{Struc} \mid \ln EDP_i] = \beta_{Struc}{}^2$, $Var[\ln \varepsilon_{ElClass_m} \mid \ln EDP_i] = \beta_{ElClass}{}^2$ for all *m*, and $Var[\ln \varepsilon_{El_k} \mid \ln EDP_i] = \beta_{El}{}^2$ for all *k*. Then the variance of $\ln DVE_k \mid \ln EDP_i$ is the sum of these variances. For this special case, a simple closed-form solution exists for the correlation coefficient (see Appendix A, Equations A.9 and A.12). Loosely speaking, the correlation coefficient between two *DVE*'s can be said to be the ratio of their shared variances to their total variance. The following examples are illustrative:

Example 1: correlation between two elements, DVE_k, DVE_l, if both elements are of class 1

$$\ln DVE_k \mid \ln EDP_i = g_k(\ln EDP_i) + \ln \varepsilon_{Struc} + \ln \varepsilon_{ElClass_1} + \ln \varepsilon_{El_k} \tag{3.12}$$

Shared Terms

$$\ln DVE_l \mid \ln EDP_j = g_l(\ln EDP_j) + \ln \varepsilon_{Struc} + \ln \varepsilon_{ElClass_1} + \ln \varepsilon_{El_l} \tag{3.13}$$

Then (see Appendix A, Equation A.9):

$$\rho(\ln DVE_k, \ln DVE_l \mid \ln EDP_i, \ln EDP_j) = \frac{\beta_{Struc}{}^2 + \beta_{ElClass}{}^2}{\beta_{Struc}{}^2 + \beta_{ElClass}{}^2 + \beta_{El}{}^2} \tag{3.14}$$

Example 2: correlation between two elements, DVE_k, DVE_l, if DVE_k is of element class 1, and DVE_l is of element class 2

$$\ln DVE_k \mid \ln EDP_i = g_k(\ln EDP_i) + \ln \varepsilon_{Struc} + \ln \varepsilon_{ElClass_1} + \ln \varepsilon_{El_k} \tag{3.15}$$

Shared Term

$$\ln DVE_l \mid \ln EDP_j = g_l(\ln EDP_j) + \ln \varepsilon_{Struc} + \ln \varepsilon_{ElClass_2} + \ln \varepsilon_{El_l} \tag{3.16}$$

Then (see Appendix A, Equation A.12):

$$\rho(\ln DVE_k, \ln DVE_l \mid \ln EDP_i, \ln EDP_j) = \frac{\beta_{Struc}{}^2}{\beta_{Struc}{}^2 + \beta_{ElClass}{}^2 + \beta_{El}{}^2} \tag{3.17}$$

Note that this formulation requires $\beta_{ElClass}{}^2$ to be equal for all element classes, and $\beta_{El}{}^2$ to be equal for all elements. If this is excessively limiting, a closed-form solution also exists that allows $\beta_{ElClass}{}^2$ to vary by class, and $\beta_{El}{}^2$ to vary by element, and be functionally dependent on the *EDP* value. This solution is also outlined in Appendix A. Note also that this model can be expanded to more than three ε terms if desired. The use of more than two uncertain terms, and the use of β^2 terms that vary by class or element are both generalizations of the basic equi-correlated model, and thus we will refer to a model incorporating any of these generalizations as a *generalized equi-correlated model*. The correlation matrix for a generalized equi-correlated model will have off-diagonal terms that vary from term to term, as opposed to the strict equi-correlated model, where all off-diagonal terms are identical.

We now have the conditional mean and variance functions of $\ln DVE_k \mid \ln EDP_i$, obtained by collapsing the distributions provided (Equations 3.9 and 3.11), and correlation coefficients determined using the generalized equi-correlated model (Equations 3.14 and 3.17). We choose for future notational clarity to denote these results as:

$E[\ln DVE_k \mid \ln EDP_i]$, denoted $g_k(\ln EDP_i)$ for all DVE_k (3.18)

$Var[\ln DVE_k \mid \ln EDP_i]$, denoted $g^*_k(\ln EDP_i)$ for all DVE_k (3.19)

$\rho(\ln DVE_k, \ln DVE_l \mid \ln EDP_i, \ln EDP_j)$, denoted $\hat{g}_{kl}(\ln EDP_i, \ln EDP_j)$ for all $\{DVE_k, DVE_l\}$ (3.20)

3.3 CALCULATE $\ln \mathbf{DVE} \mid IM$

Using information from above, we can calculate the first and second moments of $\ln \mathbf{DVE} \mid IM$. This involves collapsing out the dependence on $\mathbf{EDP}$, as suggested in Figure 3.6 below.

$$\lambda_{TC}(z) = \int_{\mathbf{u}}\int_{\mathbf{y}}\int_{x} f_{TC|\mathbf{DVE}}(z,\mathbf{u}) f_{\mathbf{DVE}|\mathbf{EDP}}(\mathbf{u},\mathbf{y}) f_{\mathbf{EDP}|IM}(\mathbf{y},x)\, d\lambda_{IM}(x) \tag{3.21}$$

$$\lambda_{TC}(z) = \int_{\mathbf{u}}\int_{x} f_{TC|\mathbf{DVE}}(z,\mathbf{u}) f_{\mathbf{DVE}|IM}(\mathbf{u},x)\, d\lambda_{IM}(x) \tag{3.22}$$

because $f_{\mathbf{DVE}|IM}(\mathbf{u},x) = \int_{\mathbf{y}} f_{\mathbf{DVE}|\mathbf{EDP}}(\mathbf{u},\mathbf{y}) f_{\mathbf{EDP}|IM}(\mathbf{y},x)$ (3.23)

Fig. 3.6: Collapsing out EDP

To maintain tractability, we shall do this in an approximate way referred to in structural reliability literature as first-order second-moment, or FOSM. To remove dependence on $\mathbf{EDP}$, we take the expectation of $\ln \mathbf{DVE}$ with respect to $\ln \mathbf{EDP}$ (given *IM*). We write this as $E[\ln DVE_k \mid IM] = E_{EDP_i|IM}\left[E[\ln DVE_k \mid \ln EDP_i]\right]$, where $E_{EDP_i|IM}[\cdot]$ denotes this particular conditional expectation operator. Substituting our notation from Equation 3.18, we have

$$E[\ln DVE_k \mid IM] = E_{EDP_i|IM}\left[g_k(\ln EDP_i)\right] \tag{3.24}$$

Taking a Taylor expansion of $g_k(\ln EDP_i)$ about $E[\ln EDP_i]$ gives:

$$\begin{aligned} E[\ln DVE_k \mid IM] = E_{EDP_i|IM}\Big[& g_k(E[\ln EDP_i]) \\ & + \left.\frac{\partial g_k(\ln EDP_i)}{\partial \ln EDP_i}\right|_{E[\ln EDP_i]} (\ln EDP - E[\ln EDP_i]) + \ldots \ \Big] \end{aligned} \tag{3.25}$$

The first-order approximation of this series is thus:

$$
\begin{aligned}
E[\ln DVE_k \mid IM] &\approx E_{EDP_i|IM}\left[g_k\left(E[\ln EDP_i]\right)\right] \\
&\quad + E_{EDP_i|IM}\left[\left.\frac{\partial g_k\left(\ln EDP_i\right)}{\partial \ln EDP_i}\right|_{E[\ln EDP_i]}\left(\ln EDP_i - E[\ln EDP_i]\right)\right] \\
&= g_k\left(E[\ln EDP_i]\right) + 0 \\
&= g_k\left(h_i(IM)\right)
\end{aligned}
\tag{3.26}
$$

where the substitution $h_i(IM) = E[\ln EDP_i \mid IM]$ has been made, as defined in Equation 3.2. Using a similar approach to conditional moments (Equation B.7, Appendix B), it can be shown that:

$$
\begin{aligned}
Var[\ln DVE_k \mid IM] &= E_{\ln EDP_i|IM}[Var_{\ln DVE_k|\ln EDPi}[\ln\ DVE_k \mid \ln\ EDP_i] \\
&\quad + Var_{\ln EDP_i|IM}[E_{\ln DVE_k|\ln EDP_i}[\ln\ DVE_k \mid \ln\ EDP_i] \\
&\approx g^*_k\ (h_i(IM)) + \left(\frac{\partial g_k}{\partial \ln EDP_i}\right)^2 \Bigg|_{h_i(IM)} h^*_i\ (IM)
\end{aligned}
\tag{3.27}
$$

and similarly,

$$
\begin{aligned}
Cov[\ln DVE_k, \ln DVE_l \mid IM] &= E[Cov[\ln DVE_k, \ln DVE_l \mid EDP_i, EDP_j] \\
&\quad + Cov[E[\ln DVE_k, \ln DVE_l \mid EDP_i, EDP_j] \\
&\approx \hat{g}_{kl}(h_i(IM), h_j(IM))\sqrt{g^*_k\ (h_i(IM))}\sqrt{g^*_l\ (h_j(IM))} \\
&\quad + \left(\frac{\partial g_k}{\partial \ln EDP_i}\right)\Bigg|_{h_i(IM)}\left(\frac{\partial g_l}{\partial \ln EDP_j}\right)\Bigg|_{h_j(IM)} \hat{h}_{ij}(IM)\sqrt{h^*_i\ (IM)}\sqrt{h^*_j\ (IM)}
\end{aligned}
\tag{3.28}
$$

Although not needed for the calculations in this framework, correlation coefficients can be easily calculated using the above information, if they are of interest:

$$
\rho(\ln DVE_k, \ln DVE_l \mid IM) = \frac{Cov[\ln DVE_k, \ln DVE_l \mid IM]}{\sqrt{Var[\ln DVE_k \mid IM]}\sqrt{Var[\ln DVE_l \mid IM]}}
\tag{3.29}
$$

Subscripts on the expectation operators were initially used on the expectation and variance terms above to emphasize which variable the expectation was being taken with respect to. For the sake of conciseness, these subscripts are dropped in future equations.

3.4 SWITCH TO THE NON-LOG FORM DVE | *IM*

Using first-order second-moment methods, as in the previous step, we have the following results:

$$
E[DVE_k \mid IM] = E[e^{\ln DVE_k|IM}] \approx e^{E[\ln DVE_k|IM]} \approx e^{g_k(h_i(IM))}
\tag{3.30}
$$

$$Var[DVE_k \mid IM] \approx \left(\frac{\partial e^{\ln DVE_k \mid IM}}{\partial \ln DVE_k} \right)^2 \Bigg|_{E[\ln DVE_k \mid IM]} Var[\ln DVE_k \mid IM]$$

$$= e^{2 g_k (h_i (IM))} Var[\ln DVE_k \mid IM] \tag{3.31}$$

$$Cov[DVE_k, DVE_l \mid IM] \approx \left(\frac{\partial e^{\ln DVE_k}}{\partial \ln DVE_k} \right) \Bigg|_{E[\ln DVE_k \mid IM]} \left(\frac{\partial e^{\ln DVE_l}}{\partial \ln DVE_l} \right) \Bigg|_{E[\ln DVE_l \mid IM]} Cov[\ln DVE_k, \ln DVE_l \mid IM]$$

$$= e^{g_k (h_i (IM)) + g_l (h_j (IM))} Cov[\ln DVE_k, \ln DVE_l \mid IM] \tag{3.32}$$

3.5 COMPUTE MOMENTS OF $TC \mid IM$

We now aggregate the results from all individual elements to compute an expectation and variance for the total cost of damage to the entire building. Using information from previous steps, we have the following results:

$$E[TC \mid IM] = E\left[\sum_{k=1}^{\#elements} DVE_k \mid IM \right] = \sum_{k=1}^{\#elements} E[DVE_k \mid IM] \text{, denoted } q(IM) \tag{3.33}$$

$$\begin{aligned} Var[TC \mid IM] &= \sum_{k=1}^{\#elements} \sum_{l=1}^{\#elements} Cov[DVE_k, DVE_l \mid IM] \\ &= \sum_{k=1}^{\#elements} Var[DVE_k \mid IM] + 2 \sum_{k=1}^{\#elements} \sum_{l=k+1}^{\#elements} Cov[DVE_k, DVE_l \mid IM], \end{aligned} \tag{3.34}$$

denoted $q^*(IM)$

3.6 ACCOUNTING FOR COLLAPSE CASES

At high *IM* levels, the potential exists for a structure to experience collapse (defined here as extreme deflections at one or more story levels). In this building state, repair costs are more likely a function of the collapse rather than individual element damage. In fact, the structure is likely not to be repaired at all. Thus, our predicted loss may not be accurate in these cases. In addition, the large deflections predicted in a few cases will skew our expected values of some *EDP*s such as interstory drifts, although collapse is only occurring in a fraction of cases. To account for the possibility of collapse, we would like to use the technique outlined above for no-collapse cases, and allow for an alternate loss estimate when collapse occurs. The following modification is suggested. Note, in the following calculations, we are conditioning on a collapse indicator variable. To communicate this, we have denoted the collapse and no collapse condition as "*C*" and "*NC*" respectively.

- At each IM level, compute the probability of collapse. This probability, $P(C \mid IM)$, is simply the fraction of analysis runs where collapse occurs.
- Calculate results using the FOSM analysis as before, but using only the runs that resulted in no collapse. We now denote these results $E[TC \mid IM, NC]$ and $Var[TC \mid IM, NC]$.
- Define an expected value and variance of total cost, given that collapse has occurred, denoted $E[TC \mid IM, C]$ and $Var[TC \mid IM, C]$. These values will likely not be functions of *IM*, but the conditioning on *IM* is still noted for consistency.
- The expected value of *TC* for a given *IM* level is now the average of the collapse and no collapse *TC*, weighted by their respective probabilities of occurring

$$E[TC \mid IM] = \left(1 - P(C \mid IM)\right) E[TC \mid IM, NC] + P(C \mid IM) E[TC \mid IM, C] \tag{3.35}$$

- The variance of *TC* for a given *IM* level can be calculated using the result from Appendix B:

$$\begin{aligned} Var[TC \mid IM] &= E[Var[TC \mid IM, NC \text{ or } C]] + Var[E[TC \mid IM, NC \text{ or } C]] \\ &= \left[\left(1 - P(C \mid IM)\right) Var[TC \mid IM, NC] + P(C \mid IM) Var[TC \mid IM, C]\right] \\ &+ \begin{bmatrix} \left(1 - P(C \mid IM)\right)\left(E[TC \mid IM] - E[TC \mid IM, NC]\right)^2 \\ + P(C \mid IM)\left(E[TC \mid IM] - E[TC \mid IM, C]\right)^2 \end{bmatrix} \end{aligned} \tag{3.36}$$

The procedure can now be implemented as before, using these moments. This collapse-case modification is probably necessary for any implementation of the model, as analysis of shaking (*IM*) levels sufficient to cause large financial loss are likely also to cause collapse in some representative ground motion records.

3.7 OTHER GENERALIZATIONS OF THE MODEL

Several other modifications to this model can potentially be used to increase the accuracy of the estimate, without fundamentally changing the approach outlined above. One such modification to the model is incorporation of demand surge (the increase in contractor costs after a major earthquake) using the following steps:

1. Determine the demand surge cost multiplier as a function of magnitude, *g*(*M*)

2. Determine the PDF $p_{M|IM}(m_i \mid im)$ from deaggregation of the hazard
3. Demand surge as a function of *IM* is $h(im) = \sum_{m_i} g(m_i) p_{M|IM}(m_i \mid im)$
4. The new total cost as a function of *IM* can be calculated as $TC(IM) = h(IM)E[TC \mid IM]$, where $E[TC \mid IM]$ is the expected total cost given *IM*, as calculated previously.

Another modification to the model is a revision to allow the calculation of an element damage measure based on a vector of *EDP*s, rather than just a scalar. A simple way to accommodate this possibility is to create additional *EDP*s that describe the vector of interest:

1. Create a new "derived" *EDP* that is a function of the vector of "basic" *EDP*s of interest (e.g., if *DM* is a function of the average of EDP_i and EDP_j, create a new *EDP*, $EDP_k = (EDP_i + EDP_j)/2$).
2. Compute mean, variance, and covariances of EDP_k using first-order approximations, and the second moment information calculated for EDP_i and EDP_j
3. The damage measure can then be a function of the scalar EDP_k

This method allows the simple scalar algebra to be used, at the expense of needing to track an increased number of *EDP*s. If many additional *EDP*s are needed, it may be preferable to develop a more complex vector-based procedure.

3.8 INCORPORATE THE GROUND MOTION HAZARD TO DETERMINE *E*[*TC*] AND *VAR*[*TC*]

Using the functions *q*(*IM*) and *q**(*IM*), and the derivative of the ground motion hazard curve, $d\lambda(IM)$, the mean and variance of *TC* per annum can be calculated by numerical integration:

$$E[TC] = \int_{IM} q(x) \left| d\lambda_{IM}(x) \right| \tag{3.37}$$

$$\begin{aligned} Var[TC] &= E[Var[TC \mid IM]] + Var[E[TC \mid IM]] \\ &= \int_{IM} q^*(x) \left| d\lambda_{IM}(x) \right| + E[q^2(x)] - E[q(x)]^2 \\ &= \int_{IM} q^*(x) \left| d\lambda_{IM}(x) \right| + \int_{IM} q^2(x) \left| d\lambda_{IM}(x) \right| - E[TC]^2 \end{aligned} \tag{3.38}$$

Note that the first term of Equation 3.38 is the contribution from uncertainty in the cost function *given IM*, and that the second two terms are the contribution from uncertainty in the *IM*.

3.9 RATE OF EXCEEDANCE OF A GIVEN *TC*

The first- and second-moment information for $TC|IM$ can also be combined with a site hazard to compute $\lambda_{TC}(z)$, the annual frequency of exceeding a given Total Cost z. For this calculation, it is necessary to assume a probability distribution for $TC|IM$ that has a conditional mean and variance equal to the values calculated previously. The rate of exceedance of a given TC is then given by

$$\lambda_{TC}(z) = \int_{IM} G_{TC|IM}(z,x)\left|d\lambda_{IM}(x)\right| \tag{3.39}$$

where $G_{TC|IM}(z,x) = P(TC > z \mid IM = x)$ is the Complementary Cumulative Distribution Function of $TC|IM$. By evaluating the integral for several values of z, a plot can be generated relating damage values to rates of exceedance.

Generally, the integral above will require a numerical integration. However, if the following simplifying assumptions are made, an analytic solution is available:

1. E[$TC|IM=im$] is approximated by a function of the form $a'(im)^b$, where a' and b are constants. Note that this is consistent with fitting the *median* of $TC|IM$ with $a(im)^b$, where

$$a = a' e^{-\frac{1}{2}\beta^*_{TC|IM}{}^2} \tag{3.40}$$

2. The uncertainty $TC|IM$ is characterized as follows: $TC \mid IM \; = E[TC \mid IM = im]\varepsilon_{TC|IM}$, where $\varepsilon_{TC|IM}$ is a lognormal random variable with median equal to 1 and logarithmic standard deviation $\sigma_{\ln(\varepsilon_{TC|IM})} = \beta^*_{TC|IM}$ (note that this is constant for all IM).
3. An approximate function of the form $\hat{\lambda}_{IM}(x) = k_0 x^{-k}$ is fit to the true *mean* site hazard curve. Note, this form for the hazard curve has been proposed previously by Kennedy and Short (1984) and Luco and Cornell (1998).

Under these conditions, the annual rate of exceeding a given Total Cost is given by:

$$\lambda_{TC}(z) = k_0 \left(\frac{z}{a'}\right)^{-k/b} \exp\left(\frac{1}{2}\frac{k}{b}\left(\frac{k}{b}-1\right)\beta^*_{TC|IM}{}^2\right) \tag{3.41}$$

We note that if the a from Equation 3.40 is substituted into Equation 3.41, then the result becomes

$$\lambda_{TC}(z) = k_0 \left(\frac{z}{a}\right)^{-k/b} \exp\left(\frac{1}{2}\frac{k^2}{b^2}\beta^*_{TC|IM}{}^2\right) \tag{3.42}$$

Equation 3.41 is useful as a simple estimate of $\lambda_{TC}(z)$, but it is also very informative as a measure of the relative importance of uncertainty in the calculation. The term

$$k_0\left(\frac{z}{a'}\right)^{-k/b} \tag{3.43}$$

in the equation would be the result if $\beta^*_{TC|IM}$ were to equal zero — that is, if we made all calculations using only expected values and neglected uncertainty. The term

$$\exp\left(\frac{1}{2}\frac{k}{b}\left(\frac{k}{b}-1\right)\beta^{*\,2}_{TC|IM}\right) \tag{3.44}$$

is an "amplification factor" that varies with the uncertainty in *TC*|*IM* present in the problem. Thus for this special case, it is simple to calculate the effect of uncertainty on the rate of exceeding a given Total Cost. As we shall show below, it may not be unreasonable for this factor to increase $\lambda_{TC}(z)$ by a factor of 10, so the effect of uncertainty may very well be significant. However, even for large values of $\beta^*_{TC|IM}$, the annual rate of exceedance is still dominated by the deterministic term. It is for this reason that it has been proposed here that the FOSM approximations of $\beta^*_{TC|IM}$ performed above are sufficient to provide an accurate result.

3.10 INCORPORATION OF EPISTEMIC UNCERTAINTY

Equations 3.37, 3.38, and 3.41 are valid for the case when there is no epistemic uncertainty in the ground motion hazard curve or *TC*|*IM*. However, we now need to extend our calculation to account for this uncertainty, which is expected to be significant. To do this, we first introduce the effect of epistemic uncertainty in *TC*|*IM*, and then introduce epistemic uncertainty in the ground motion hazard.

3.10.1 Epistemic Uncertainty in *TC*|*IM*

We have previously assumed a model which can be written $TC \mid IM = E[TC \mid IM]\varepsilon_R$, where ε_R is a random variable representing aleatory uncertainty. Now, we extend that model to incorporate epistemic uncertainty. We now assume a simplified (first-order) model of epistemic uncertainty in which that uncertainty is attributed only to the central or mean value of a random variable and not for example, its variance or distribution shape. (In practice, one may inflate somewhat this uncertainty in the mean to reflect these second-order elements of epistemic uncertainty.) This model means that we represent the total uncertainty in TC as $TC \mid IM = \overline{E[TC \mid IM]}\varepsilon_R \varepsilon_U$, where $\overline{E[TC \mid IM]}$ is the best estimate of the (conditional) mean and ε_R and ε_U are uncorrelated random variables representing aleatory uncertainty and epistemic uncertainty, respectively. Note that $\overline{E[TC \mid IM]}\varepsilon_U$ is a random variable representing the (uncertain) estimate of the mean value of $TC \mid IM$, with variance $Var\left[E[TC \mid IM]\right]$.

We have a total model of the form $TC \mid IM = q(IM)\varepsilon_R \varepsilon_U$. So taking logs gives us $\ln TC \mid IM = \ln q(IM) + \ln \varepsilon_R(IM) + \ln \varepsilon_U(IM)$. The random variables ε_R and ε_U are uncorrelated, so we may deal with them in separate steps. The above procedure, described in Sections 3.1 through 3.7, accounted for aleatory uncertainty and allowed us to find the variance due to ε_R. We must now repeat the procedure to calculate the variance due to ε_U. Note that we have switched to logs again to allow use of sums rather than products. The change can be made using the following relationship:

$$Var[\ln \varepsilon_R(IM)] \cong \ln\left(1 + \frac{Var_R[TC \mid IM]}{E_R[TC \mid IM]^2}\right) \tag{3.45}$$

We denote $Var[\ln \varepsilon_R]$ and $Var[\ln \varepsilon_U]$ as $\beta_R{}^2$ as $\beta_U{}^2$, respectively. Note that in the previous sections, the uncertainty that we have denoted as $\beta_{TC|IM}{}^2$ is now referred to as $\beta_R{}^2$, to distinguish it from the $\beta_U{}^2$ that we are now adding.

Representation of Conditional Variables in the Framework Equation

To distinguish between aleatory and epistemic uncertainties of various conditional random variables, we introduce an additional notation. For example, we denote the epistemic and aleatory uncertainty of $\ln EDP \mid IM$, as:

$Var[\ln EDP \mid IM] = \beta^2{}_{R;EDP|IM}$, for variance due to aleatory uncertainty in $\ln EDP \mid IM$ (3.46)

$Var\left[E[\ln EDP \mid IM]\right] = \beta^2{}_{U;EDP|IM}$, for variance due to epistemic uncertainty in the estimate of the mean of $\ln EDP \mid IM$ (3.47)

These values are equivalent to $h^*_i(IM)$ in Equation 3.3. This notation is introduced simply to distinguish between aleatory and epistemic uncertainty. As a guideline for estimating uncertainty, several references are included in later sections. Example results for $\beta_{R;EDP|IM}$ are presented in Section 6.2, and guidelines for estimating $\beta_{U;EDP|IM}$ are presented in Section 6.3.

As a final note, one should be aware of the potential for double-counting of a source of uncertainty when constructing these models and classifying sources of uncertainty as epistemic or aleatory. Any single source of uncertainty should be accounted for as either aleatory or epistemic uncertainty, but it should not be included in both.

Accounting for Correlations in $\mathbf{EDP}\,|\,IM$

Estimates of correlations need to be made at each step of the PEER equation (i.e., $\mathbf{EDP}\,|\,IM$, $\mathbf{DVE}\,|\,\mathbf{EDP}$ after DM has been collapsed out, and $TC\,|\,\mathbf{DVE}$). In this section, we propose a model, and demonstrate its use for correlations in $\mathbf{EDP}\,|\,IM$. The same model is generally applicable to the other variables as well. Consider the following model for $\ln \mathbf{EDP}\,|\,IM$:

$$\ln(\mathbf{EDP}\,|\,IM) = \overline{E[\ln(\mathbf{EDP}\,|\,IM)]} + \boldsymbol{\varepsilon}_{R;EDP|IM} + \boldsymbol{\varepsilon}_{U;EDP|IM} \tag{3.48}$$

where $\overline{E[\ln(\mathbf{EDP}\,|\,IM)]}$ is the mean estimate of $E[\ln(\mathbf{EDP}\,|\,IM)]$ and $\boldsymbol{\varepsilon}_{R;EDP|IM}$ and $\boldsymbol{\varepsilon}_{U;EDP|IM}$ are random variables representing aleatory and epistemic uncertainty, respectively. Both random variables have an expected value of zero. Remember that we are using boldface notation because **EDP** is a vector of random variables. The aleatory uncertainty term ($\boldsymbol{\varepsilon}_{R;EDP|IM}$) can be estimated directly from data (see Section 6.2.2), so now we need to address the epistemic uncertainty term ($\boldsymbol{\varepsilon}_{U;EDP|IM}$).

Some of our epistemic uncertainty comes from model uncertainty (uncertainty about the accuracy of the structural model we are using—see Section 6.1 and Appendix E). Another portion of our uncertainty comes from estimation error—we are estimating the moments of $\ln(\mathbf{EDP}\,|\,IM)$ by using the sample averages of a finite set of records. This "estimation uncertainty" is most famously seen when estimating a mean of a distribution by the average of n samples, each with variance σ^2. The variance of this estimate is $\sigma_{\hat{\mu}}^{\ 2} = \sigma^2 / n$. This σ^2 / n is an epistemic uncertainty that we have referred to as "estimation uncertainty." So we now split our epistemic uncertainty into two terms:

$$\boldsymbol{\varepsilon}_{U;EDP|IM} = \boldsymbol{\varepsilon}_{U_{model};EDP|IM} + \boldsymbol{\varepsilon}_{U_{estimate};EDP|IM} \tag{3.49}$$

where $\boldsymbol{\varepsilon}_{U_{model};EDP|IM}$ is a random variable representing model uncertainty and $\boldsymbol{\varepsilon}_{U_{estimate};EDP|IM}$ is a random variable representing estimation uncertainty, and both random variables have a mean of zero. These two random variables are assumed to be uncorrelated, so they can be analyzed separately.

When calculating the epistemic uncertainty, we will also need to calculate correlations between estimates of means at differing *IM* levels (e.g., correlation of estimates of the expected value of ln*EDP* at $IM = im_1$ and $IM = im_2$: $\rho_{E[\ln EDP|IM=im_1],E[\ln EDP|IM=im_2]}$). Although there is no correlation between aleatory uncertainties, epistemic uncertainty (representing our uncertainty about the mean values) will potentially be correlated. The modeling uncertainty, represented by $\boldsymbol{\varepsilon}_{U_{model};EDP|IM}$, may presumably, to a first approximation, be assumed to have a perfect correlation at two *IM* levels, because the models tend be common at least within the linear and nonlinear ranges. The same perfect correlation could be applied to two different E[ln*EDP*]'s at a single given *IM* level. Our estimation uncertainty, represented by $\boldsymbol{\varepsilon}_{U_{estimate};EDP|IM}$, may also be correlated at two *IM* levels. For instance, if we use the same set of ground motion records to estimate the E[ln*EDP*]'s at more than one *IM* level by using scaling, our estimates at the varying *IM* levels will be correlated. In order to measure this correlation, we can utilize the bootstrap. (Efron and Tibshirani, 1998). The use of bootstrapping to calculate the correlation for a given *EDP* at two *IM* levels is outlined in Appendix D. The variance of $\boldsymbol{\varepsilon}_{U_{estimate};EDP|IM}$ can also be calculated from the bootstrap, as mentioned in the Appendix.

Once we have measured the variance and correlation of $\boldsymbol{\varepsilon}_{U_{model};EDP|IM}$ and $\boldsymbol{\varepsilon}_{U_{estimate};EDP|IM}$ at two *IM* levels, we can combine them to find the correlation of $\boldsymbol{\varepsilon}_{U;EDP|IM}$ at two *IM* levels. If the variance of $\boldsymbol{\varepsilon}_{U_{model};EDP|IM}$, denoted $\beta^2_{U_{model};EDP|IM}$ is equal at both *IM* levels, and the variance of $\boldsymbol{\varepsilon}_{U_{estimate};EDP|IM}$, denoted $\beta^2_{U_{estimate};EDP|IM}$ is equal at both *IM* levels, then the correlation of $\boldsymbol{\varepsilon}_{U;EDP|IM}$ at two *IM* levels, denoted $\rho_{U;EDP|IM_1,IM_2}$ is:

$$\rho_{U;EDP|IM_1,IM_2} = \frac{\beta^2_{U_{model};EDP|IM} + \rho \cdot \beta^2_{U_{estimate};EDP|IM}}{\beta^2_{U_{model};EDP|IM} + \beta^2_{U_{estimate};EDP|IM}} \tag{3.50}$$

where ρ is the correlation between $E[\ln EDP \mid IM]$ at two *IM* levels due to estimation uncertainty (the correlation we measured from the bootstrap). We note, however, that if ρ is expected to be near one, or if $\beta^2_{U_{model};EDP|IM}$ is much greater than $\beta^2_{U_{estimate};EDP|IM}$, then $\rho_{U;EDP|IM_1,IM_2}$ will be nearly one. Under these conditions, it is thus reasonable to simply assume a perfect correlation, and thus skip the computations of the bootstrap. Note that if the more complex model of Equation 3.50 is used, it is not necessary that the variance of $\boldsymbol{\varepsilon}_{U_{estimate};EDP|IM}$ is equal at both *IM* levels as we have assumed above. An equation following the form of Equation A.5 will allow for the variance of $\boldsymbol{\varepsilon}_{U_{estimate};EDP|IM}$ to be different at the two *IM* levels.

Additional Correlations

In addition to correlations between one $E[\ln EDP \mid IM]$ at two *IM* levels, it is also necessary to find correlations between two $E[\ln EDP \mid IM]$'s at the same *IM* level (e.g., $Corr[E[\ln EDP_i \mid IM = im_1], E[\ln EDP_j \mid IM = im_1]]$). For aleatory uncertainty, it was possible to make a direct estimate from the data available, but it is slightly more complicated for epistemic uncertainty. However, the model of Equation 3.50 is suitable for this situation as well. The correlation coefficient from estimation uncertainty can be computed from the bootstrap, and for model uncertainty, a perfect correlation could again be assumed. Then the estimate of total correlation can be calculated in a similar manner to Equation 3.50.

We also need an estimate of $Cov\left[E[\ln DVE_i \mid \ln EDP_1], E[\ln DVE_i \mid \ln EDP_2]\right]$. The conditional random variable $E[\ln DVE_i \mid \ln EDP_1]$ will have epistemic uncertainty. The model developed in the previous section for $E[\ln EDP \mid IM]$ could be applied in the same way to $E[\ln DVE_i \mid \ln EDP_1]$. Again, it is worth considering whether the simple assumption of a perfect correlation model may be appropriate before using the slightly more complex model proposed here.

Propagation of Uncertainty, Accounting for Correlations at Two *IM* Levels

Earlier, in Equation 3.38, we did not consider correlations in *TC* at two *IM* levels. However, if there is correlation in $E[\ln EDP \mid IM]$ at two *IM* levels, as we have introduced in this section, then this correlation will propagate through to $E[TC \mid IM]$, and result in correlation between $E[TC \mid IM]$ at two *IM* levels. We now show the FOSM approximation that accounts for this correlation.

Once we have specified the correlation between $E[\ln EDP_i \mid IM_1]$ and $E[\ln EDP_i \mid IM_2]$, and the correlation between $E[\ln DVE_k \mid \ln EDP_1]$ and $\mathrm{E}[\ln DVE_k \mid \ln EDP_2]$, as described in this section, and we have the expected values from Sections 3.1 and 3.2, we can use FOSM to combine this information and compute the covariance between $E[\ln DVE_k \mid IM_1]$ and $\mathrm{E}[\ln DVE_k \mid IM_2]$ as shown below:

$$\begin{aligned} &Cov[E[\ln DVE_k \mid IM_1], E[\ln DVE_k \mid IM_2]] \\ &\quad \approx Cov[E[\ln EDP_i] \mid IM_1, E[\ln EDP_i \mid IM_2]] \\ &\quad\quad * \left.\frac{\partial E[\ln DVE_k \mid \ln EDP_i]}{\partial \ln EDP_i}\right|_{\overline{E[\ln EDP_i|IM_1]}} \left.\frac{\partial E[\ln DVE_k \mid \ln EDP_i]}{\partial \ln EDP_i}\right|_{\overline{E[\ln EDP_i|IM_2]}} \\ &\quad\quad + Cov[E[\ln DVE_k \mid \ln EDP_i], E[\ln DVE_k \mid \ln EDP_i]]\Big|_{\overline{E[\ln EDP_i|IM_1]}, \overline{E[\ln EDP_i|IM_2]}} \end{aligned} \tag{3.51}$$

In the same way, we can compute the covariance between $E[\ln DVE_k \mid IM_1]$ and $\mathrm{E}[\ln DVE_l \mid IM_2]$:

$$\begin{aligned} &Cov[E[\ln DVE_k \mid IM_1], E[\ln DVE_l \mid IM_2]] \\ &\quad \approx Cov[E[\ln EDP_i] \mid IM_1, E[\ln EDP_j \mid IM_2]] \\ &\quad\quad * \left.\frac{\partial E[\ln DVE_k \mid \ln EDP_i]}{\partial \ln EDP_i}\right|_{\overline{E[\ln EDP_i|IM_1]}} \left.\frac{\partial E[\ln DVE_l \mid \ln EDP_j]}{\partial \ln EDP_j}\right|_{\overline{E[\ln EDP_j|IM_2]}} \\ &\quad\quad + Cov[E[\ln DVE_k \mid \ln EDP_i], E[\ln DVE_l \mid \ln EDP_j]]\Big|_{\overline{E[\ln EDP_i|IM_1]}, \overline{E[\ln EDP_j|IM_2]}} \end{aligned} \tag{3.52}$$

As in Equation 3.32, we must convert the covariance of the E[ln*DVE*]'s to the covariance of the (non-log) E[*DVE*]'s:

$$\begin{aligned} &Cov[E[DVE_k \mid IM_1], E[DVE_l \mid IM_2]] \\ &\quad \approx \left.\frac{\partial e^{E[\ln DVE_k|IM_1]}}{\partial E[\ln DVE_k \mid IM_1]}\right|_{\overline{E[\ln DVE_k|IM_1]}} \left.\frac{\partial e^{E[\ln DVE_l|IM_2]}}{\partial E[\ln DVE_l \mid IM_2]}\right|_{\overline{E[\ln DVE_l|IM_2]}} \\ &\quad\quad * Cov[E[\ln DVE_k \mid IM_1], E[\ln DVE_l \mid IM_2]] \end{aligned} \tag{3.53}$$

We sum the E[*DVE*|*IM*] random variables to get E[*TC*|*IM*] (e.g., $E[TC \mid IM] = \sum_{\text{all } i} E[DVE_i \mid IM]$). So given the values from the above equation, we can compute the covariance of E[*TC*|*IM*] at two *IM* levels:

$$
\begin{aligned}
Cov[E[TC \mid IM_1], E[TC \mid IM_2]] &= Cov\left[\left(\sum_k E[DVE_k \mid IM_1]\right), \left(\sum_l E[DVE_l \mid IM_2]\right)\right] \\
&= \sum_k Cov[E[DVE_k \mid IM_1], E[DVE_k \mid IM_2]] \\
&\quad + 2\sum_{k<l}\sum Cov\left[E[DVE_k \mid IM_1], E[DVE_l \mid IM_2]\right]
\end{aligned} \tag{3.54}
$$

This covariance is merely a summation of many covariance terms that we can now calculate. Note that we will need to repeat this calculation for different $\{IM_1, IM_2\}$ pairs. We will use these values in the sections below (e.g., in Equation 3.61).

3.10.2 Epistemic Uncertainty in the Ground Motion Hazard

It is now necessary to account for epistemic uncertainty in the ground motion hazard. This uncertainty is often displayed qualitatively as the fractile uncertainty bands about the mean estimate of the hazard curve, as shown in Fig. 3.7.

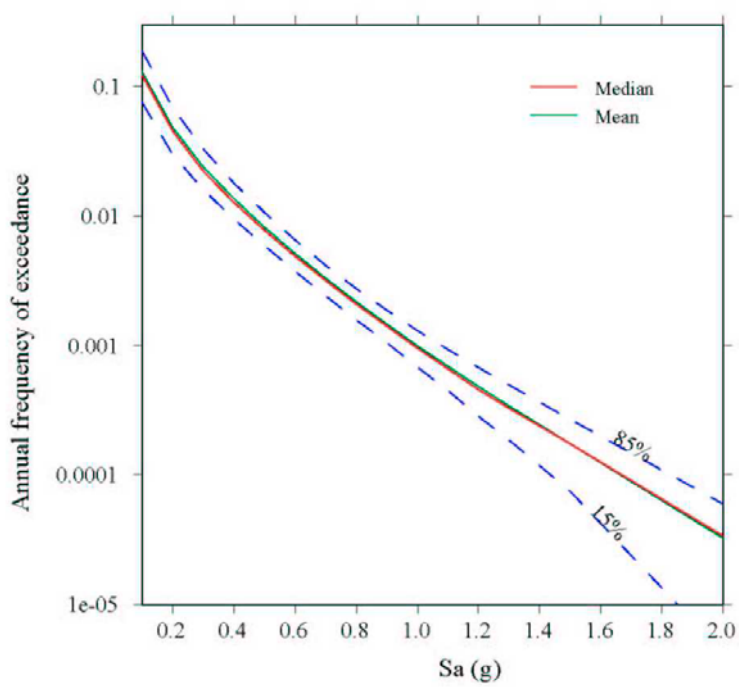

Fig. 3.7: Sample Hazard Curve for the Van Nuys Site

Formally, we represent the ground motion hazard at a given *IM* level as a random variable.

$$\lambda_{IM}(im) = \overline{\lambda}_{IM}(im)\varepsilon_{UIM}(im) \tag{3.55}$$

where $\bar{\lambda}_{IM}(im)$ is our best estimate or *mean estimate* of $\lambda_{IM}(im)$, and $\varepsilon_{UIM}(im)$ is a random variable with a mean of 1. Considering the entire range of *IM* levels implies that $\varepsilon_{UIM}(im)$ is in fact a random *function* of *IM*. We will again need to consider correlations, because the first and second moment representation of this function will involve a covariance function that is parameterized by the two *im* levels considered, $Cov[\lambda_{IM}(im_1), \lambda_{IM}(im_2)]$. However, before we discuss the random function theory solution to this problem (e.g., Nigam, 1983), let us realize that we will be performing all of our calculations using numerical integration. For example, the integral of Equation 3.37 will in practice be calculated as a discrete summation, which we shall discuss further in the next section:

$$E[TC] = \int_{IM} E[TC \mid IM = x] \left| d\lambda_{IM}(x) \right| \cong \sum_{i=1}^{n} E[TC \mid IM = x_i] \cdot \left(-\frac{\lambda_{IM}(x_i) - \lambda_{IM}(x_{i-1})}{x_i - x_{i-1}} \right) \quad \text{where } 0 \equiv x_0 < x_1 < \ldots < x_n \tag{3.56}$$

Here it is only important to recognize that we are now dealing with a *discrete* set of $\lambda_{IM}(x)$. Therefore we define a new random vector:

$$\Delta\lambda_{IM}(x_i) = -\frac{\lambda_{IM}(x_i) - \lambda_{IM}(x_{i-1})}{x_i - x_{i-1}} \tag{3.57}$$

The mean and covariance of the array $\Delta\lambda_{IM}(x_i), i = 1, \ldots, n$ can be computed if we know the mean and covariance of the array of $\lambda_{IM}(x_i), i = 1, \ldots, n$. We have previously used the mean value of this array, $\bar{\lambda}_{IM}(im)$, which we get from PSHA software. The variances can be estimated from the fractile uncertainty typically displayed in a graph of the seismic hazard curve (e.g., Fig. 3.7). The covariances of the array are potentially available from the output of PSHA software as well (see Appendix F). Using this formulation, the random variable *E*[*TC*] can be represented as:

$$E[TC] = E\left[\sum_{i=1}^{n} E[TC \mid IM = x_i] \cdot \Delta\lambda_{IM}(x_i) \right] \tag{3.58}$$

in which we understand that there is now epistemic uncertainty in $E[TC \mid IM = x_i]$ and $\Delta\lambda_{IM}(x_i)$. Further, we assume that there is no stochastic dependence between the epistemic aspects of $E[TC \mid IM = x_i]$ and $\Delta\lambda_{IM}(x_i)$.

Now that we have quantified the epistemic uncertainty in $E[TC \mid IM = im]$ and in $\lambda_{IM}(im)$, we will apply this model to assessment of epistemic uncertainty in $E[TC]$, $\lambda_{collapse}$ and $\lambda_{TC}(z)$.

3.11 EXPECTATION AND VARIANCE OF $E[TC]$, ACCOUNTING FOR EPISTEMIC UNCERTAINTY

Consider first the effect of epistemic uncertainty on the mean estimate ($\overline{E[TC]}$) and epistemic variance ($Var\left[E[TC]\right]$) of $E[TC]$. We calculate $\overline{E[TC]}$ by taking advantage of the independence of $E[TC \mid IM = x_i]$ and $\Delta\lambda_{IM}(x_i)$, and using the linearity of the expectation operator:

$$\begin{aligned}
E[TC] &= E\left[\sum_{i=1}^{n} E\left[TC \mid IM = x_i\right] \cdot \Delta\lambda_{IM}(x_i)\right] \\
&= \sum_{i=1}^{n} \overline{E\left[TC \mid IM = x_i\right]} \cdot \left(-\frac{E\left[\lambda_{IM}(x_i) - \lambda_{IM}(x_{i-1})\right]}{x_i - x_{i-1}} \right) \\
&= \sum_{i=1}^{n} \overline{E\left[TC \mid IM = x_i\right]} \cdot \left(-\frac{\overline{\lambda}_{IM}(x_i) - \overline{\lambda}_{IM}(x_{i-1})}{x_i - x_{i-1}} \right) \\
&= \sum_{i=1}^{n} \overline{E\left[TC \mid IM = x_i\right]} \cdot \Delta\overline{\lambda}_{IM}(x_i)
\end{aligned} \tag{3.59}$$

This is the discrete analog of Equation 3.37 (where we used the mean hazard curve in the calculation). So we see that our estimate of expected total cost is unchanged when we include epistemic uncertainty in the analysis, provided that we use the mean estimate of the ground motion hazard curve.

However, because we are now uncertain about $E[TC \mid IM = x_i]$ and $\Delta\lambda_{IM}(x_i)$ (for all *i*), $E[TC]$ is now uncertain. So we would like to calculate the epistemic variance in $E[TC]$. This calculation involves a summation of products of random variables. Consider equation 3.58. If we denote $E[TC \mid IM = x_i]$ as X_i, and $\Delta\lambda_{IM}(x_i)$ as Y_i, then $E[TC]$ is of the form:

$$E[TC] = \sum_{i=1}^{n} X_i \cdot Y_i \tag{3.60}$$

where X, and Y are random arrays. There is no correlation between X_i, and Y_i, but there is quite likely to be a correlation between X_i and X_j, and also between Y_i and Y_j ($i \neq j$), as discussed above. We have calculated $Cov[X_i, X_j]$ in Equation 3.54 and $Cov[Y_i, Y_j]$ is discussed in Section

3.10.2 and Appendix F. Given that the needed covariance matrices have been calculated, from Ditlevsen (1981) we have the following result for a product of random arrays:

$$\begin{aligned} Var\left[\sum_i X_i Y_i\right] &= E\left[\sum_i \sum_j X_i X_j Cov[Y_i, Y_j]\right] + Var\left[\sum_i X_i E[Y_i]\right] \\ &= \sum_i \sum_j \begin{bmatrix} Cov[X_i, X_j]Cov[Y_i, Y_j] \\ +E[X_i]E[X_j]Cov[Y_i, Y_j] \\ +E[Y_i]E[Y_j]Cov[X_i, X_j] \end{bmatrix} \end{aligned} \tag{3.61}$$

The above equation becomes very long when $X_i = E[TC \mid IM = x_i]$ and $Y_i = \Delta\lambda_{IM}(x_i)$ is substituted back in. It is left to the reader to make this change of notation at the time of implementation in a computer program.

To match the other computations in this report, we would hope to have an analytical solution for the expectation and variance of $E[TC]$. However, it can be shown that under similar analytical assumptions to those made elsewhere in this report, an analytical solution does not exist[3].

---

[3] **An Analytical Solution?** Equation 3.62 should be easy to implement in a simple computer program, although it is not feasible for "back-of-the-envelope" calculations. So we would hope to have a closed form solution for $\overline{E[TC]}$ and $Var[E[TC]]$. We can compute $\overline{E[TC]}$ using the following integral:

$$\overline{E[TC]} = \int_0^\infty z \left| d\lambda_{TC}(z) \right| dz$$

We note however, that when we try to evaluate this integral using our analytic functional forms, we have a problem. Substituting our analytical solution for $\overline{\lambda}_{TC}(z)$ from Equation 3.72 (developed in Section 3.13 below), we have:

$$\begin{aligned} \overline{E[TC]} &= \int_0^\infty z \left| \frac{d}{dz}\left[k_0 \left(\frac{z}{a'}\right)^{-k/b} \exp\left(\frac{1}{2}\frac{k}{b}\left(\frac{k}{b}-1\right)\left(\beta_R^2 + \beta_U^2\right)\right)\right]\right| dz \\ &= \int_0^\infty z \left| -k_0 \frac{k}{b}\left(\frac{z}{a'}\right)^{-k/b-1}\left(\frac{1}{a'}\right)\exp\left(\frac{1}{2}\frac{k}{b}\left(\frac{k}{b}-1\right)\left(\beta_R^2 + \beta_U^2\right)\right)\right| dz \\ &= k_0 \frac{k}{b}\left(\frac{1}{a'}\right)^{-k/b} \exp\left(\frac{1}{2}\frac{k}{b}\left(\frac{k}{b}-1\right)\left(\beta_R^2 + \beta_U^2\right)\right) \int_0^\infty z^{-k/b} dz \\ &= K_0 \, z^{-k/b+1}\Big|_0^\infty \end{aligned}$$

where

$$K_0 = k_0 \frac{k}{b}\left(\frac{1}{a'}\right)^{-k/b} \exp\left(\frac{1}{2}\frac{k}{b}\left(\frac{k}{b}-1\right)\left(\beta_R^2 + \beta_U^2\right)\right)$$

Note that we have not yet included costs due to collapse, so we will address collapses in the following section, and then revisit the $E[TC]$ calculation with collapses in mind.

3.12 EXPECTATION AND VARIANCE OF THE ANNUAL FREQUENCY OF COLLAPSE, ACCOUNTING FOR EPISTEMIC UNCERTAINTY

Within the framework outlined here, we have acquired the information necessary to compute the mean and variance in annual probability of collapse—another decision variable of interest to project stakeholders. This process is described below, as a relevant aside to our repair cost calculations. The notation follows that proposed in Section 3.7. To compute the mean annual frequency of collapse, we use the following equation:

$$\lambda_{collapse} = \sum_{i=1}^{n} P\left[C \mid IM = x_i\right] \cdot \Delta\lambda_{IM}(x_i) \tag{3.62}$$

We have already determined the mean and covariance $\Delta\lambda_{IM}(x_i)$ in Section 3.10.2, so now we need information about the mean and covariance of $P(C \mid IM)$. We can take the mean value of $P(C \mid IM)$ to be the fraction of records that collapse at a given *IM* level. To estimate the variance, we will consider variance due to model uncertainty, and variance due to our estimation uncertainty. This is the same problem as we outlined in the section titled "Accounting for Correlations in $\mathbf{EDP} \mid IM$" on page 21. In order to quantify the estimation uncertainty, we could again take advantage of the bootstrap, as we did in Section 3.10.1. That is, we could make bootstrap replicates of the records used in the analysis. We could then use these replicates to make new estimates of *p*. These replicates of *p* will help us determine both the epistemic

---

This integral does not converge unless k = b, so the simplified analytical solution is not a useful method for obtaining $\overline{E[TC]}$. We find the same problem when we try to calculate *Var*[*E*[*TC*]]. The reason that this does not converge is due to inadequacies of the analytical model at extreme values of *IM*. The integral given in this footnote is the product of the rate of equaling and *IM* and the repair cost given that *IM*. At extreme values of this problem, the analytical forms result in infinities. If $k>b$, then as $IM \rightarrow 0$, the rate of equaling *IM* goes to infinity faster than the repair cost goes to zero. If $k<b$, then as $IM \rightarrow \infty$, repair costs go to infinity faster than the rate of equaling IM goes to zero. In either case, the expected value goes to infinity. For this reason, it is recommended that the numerical integration technique be used to calculate $\overline{E[TC]}$ and *Var*[*E*[*TC*]], rather than a simplified analytic solution. In the numerical integration case, we are not limited by the functional forms that are inadequate in this analytical solution case.

uncertainty in our estimate, $Var\left[P(C \mid IM)\right]$, and also the covariance for differing IM levels, $Cov\left[P(C \mid IM_i), P(C \mid IM_j)\right]$. This bootstrap method accounts for uncertainty due to the records used in estimation. However, additional epistemic uncertainty should be added to account for modeling uncertainty, etc (see Section 6.1 and Appendix E). Then the total covariance could be calculated using the form of equation 3.50 as described above.

These values are now put in the notation of Equation 3.61, to show the parallel with this previous calculation. Define $P(C \mid IM_i)$ as X_i. We know that $E[X_i] = p$, and we have discussed how to find $Var[X_i]$ and $Cov[X_i, X_j]$ above. And the mean and covariance of the ground motion hazard, Y_i, remain identical to the results needed for Section 3.11. So now we have reduced the problem to one that was previously solved in Equations 3.59 and 3.61:

$$E[\lambda_{collapse}] = \bar{\lambda}_{collapse} = \sum_{i=1}^{n} E[X_i] \cdot E[Y_i] \tag{3.63}$$

$$\begin{aligned} Var\left[\lambda_{collapse}\right] &= E\left[\sum_i \sum_j X_i X_j Cov[Y_i, Y_j]\right] + Var\left[\sum_i X_i E[Y_i]\right] \\ &= \sum_i \sum_j \begin{bmatrix} Cov[X_i, X_j] Cov[Y_i, Y_j] \\ +E[X_i] E[X_j] Cov[Y_i, Y_j] \\ +E[Y_i] E[Y_j] Cov[X_i, X_j] \end{bmatrix} \end{aligned} \tag{3.64}$$

where we have denoted the mean estimate of the annual frequency of collapse as $\bar{\lambda}_{collapse}$.

Analytical Solution

Additionally, we can make the several functional form assumptions to take advantage of a closed form analytical solution:

1. Consider a random variable for capacity, *C*, of the form $C = \eta_C \varepsilon_{UC} \varepsilon_{RC}$, where η_C is the median value of *C* (expressed in units of *IM*), and ε_{UC} and ε_{RC} are lognormal random variables. ε_{RC} accounts for aleatory uncertainty in the capacity, and ε_{UC} accounts for epistemic uncertainty in the median value of *C*. These random variables have the following properties:

$$median(\varepsilon_{UC}) = median(\varepsilon_{RC}) = e^{mean(\varepsilon)} = 1$$

$$\sigma_{\ln(\varepsilon_{RC})} = \beta_{RC} \tag{3.65}$$

$\sigma_{\ln(\varepsilon_{UC})} = \beta_{UC}$

We can find the moments of these random variables using information previously calculated. First, we note that what we previously called P(C|IM) is in fact the CDF of a random variable for collapse: $F_C(IM)$. So we can fit a lognormal CDF P(C|IM) (we would actually fit a normal CDF to P(C|lnIM)). We can find the mean and standard deviation of this normal distribution, and these values represent η_C and β_{RC}, respectively. We can estimate β_{UC} in the same way that we estimated variance due to epistemic uncertainty earlier in this section. We now need to specify a constant β_{UC}, however.

2. The ground motion hazard is fit in the same way as described above. That is, $\lambda_{IM}(x) = k_0 x^{-k} \varepsilon_{UIM}$, where ε_{UIM} is a lognormal random variable with *mean* equal to 1 and standard deviation $\sigma_{\ln(\varepsilon_{UIM})} = \beta_{UIM}$.

Under these assumptions, the mean estimate of the annual frequency of collapse is given by:

$$E\left[\lambda_{collapse}\right] = \overline{\lambda}_{collapse} = k_0 \eta_C^{\,-k} \cdot e^{\frac{1}{2}k^2\beta_{UC}^2} \cdot e^{\frac{1}{2}k^2\beta_{RC}^2} \tag{3.66}$$

We also have a result for the uncertainty in this estimate. Under the assumptions above, $\lambda_{collapse}$ is a lognormal random variable. So we represent its uncertainty by a lognormal standard deviation, as we have throughout this exercise:

$$\sigma_{\ln(\lambda_{collapse})} \equiv \beta_{\lambda_{collapse}} = \sqrt{\beta_{UIM}^2 + k^2 \beta_{UC}^2} \tag{3.67}$$

This result is derived from related problems that have previously been solved (e.g., Cornell et al., 2002).

3.13 REVISED CALCULATION FOR E[*TC*] , ACCOUNTING FOR COSTS DUE TO COLLAPSES

In Section 3.11 we calculated the mean and variance of $E[TC]$. However, we did not include the effect of collapses on Total Cost, in the way that we outlined in Section 3.6. Now that we have information about the mean and variance of $P(C \mid IM)$ from Section 3.12, we can revise our calculation of $E[TC]$ to include this generalization.

In Section 3.11 (Equation 3.59), we represented TC as:

$$E[TC] = \sum E\left[TC \mid IM = x_i\right] \cdot \Delta\lambda_{IM}(x_i) \tag{3.68}$$

But incorporating the possibility of collapse into the calculation results in the equation:

$$E[TC] = \sum \left\{ \left[1 - P(C \mid IM = x_i)\right] E\left[TC \mid NC, IM = x_i\right] + P(C \mid IM = x_i) E\left[TC \mid C\right] \right\} \cdot \Delta\lambda_{IM}(x_i) \tag{3.69}$$

To calculate the mean and variance of $E[TC]$, we need the mean and variance of $E[TC|IM]$ and $P(C \mid IM)$, which have been calculated above. We will also need to specify a mean and variance of $E[TC|C]$ (i.e., the Total Cost given that the structure has collapsed) as noted in Section 3.6. We then assume that the random variables $E\left[TC \mid NC, IM = x_i\right]$, $E[TC|C]$, $P(C \mid IM = x_i)$, and $\Delta\lambda_{IM}(x_i)$ are mutually uncorrelated. It is now necessary to compute means, variances, and covariances of $E[TC|IM]$:

$$E[TC \mid IM] = \left[1 - P(C \mid IM = x_i)\right] E\left[TC \mid NC, IM = x_i\right] + P(C \mid IM = x_i) E\left[TC \mid C\right] \tag{3.70}$$

The mean and variance of $E[TC|IM]$ can be calculated in a manner closely related to Equations 3.35 and 3.36, as before:

$$\begin{aligned} E\left[E[TC \mid IM]\right] &= \left[1 - E\left[P(C \mid IM = x_i)\right]\right] E\left[E\left[TC \mid NC, IM = x_i\right]\right] \\ &\quad + E\left[P(C \mid IM = x_i)\right] E\left[E\left[TC \mid C\right]\right] \end{aligned} \tag{3.71}$$

$$\begin{aligned} Var\left[E[TC \mid IM]\right] &= \left(1 - E\left[P(C \mid IM)\right]\right) Var\left[E\left[TC \mid IM, NC\right]\right] \\ &\quad + E\left[P(C \mid IM)\right] Var\left[E\left[TC \mid C\right]\right] \\ &\quad + \left(1 - E\left[P(C \mid IM)\right]\right)\left(E\left[E[TC \mid IM]\right] - E[TC \mid IM, NC]\right)^2 \\ &\quad + E\left[P(C \mid IM)\right]\left(E\left[E[TC \mid IM]\right] - E[TC \mid C]\right)^2 \end{aligned} \tag{3.72}$$

In addition, we now need to calculate the covariance in $TC|IM$ at two IM levels. This result is shown in the footnote below[4]. Once we have calculated these moments of $E[TC|IM]$, we simply

---

[4] We derive the covariance calculation by switching to the following notation for brevity:

$$\begin{aligned} E[TC \mid IM_1] &\equiv (1 - P_1) NC_1 + P_1 C_{TC} \\ E[TC \mid IM_2] &\equiv (1 - P_2) NC_2 + P_2 C_{TC} \end{aligned} \qquad \text{where} \quad \begin{aligned} P_i &\equiv P(C \mid IM_i) \\ NC_i &\equiv E[TC \mid IM_i, NC] \\ C_{TC} &\equiv E[TC \mid C] \end{aligned}$$

Then the covariance calculation is (after expansion and collection of terms):

$$\begin{aligned} Cov\left[E[TC \mid IM_1], E[TC \mid IM_2]\right] &= E\left[E[TC \mid IM_1] E[TC \mid IM_2]\right] - E\left[E[TC \mid IM_1]\right] E\left[E[TC \mid IM_2]\right] \\ &= E\left[\left\{(1 - P_1) NC_1 + P_1 C_{TC}\right\}\left\{(1 - P_2) NC_2 + P_2 C_{TC}\right\}\right] \\ &\quad - E[(1 - P_1) NC_1 + P_1 C_{TC}] E[(1 - P_2) NC_2 + P_2 C_{TC}] \\ &= E\left[P_1 P_2\right]\left\{E\left[NC_1 NC_2\right] + E\left[C_{TC}^{\ 2}\right]\right\} + E\left[P_1\right] E\left[P_2\right]\left(E\left[NC_1\right] E\left[NC_2\right] - E\left[C_{TC}\right]^2\right) \\ &\quad + \left(1 - E\left[P_2\right] - E\left[P_1\right]\right) Cov\left[NC_1, NC_2\right] - \left(E\left[NC_2\right] + E\left[NC_1\right]\right) E\left[C_{TC}\right] Cov\left[P_1, P_2\right] \end{aligned}$$

combine them with our previous ground motion hazard curve and use Equations 3.60 and 3.61 as before.

3.14 RATE OF EXCEEDANCE OF A GIVEN *TC*, ACCOUNTING FOR EPISTEMIC UNCERTAINTY

With epistemic uncertainty in $E[TC \mid IM = im]$ and $\Delta\lambda_{IM}(im)$ described in Section 3.10, we can set about to compute the mean annual frequency of exceeding a level of *TC*, say *z* (i.e., $\lambda_{TC}(z)$). Recall that the integral of defining $\lambda_{TC}(z)$ (Equation 3.39) is given by $\lambda_{TC}(z) = \int_{IM} G_{TC|IM}(z,x)\left|d\lambda_{IM}(x)\right|$. Now, because $G_{TC|IM}(z,x)$ and $\lambda_{IM}(x)$ are uncertain (random) functions, the expected value of $\lambda_{TC}(z)$ is given by:

$$E\left[\lambda_{TC}(z)\right] \equiv \bar{\lambda}_{TC}(z) = \sum_{x_i} \bar{G}_{TC|IM}(z,x_i)\left|\Delta\bar{\lambda}_{IM}(x_i)\right| \tag{3.73}$$

where $\bar{G}_{TC|IM}(z,x) = E\left[P(TC > z \mid IM = x)\right]$ is our *mean estimate* of the Complementary Cumulative Distribution Function of *TC*|*IM*, and $d\bar{\lambda}_{IM}(x)$ is our *mean estimate* of the derivative of the hazard curve. This problem is more difficult than the analogous problem of Equation 3.62, because we don't have the epistemic variance in the *probability* ($G_{TC|IM}(z,x)$). We only know the epistemic variance in $E[TC \mid IM]$, and we need to propagate this variance to $G_{TC|IM}(z,x)$.

To compute the epistemic variance in $\lambda_{TC}(z)$, we will need to know both the variance in our estimate of $d\lambda_{IM}(x)$ (which we have already addressed in Section 3.10.2) and the epistemic variance of $G_{TC|IM}(z,x)$. If we assume that $G_{TC|IM}(z,x)$ is a lognormal distribution (with the mean of ln(*x*) equal to $E[\ln TC \mid IM = x]$ and the standard deviation of ln(*x*) equal to β_R), then $G_{TC|IM}(z,x)$ is defined by:

$$G_{TC|IM}(z,x) = \Phi\left(\frac{\ln z - E[\ln TC \mid IM = x]}{\beta_R}\right) \tag{3.74}$$

---

where $E[P_1P_2] = Cov[P_1,P_2] + E[P_1]E[P_2]$ and $E[NC_1NC_2] = Cov[NC_1,NC_2] + E[NC_1]E[NC_2]$

We are assuming that the epistemic uncertainty is limited (in a first-order sense) to $E[\ln TC \mid IM = x]$, which has standard deviation $\beta_U(x)$ (as calculated in Section 3.10.1). Using a first-order expansion, we can estimate the variance in $G_{TC|IM}(z,x)$ as:

$$
\begin{aligned}
Var[G_{TC|IM}(z,x)] &\approx \left(\frac{\partial \Phi\left(\dfrac{\ln z - E[\ln TC \mid IM = x]}{\beta_R(x)} \right)}{\partial E[\ln TC \mid IM = x]} \right)^2 \Bigg|_{\overline{E[\ln TC|IM=x]}} \cdot \beta_U^2(x) \\
&= \phi\left(\frac{\ln z - E[\ln TC \mid IM = x]}{\beta_R(x)} \right)^2 \cdot \frac{\beta_U^2(x)}{\beta_R^2(x)}
\end{aligned}
\tag{3.75}
$$

where $\beta_R(x)$ is the *aleatory* dispersion of $TC \mid IM$ (as calculated in Section 3.10.1). Similarly, the covariance between $G_{TC|IM}(z,x)$ evaluated at two *IM* levels is:

$$
\begin{aligned}
Cov[G_{TC|IM}(z,x_1), G_{TC|IM}(z,x_2)] &\approx \left(\frac{\partial \Phi\left(\dfrac{\ln z - E[\ln TC \mid IM = x_1]}{\beta_R(x_1)} \right)}{\partial E[\ln TC \mid IM = x_1]} \right) \Bigg|_{\overline{E[\ln TC|IM=x_1]}} \\
&\quad * \left(\frac{\partial \Phi\left(\dfrac{\ln z - E[\ln TC \mid IM = x_2]}{\beta_R(x_2)} \right)}{\partial E[\ln TC \mid IM = x_2]} \right) \Bigg|_{\overline{E[\ln TC|IM=x_2]}} \\
&\quad * Cov\left[E[\ln TC \mid IM = x_1], E[\ln TC \mid IM = x_2] \right] \\
&= \phi\left(\frac{\ln z - E[\ln TC \mid IM = x_1]}{\beta_R(x_1)} \right) \phi\left(\frac{\ln z - E[\ln TC \mid IM = x_2]}{\beta_R(x_2)} \right) \\
&\quad * \frac{Cov\left[E[\ln TC \mid IM = x_1], E[\ln TC \mid IM = x_2] \right]}{\beta_R^2(x_1) \beta_R^2(x_2)}
\end{aligned}
\tag{3.76}
$$

Note that the needed covariance between $E[\ln TC \mid IM = x]$ at two levels of *IM* was given in Equation 3.54. We have now reduced this problem to the one solved earlier in Equation 3.61:

$$
Var\left[\overline{\lambda}_{TC}(z) \right] = \sum_i \sum_j \begin{bmatrix} Cov[X_i, X_j] Cov[Y_i, Y_j] \\ + E[X_i] E[X_j] Cov[Y_i, Y_j] \\ + E[Y_i] E[Y_j] Cov[X_i, X_j] \end{bmatrix}
\tag{3.77}
$$

Where $X_i = G_{TC|IM}(z, x_i)$ and $Y_i = \Delta\lambda_{IM}(x_i)$. Again, although these computations look complicated, once implemented in a computer program, they should not be difficult to evaluate.

Accounting for Collapses

Again, we can modify our result to account for costs due to collapse, if collapses are expected to contribute significantly to total repair costs in the structure. We could simply increase the variance in ln*TC*|*IM* (β_R^2) and *E*[ln*TC*|*IM*] (β_U^2) to account for the additional variability in *TC*. However, it may no longer be reasonable to assume that E[ln*TC*|*IM*] is Gaussian, because the collapse and non-collapse cases may result in a bimodal distribution (see **Fig. 3.8**). In this case, we could model the distribution as the sum of two Gaussian distributions, weighted by the probabilities of collapse and non-collapse:

$$\begin{aligned} G_{TC|IM}(z,x) &= \left(1 - P(C \mid IM = x)\right)\Phi\left(\frac{\ln z - E[\ln TC \mid IM = x, NC]}{\sqrt{Var[\ln TC \mid IM = x, NC]}}\right) \\ &\quad + P(C \mid IM = x)\Phi\left(\frac{\ln z - E[\ln TC \mid C]}{\sqrt{Var[\ln TC \mid C]}}\right) \end{aligned} \tag{3.78}$$

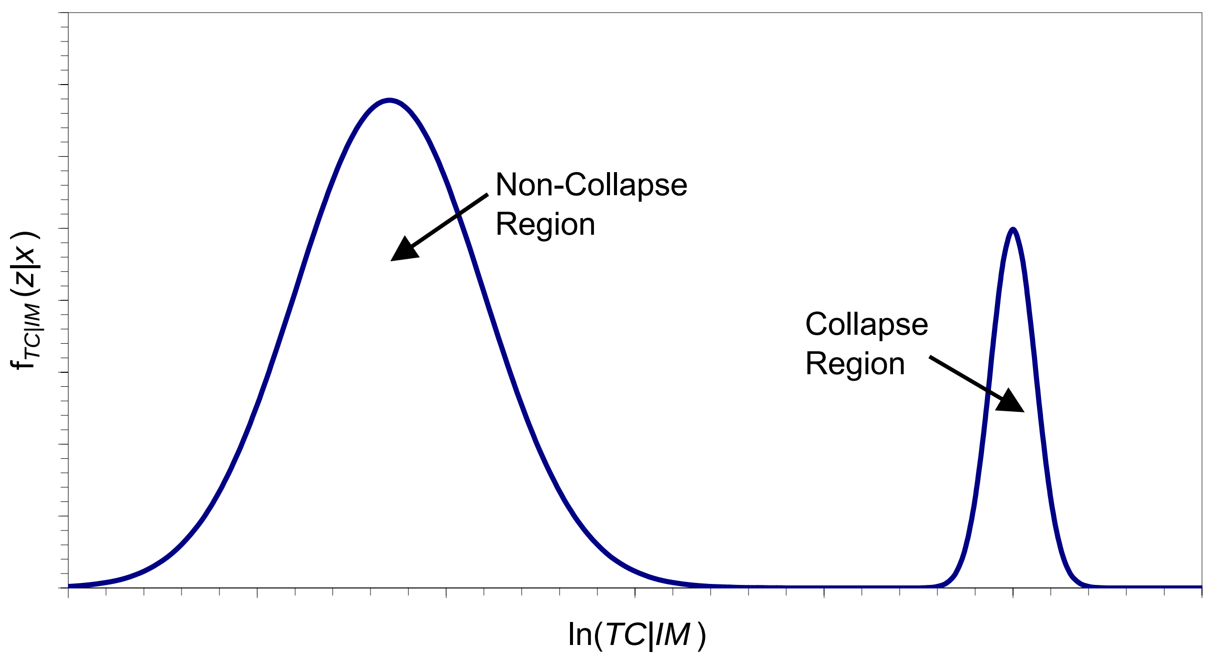

Fig. 3.8: Hypothetical distribution of *TC*|*IM* when both non-collapse and collapse cases are considered

Then we have:

$$
\begin{aligned}
Var[G_{TC|IM}(z,x)] \approx & \left(\frac{\partial G_{TC|IM}(z,x)}{\partial E[\ln TC \mid IM=x,NC]}\right)^2\Bigg|_{\overline{E[\ln TC|IM=x,NC]}} \cdot \beta_U^2(x) \\
& + \left(\frac{\partial G_{TC|IM}(z,x)}{\partial E[\ln TC \mid C]}\right)^2\Bigg|_{\overline{E[\ln TC|C]}} \cdot Var[E[\ln TC \mid C]] \\
& + \left(\frac{\partial G_{TC|IM}(z,x)}{\partial P(C \mid IM=x)}\right)^2\Bigg|_{E[P(C|IM=x)]} \cdot Var[P(C \mid IM=x)] \\
= & \left(1-P(C \mid IM=x)\right)^2 \phi\left(\frac{\ln z - E[\ln TC \mid IM=x]}{\beta_R(x)}\right)^2 \cdot \frac{\beta_U^2(x)}{\beta_R^2(x)} \\
& + P(C \mid IM=x)^2 \phi\left(\frac{\ln z - E[\ln TC \mid C]}{Var[\ln TC \mid C]}\right)^2 \cdot \frac{Var[E[\ln TC \mid C]]}{Var[\ln TC \mid C]} \\
& + \left(\Phi\left(\frac{\ln z - \ln E[TC \mid C]}{\sqrt{Var[\ln TC \mid C]}}\right) - \Phi\left(\frac{\ln z - \ln E[TC \mid IM=x,NC]}{\sqrt{Var[\ln TC \mid IM=x,NC]}}\right)\right)^2 \cdot Var[P(C \mid IM=x)]
\end{aligned} \tag{3.79}
$$

$$
\begin{aligned}
Cov[G_{TC|IM}(z,x_1), G_{TC|IM}(z,x_2)] \approx & \left(1-P(C \mid IM=x_1)\right)\left(1-P(C \mid IM=x_2)\right) \\
& \cdot \phi\left(\frac{\ln z - E[\ln TC \mid IM=x_1]}{\beta_R(x_1)}\right)\phi\left(\frac{\ln z - E[\ln TC \mid IM=x_2]}{\beta_R(x_2)}\right) \\
& \cdot \frac{Cov\left[E[\ln TC \mid IM=x_1], E[\ln TC \mid IM=x_2]\right]}{\beta_R(x_1)\beta_R(x_2)} \\
& + P(C \mid IM=x_1)P(C \mid IM=x_2) \\
& \cdot \phi\left(\frac{\ln z - E[\ln TC \mid C]}{\sqrt{Var[\ln TC \mid C]}}\right)^2 \cdot \frac{Var[E[\ln TC \mid C]]}{Var[\ln TC \mid C]} \\
& + \left(\Phi\left(\frac{\ln z - E[\ln TC \mid C]}{\sqrt{Var[\ln TC \mid C]}}\right) - \Phi\left(\frac{\ln z - E[\ln TC \mid IM=x_1,NC]}{\sqrt{Var[\ln TC \mid IM=x_1,NC]}}\right)\right) \\
& \cdot \left(\Phi\left(\frac{\ln z - E[\ln TC \mid C]}{\sqrt{Var[\ln TC \mid C]}}\right) - \Phi\left(\frac{\ln z - E[\ln TC \mid IM=x_2,NC]}{\sqrt{Var[\ln TC \mid IM=x_2,NC]}}\right)\right) \\
& \cdot Cov\left[P(C \mid IM=x_1), P(C \mid IM=x_2)\right]
\end{aligned} \tag{3.80}
$$

Again, we are assuming no cross-correlation between $E\left[\ln TC \mid NC, IM = x_i\right]$, E[ln$TC$|$C$], $P(C \mid IM = x_i)$ and $\Delta\lambda_{IM}(x_i)$, as stated above in Section 3.13. These new values can now be used in Equations 3.73 and 3.77 as before.

Analytical Solution

As an alternative to the computation described above, there is a less complicated closed form solution that follows the analytic solutions in preceding sections. The result is very similar to Equation 3.41, but includes epistemic uncertainty as well. Using the functional form assumptions of the analytic solutions in previous sections, we have the result:

$$E\left[\lambda_{TC}(z)\right] = k_0 \left(\frac{z}{a'}\right)^{-k/b} \exp\left(\frac{1}{2}\frac{k}{b}\left(\frac{k}{b}-1\right)\left(\beta_R{}^2 + \beta_U{}^2\right)\right) \tag{3.81}$$

We see that this is identical to Equation 3.41, except that both $\beta_R{}^2$ and $\beta_U{}^2$ are included to represent both epistemic and aleatory uncertainty, rather than just aleatory uncertainty as we had before. In addition, we can compute the lognormal epistemic standard deviation of $\lambda_{TC}(z)$:

$$\beta_{\lambda_{TC}(z)} = \sqrt{\beta_{UIM}{}^2 + \frac{k^2}{b^2}\beta_U{}^2} \tag{3.82}$$

This standard deviation allows us to put error bounds on our TC hazard curve, but it is only valid when specified analytic assumptions are made. There are several limitations to this analytical model: it assumes perfect correlation in $\ln E[TC \mid IM = x]$ and $\lambda_{IM}(x)$ over varying levels of *x*, there is no consideration of collapse and the epistemic and aleatory variances $\beta_R{}^2$ and $\beta_U{}^2$ are assumed to be constant at all levels. Nonetheless, this analytical formulation is clearly simpler than the full numeric solution outlined in this section, and so it may be useful in some situations.

This completes our calculations of the means and variances of TC, $\lambda_{collapse}$, and $\lambda_{TC}(z)$ when accounting for epistemic uncertainty. Some of these calculations may be unfamiliar to or conceptually difficult for the reader, and so a very simple numerical example is presented in the next section for illustration.

4 Simple Numerical Example

It is anticipated that the method outlined above will be implemented as a computer algorithm to facilitate bookkeeping. However, to demonstrate the mechanics of the methods presented, a simple analytical calculation is performed. No collapse cases are included, and the procedure is performed for only aleatory uncertainty (rather than for aleatory and epistemic uncertainty, as would be required in a complete analysis).

Consider a two-story frame with two elements per floor. Let EDP_1 and EDP_2 be the *EDP*s for the first and second floor, respectively. Let DVE_1 and DVE_2 be the damage values of the elements on the first floor, and DVE_3 and DVE_4 be the damage values for the elements on the second floor. To demonstrate the generalized equi-correlated model, we will assume that DVE_1 and DVE_2 are of element class 1, and DVE_3 and DVE_4 are of element class 2. Example functions are included as needed, to facilitate demonstration of the method.

4.1 SPECIFY ln EDP | *IM*

Assume a function of the form $EDP_i \mid IM = aIM^b\varepsilon_i$ for both *EDP*s. Then $\ln EDP_i \mid IM = \ln a + b\ln IM + \ln\varepsilon_i$. Let a = 2, b = 2, and define the other needed information as follows (these functions are referred to as Equations 3.2, 3.3, and 3.4 in the above procedure):

$$E[\ln EDP_i \mid IM] = h_i(IM) = \ln 2 + 2\ln IM \quad \text{for i} = 1,2 \tag{4.1}$$

$$Var[\ln EDP_i \mid IM] = h^*_i(IM) = 0.2 \quad \text{for i} = 1,2 \tag{4.2}$$

$$\rho(\ln EDP_1, \ln EDP_2 \mid IM) = \hat{h}_{12}(IM) = 0.8 \tag{4.3}$$

$$\text{thus, } \rho = \begin{bmatrix} \hat{h}_{11} & \hat{h}_{12} \\ \hat{h}_{21} & \hat{h}_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix} \tag{4.4}$$

4.2 SPECIFY COLLAPSED FUNCTION $\ln \mathbf{DVE} \mid \ln \mathbf{EDP}$

For the sake of simplicity in the example, we assume that the collapse of $\mathbf{DM} \mid \ln \mathbf{EDP}$ and $\ln \mathbf{DVE} \mid \mathbf{DM}$ has already been performed, and that we have a function of the form $DVE_k \mid EDP_i = d(1-e^{-mEDP_i})\varepsilon_{DS}\varepsilon_{DFi}\varepsilon_{DEk}$.

Then $\ln DVE_k \mid \ln EDP_i = \ln(d(1-e^{-m(e^{\ln EDP_i})})) + \ln\varepsilon_{Struc} + \ln\varepsilon_{ElClass_m} + \ln\varepsilon_{El_k}$. Let $d = 0.25$, $m = 1$, $Var[\ln\varepsilon_{Struc}] = 0.2$, $Var[\ln\varepsilon_{ElClass_m}] = 0.2$, $Var[\ln\varepsilon_{El_k}] = 0.2$. Note that we are choosing to express cost as a function of replacement cost, so that when IM becomes large, TC is equal to one. Conditional expectations and variances are defined, using the notation in Equations 3.18 and 3.19, as:

$$E[\ln DVE_k \mid \ln EDP_i] = g_k(\ln EDP_i) = \ln(0.25 - 0.25e^{-e^{\ln EDP_i}}) \quad \text{for k =1,2,3,4} \tag{4.5}$$

$$Var[\ln DVE_k \mid \ln EDP_i] = g^*_k(\ln EDP_i) = 0.2 + 0.2 + 0.2 = 0.6 \quad \text{for k =1,2,3,4} \tag{4.6}$$

Using the procedure demonstrated in Equations 3.12 through 3.17, we can calculate correlation coefficients as follows:

$$\begin{aligned}\rho(\ln DVE_1, \ln DVE_2 \mid \ln EDP_1, \ln EDP_1) &= \hat{g}_{12}(\ln EDP_1, \ln EDP_1) \\ &= \frac{Var[\ln\varepsilon_{Struc}] + Var[\ln\varepsilon_{ElClass_1}]}{Var[\ln\varepsilon_{Struc}] + Var[\ln\varepsilon_{ElClass_1}] + Var[\ln\varepsilon_{El_k}]} \\ &= \frac{0.2+0.2}{0.2+0.2+0.2} \\ &= 0.67\end{aligned} \tag{4.7}$$

(Note that $\rho_{12} = \rho_{34}$)

$$\begin{aligned}\rho(\ln DVE_1, \ln DVE_3 \mid \ln EDP_1, \ln EDP_2) &= \frac{Var[\ln\varepsilon_{Struc}]}{Var[\ln\varepsilon_{Struc}] + Var[\ln\varepsilon_{ElClass_m}] + Var[\ln\varepsilon_{El_k}]} \\ &= 0.33\end{aligned} \tag{4.8}$$

(Note that $\rho_{13} = \rho_{14} = \rho_{23} = \rho_{24}$)

These results can be summarized in a correlation matrix, using the notation from Equation 3.20:

$$\rho = \begin{bmatrix} \hat{g}_{11} & \hat{g}_{12} & \hat{g}_{13} & \hat{g}_{14} \\ \hat{g}_{21} & \hat{g}_{22} & \hat{g}_{23} & \hat{g}_{24} \\ \hat{g}_{31} & \hat{g}_{31} & \hat{g}_{31} & \hat{g}_{31} \\ \hat{g}_{41} & \hat{g}_{42} & \hat{g}_{43} & \hat{g}_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0.67 & .033 & 0.33 \\ 0.67 & 1 & 0.33 & 0.33 \\ 0.33 & 0.33 & 1 & 0.67 \\ 0.33 & 0.33 & 0.67 & 1 \end{bmatrix} \tag{4.9}$$

4.3 CALCULATE $\ln$ DVE | IM

Using the FOSM technique and result from Equation 3.26, we can determine the expected value:

$$
\begin{aligned}
E[\ln DVE_1 \mid IM] &\approx g_1\left(h_1(IM)\right) \\
&= \ln\left(0.25 - 0.25e^{-e^{\ln 2 + 2\ln IM}}\right) \\
&= \ln\left(0.25 - 0.25e^{-2IM^2}\right)
\end{aligned}
\tag{4.10}
$$

(Note E[ln DVE_1 | IM] = E[ln DVE_2 | IM] = E[ln DVE_3 | IM] = E[ln DVE_4 | IM])

Variance can be calculated from Equation 3.27:

$$
\begin{aligned}
Var[\ln DVE_1 \mid IM] &\approx g^*_1\,(h_1(IM)) + \left(\frac{\partial g_1}{\partial \ln EDP_1}\right)^2 \Bigg|_{h_1(IM)} h^*_1\,(IM) \\
&= 0.6 + \left(\frac{e^{\ln EDP_1 - e^{\ln EDP_1}}}{1 - e^{-e^{\ln EDP_1}}}\right)^2 \Bigg|_{\ln 2 + 2\ln IM} (0.2) \\
&= 0.6 + 0.2\left(\frac{2e^{2\ln IM - 2e^{2\ln IM}}}{1 - e^{-2e^{2\ln IM}}}\right)^2 \\
&= 0.6 + 0.8\left(\frac{IM^2 e^{-2IM^2}}{1 - e^{-2IM^2}}\right)^2
\end{aligned}
\tag{4.11}
$$

(*Note Var*[ln DVE_1 | IM]=*Var* [ln DVE_2 | IM]=*Var* [ln DVE_3 | IM]=*Var* [ln DVE_4 | IM])

Covariance can be calculated from Equation 3.28:

$$
\begin{aligned}
Cov[\ln DVE_1, \ln DVE_2 \mid IM] &= 0.67\sqrt{0.6}\sqrt{0.6} + \left(\frac{2IM^2 e^{-2IM^2}}{1 - e^{-2IM^2}}\right)\left(\frac{2IM^2 e^{-2IM^2}}{1 - e^{-2IM^2}}\right) 1.0\sqrt{0.2}\sqrt{0.2} \\
&= 0.4 + 0.8\left(\frac{IM^2 e^{-2IM^2}}{1 - e^{-2IM^2}}\right)^2
\end{aligned}
\tag{4.12}
$$

(*Note* $Cov_{12} = Cov_{34}$)

$Cov[\ln DVE_1, \ln DVE_3 \mid IM]$

$$\approx \hat{g}_{13}\sqrt{g^*_1(h_1(IM))}\sqrt{g^*_3(h_2(IM))} + \left(\frac{\partial g_1}{\partial \ln EDP_1}\right)\Bigg|_{h_1(IM)}\left(\frac{\partial g_3}{\partial \ln EDP_2}\right)\Bigg|_{h_1(IM)}\hat{h}_{12}(IM)\sqrt{h^*_1(IM)}\sqrt{h^*_2(IM)}$$

$$= 0.33\sqrt{0.6}\sqrt{0.6} + \left(\frac{2IM^2e^{-2IM^2}}{1-e^{-2IM^2}}\right)\left(\frac{2IM^2e^{-2IM^2}}{1-e^{-2IM^2}}\right)0.8\sqrt{0.2}\sqrt{0.2}$$

$$= 0.2 + 0.64\left(\frac{IM^2e^{-2IM^2}}{1-e^{-2IM^2}}\right)^2 \tag{4.13}$$

(*Note* $\mathrm{Cov}_{13} = \mathrm{Cov}_{14} = \mathrm{Cov}_{23} = \mathrm{Cov}_{24}$)

4.4 SWITCH TO THE NON-LOG FORM DVE | *IM*

From Equations 3.30, 3.31, and 3.32, we can compute the following:

$$\begin{aligned} E[DVE_1 \mid IM] &= E[e^{\ln DVE_1} \mid IM] \\ &\approx e^{g_1(h_1(IM))} \\ &= e^{\ln\left(0.25-0.25e^{-2IM^2}\right)} \\ &= 0.25 - 0.25e^{-2IM^2} \end{aligned} \tag{4.14}$$

$$\begin{aligned} Var[DVE_1 \mid IM] &\approx \left(\frac{\partial e^{\ln DVE_1}}{\partial \ln DVE_1}\right)^2 \Bigg|_{E[\ln DVE_1|IM]} Var[\ln DVE_1 \mid IM] \\ &\approx e^{2g_1(h_1(IM))}Var[\ln DVE_1 \mid IM] \\ &= \left(0.25-0.25e^{-2IM^2}\right)^2\left(0.6+0.8\left(\frac{IM^2e^{-2IM^2}}{1-e^{-2IM^2}}\right)^2\right) \\ &= \left(1-e^{-2IM^2}\right)^2\left(0.0375+0.05\left(\frac{IM^2e^{-2IM^2}}{1-e^{-2IM^2}}\right)^2\right) \end{aligned} \tag{4.15}$$

$$\begin{aligned} Cov[DVE_1, DVE_3 \mid IM] &= e^{g_1(h_1(IM))+g_3(h_2(IM))}Cov[\ln DVE_1, \ln DVE_3 \mid IM] \\ &= \left(0.25-0.25e^{-2IM^2}\right)^2\left(0.2+0.64\left(\frac{IM^2e^{-2IM^2}}{1-e^{-2IM^2}}\right)^2\right) \\ &= \left(1-e^{-2IM^2}\right)^2\left(0.0125+0.04\left(\frac{IM^2e^{-2IM^2}}{1-e^{-2IM^2}}\right)^2\right) \end{aligned} \tag{4.16}$$

$$Cov[DVE_1, DVE_2 \mid IM] = \left(0.25 - 0.25e^{-2IM^2}\right)^2 \left(0.4 + 0.8\left(\frac{IM^2 e^{-2IM^2}}{1-e^{-2IM^2}}\right)^2\right)$$

$$= \left(1 - e^{-2IM^2}\right)^2 \left(0.025 + 0.05\left(\frac{IM^2 e^{-2IM^2}}{1-e^{-2IM^2}}\right)^2\right) \tag{4.17}$$

4.5 COMPUTE MOMENTS OF *TC|IM*

From Equations 3.33 and 3.34, we can compute the following:

$$\begin{aligned} E[TC \mid IM] &= \sum_{k=1}^{4} E[DVE_k \mid IM] \\ &= 4\left(0.25 - 0.25e^{-2IM^2}\right) \\ &= 1 - e^{-2IM^2} \end{aligned} \tag{4.18}$$

$$\begin{aligned} Var[TC \mid IM] &= \sum_{k=1}^{4} Var[DVE_k \mid IM] + 2\sum_{k=1}^{4}\sum_{l=k+1}^{4} Cov[DVE_k, DVE_l \mid IM] \\ &= 4Var[DVE_1 \mid IM] + 2(4)Cov[DVE_1, DVE_3 \mid IM] \\ &\quad + 2(2)Cov[DVE_1, DVE_2 \mid IM] \\ &= 4\left(1 - e^{-2IM^2}\right)^2 \left(0.0375 + 0.05\left(\frac{2IM^2 e^{-2IM^2}}{1-e^{-2IM^2}}\right)^2\right) \\ &\quad + 2(4)\left(1 - e^{-2IM^2}\right)^2 \left(0.0125 + 0.04\left(\frac{IM^2 e^{-2IM^2}}{1-e^{-2IM^2}}\right)^2\right) \\ &\quad + 2(2)\left(1 - e^{-2IM^2}\right)^2 \left(0.025 + 0.05\left(\frac{IM^2 e^{-2IM^2}}{1-e^{-2IM^2}}\right)^2\right) \\ &= \left(1 - e^{-2IM^2}\right)^2 \left(0.35 + 0.72\left(\frac{IM^2 e^{-2IM^2}}{1-e^{-2IM^2}}\right)^2\right) \end{aligned} \tag{4.19}$$

For illustration, we can calculate the coefficient of variation:

$$\delta_{TC \mid IM} = \sqrt{\frac{Var[TC \mid IM]}{E[TC \mid IM]^2}}$$

$$= \sqrt{\frac{\left(1-e^{-2IM^2}\right)^2 \left(0.35 + 0.72\left(\frac{IM^2 e^{-2IM^2}}{1-e^{-2IM^2}}\right)^2\right)}{\left(1-e^{-2IM^2}\right)^2}}$$

$$= \sqrt{0.35 + 0.72\left(\frac{IM^2 e^{-2IM^2}}{1-e^{-2IM^2}}\right)^2} \tag{4.20}$$

It can be shown that in Equation 4.20, the "0.35" term is the contribution due to uncertainty in the cost given the structural response, and the $0.72\left(\frac{IM^2 e^{-2IM^2}}{1-e^{-2IM^2}}\right)^2$ term is the contribution due to uncertainty in the structural response given *IM*. For illustration, the mean of *TC*|*IM* plus/minus one sigma are plotted in Figures 4.1, 4.2, and 4.3, with the various uncertainties present. We can see from these plots that for this example the uncertainty in *DVE*|*EDP* is dominant, and the uncertainty in *EDP*|*IM* has negligible effect. This type of result is quickly calculated using the method outlined above, without any need to re-run records, as would be required with a Monte Carlo technique.

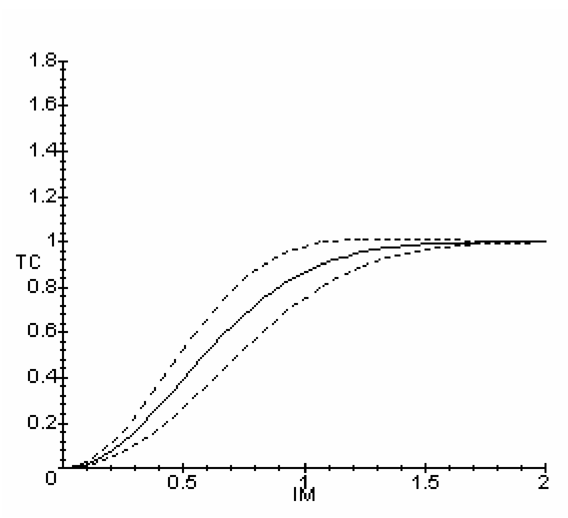

Fig. 4.1: E[*TC*|*IM*], plus/minus one sigma, with uncertainty only in *EDP*|*IM*

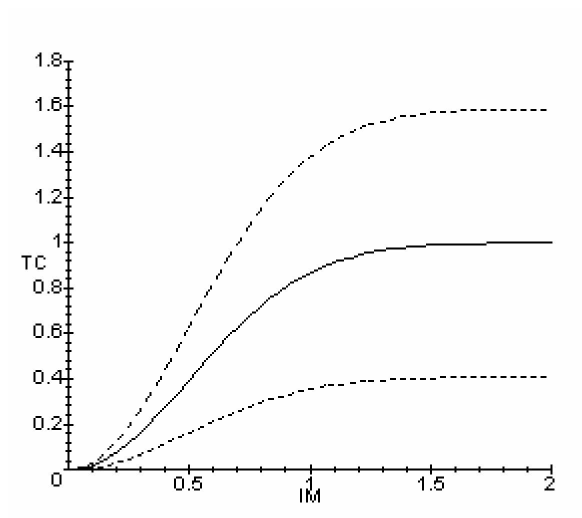

Fig. 4.2: E[*TC*|*IM*], plus/minus one sigma, with uncertainty only in *DVE*|*EDP*

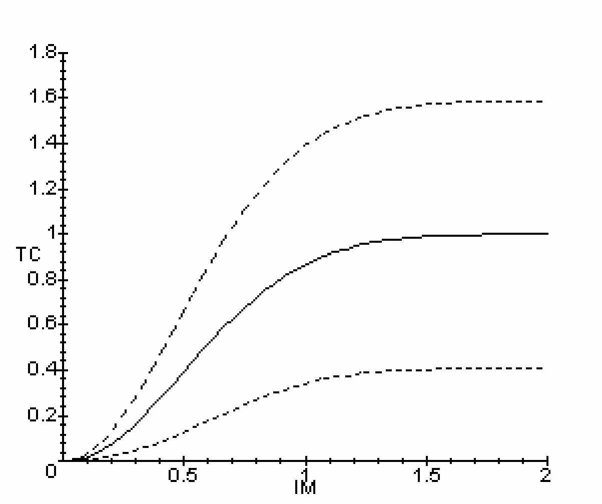

Fig. 4.3: E[*TC*|*IM*], plus/minus one sigma, with all uncertainties

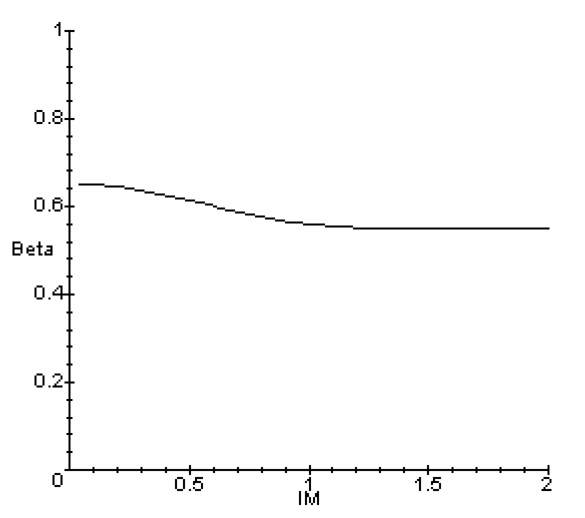

Fig. 4.4: $\sqrt{Var[\ln TC \mid IM]}$**, denoted** $\boldsymbol{\beta_{TC|IM}}$

In Figure 4.4, we have plotted $\beta_{TC|IM}$ vs. *IM* to see how the uncertainty varies as ground motion levels change. For this example, $\beta_{TC|IM}$ is approximately constant and equal to 0.6. We will take advantage of this in our analytical solution to follow. Note that we usually consider $\beta_{TC|IM}$ as approximately equal to the coefficient of variation (e.g., from Equation 4.20), for beta less than 0.3, but this relation is not true for large values of beta, such as in this case.

4.6 INCORPORATE THE SITE HAZARD

For this example, we use a ground motion hazard from the Van Nuys Testbed site, where the *IM* used is S_a at T=0.85s (Fig. 4.6). Then we can use the local slopes of this hazard curve, along with the expected cost function $E[TC|IM]$ to calculate the mean annual value of *TC* (Equation 3.37).

$$\begin{aligned} E[TC] &= \int_{IM} 1 - e^{-2x^2} \left| d\lambda_{IM}(x) \right| \\ &= .0318 \end{aligned} \tag{4.21}$$

In a similar way, the variance can be calculated using Equation 3.38:

$$
\begin{aligned}
Var[TC] &= \int_{IM} q^*(x)\left|d\lambda_{IM}(x)\right| + \int_{IM} q^2(x)\left|d\lambda_{IM}(x)\right| - E[TC]^2 \\
&= \int_{IM} \left(1-e^{-2x^2}\right)^2 \left(0.35 + 0.72\left(\frac{x^2 e^{-2x^2}}{1-e^{-2x^2}}\right)^2\right)\left|d\lambda_{IM}(x)\right| \\
&\quad + \int_{IM} \left(1-e^{-2x^2}\right)^2 \left|d\lambda_{IM}(x)\right| - E[TC]^2 \\
&= 0.0060 + 0.0109 - (0.0318)^2 \\
&= 0.0159
\end{aligned}
\tag{4.22}
$$

and also,

$$
\begin{aligned}
\sigma_{TC} &= \sqrt{Var[TC]} \\
&= 0.1262
\end{aligned}
\tag{4.23}
$$

The above integrals have been computed numerically. As discussed in Section 3.11, there is not an analogous analytic solution, using the functional forms used for the analytic solution of $\lambda_{TC}(z)$.

We can also calculate the annual rate of exceeding a given value of TC by numerical integration using Equation 3.39. Let us assume that $TC \mid IM$ has a lognormal distribution with mean equal to $E[TC \mid IM]$ and variance equal to $Var[TC \mid IM]$, as calculated above. Under this assumption, a plot of $\lambda_{TC}(z)$ for several values of TC is shown in Figure 4.7 on page 48.

The annual rate of exceeding a given TC can also be calculated using the analytical solution of Equation 3.41, if more assumptions are made. Let us approximate $\beta_{TC|IM}$ as constant and equal to 0.6. This approximation is reasonable given the plot of $\beta_{TC|IM}$ shown in Figure 4.4. We then fit $E[TC|IM]$ by the function $E[TC|IM] = 1.4\ IM^{1.8}$. As seen in Figure 4.5, this fit is good over the range $0 < IM < 0.5$.

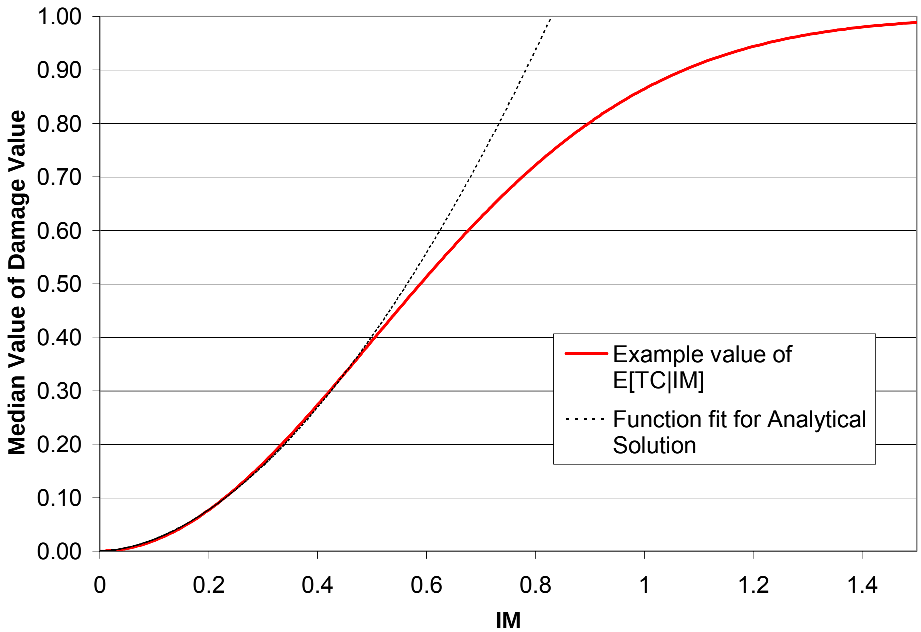

Fig. 4.5: Expected $TC \mid IM$, for example solution and $1.4IM^{1.8}$ fit

Finally, we use a ground motion hazard curve of the form $\lambda_{IM}(x) = k_0 x^{-k}$, where k_0 and k are constants equal to 0.00322 and 3.83, respectively. This hazard curve was obtained by fitting the actual hazard curve at the 2/50 and 10/50 hazard levels (Fig. 4.6).

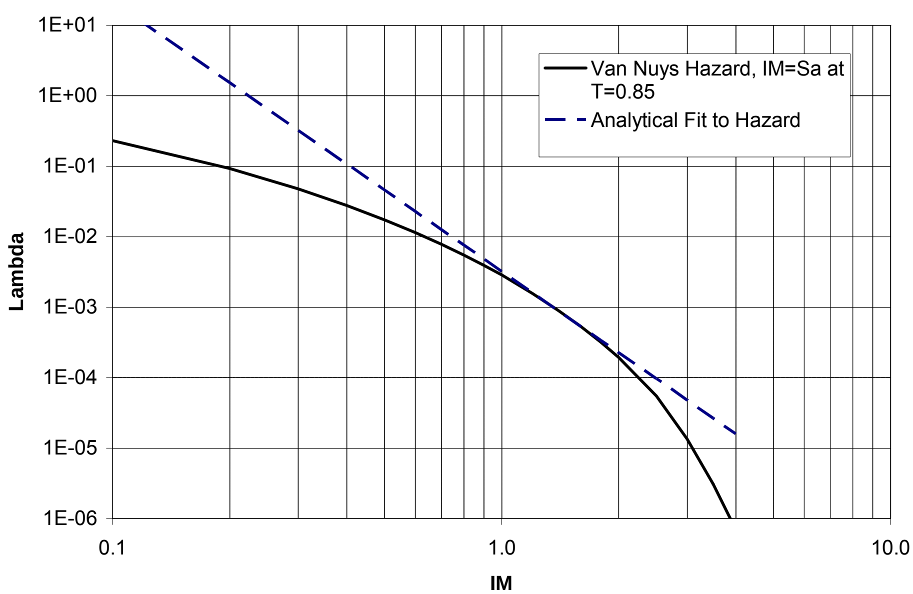

Fig. 4.6: Van Nuys ground motion hazard, and analytical fit

Then the annual rate of exceeding a given value of TC can be estimated analytically using Equation 3.41:

$$\begin{aligned}\lambda_{TC}(z) \quad &= k_0\left(\frac{z}{a'}\right)^{-k/b} \exp\left(\frac{1}{2}\frac{k}{b}\left(\frac{k}{b}-1\right)\beta^*{}_{TC|IM}{}^2\right) \\ &= 0.00322\left(\frac{z}{1.4}\right)^{-3.83/1.8} \exp\left(\frac{1}{2}\left(\frac{3.83}{1.8}\right)\left(\frac{3.83}{1.8}-1\right)0.6^2\right) \\ &= 0.01015 z^{-2.128} \end{aligned} \tag{4.24}$$

A plot of $\lambda_{TC}(z)$ obtained by numerical integration is plotted against the solution of Equation 4.24 in Figure 4.7. We see that the results of the analytical and numerical solutions are in good agreement.

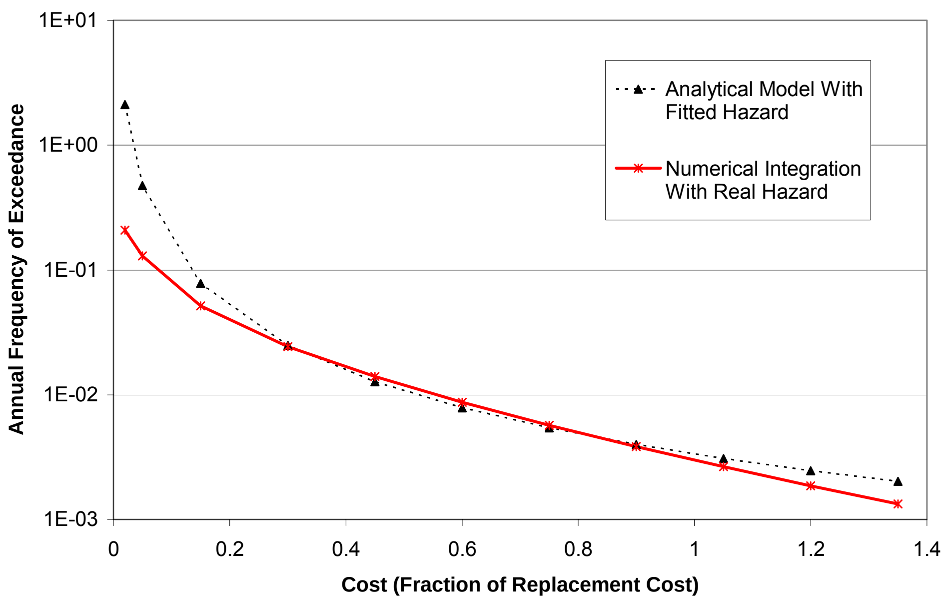

Fig. 4.7: $\lambda_{TC}(z)$: Comparison of numerical integration and analytical solution

Note that the results are in good agreement even for high values of TC, where the analytical fit of $E[TC|IM]$ was not good. This is because low values of expected $TC|IM$ have a significant impact on high TC values due to the high variance in $TC|IM$. High values of expected $TC|IM$ are not as significant due to the infrequent occurrence of high values of IM.

5 The Role of Variance in *TC* given *IM*

Most studies have limited themselves to $E[TC|IM]$ (neglecting $Var[TC|IM]$) with the objective of estimating $E[TC]$ itself. We now examine the role of uncertainty in *TC* given *IM* (which we choose to measure with $\beta_{TC|IM}$), to understand its effect on our results.

We first examine the analytical solution of the above example. The exponential term of Equation 4.24, which we previously called the amplification factor (Equation 3.44), has a value of 1.22. Equivalently, we can say that the annual rate of exceeding a given total cost is increased 22% due to the effects of uncertainty in *TC* (for a given *IM*) in this problem.

More generally, the effect of uncertainty in *TC* given *IM* can easily be seen in Figure 5.1 below. The $E[TC|IM]$ from equation 4.18 above was used, with three constant values of $\beta^*_{TC|IM}$ assumed. Numerical integration was used to calculate $\lambda_{TC}(z)$, per Equation 3.39. We see that for $\beta^*_{TC|IM}$ =1, the result is not dramatically different until we reach high values of *TC* (because our $E[TC|IM]$ never exceeds one, $\lambda_{TC}(z) = 0$ for z>1 when we have no uncertainty). Note that the $\beta^*_{TC|IM}$=0.6 case from Figure 4.7 would be in between the $\beta^*_{TC|IM}$ =0 and $\beta^*_{TC|IM}$=1 in this plot. For $\beta^*_{TC|IM}$ =2, the $\lambda_{TC}(z)$ curve is shifted upward by a factor of three to ten. Also, if we hold $\lambda_{TC}(z)$ constant, the *TC* we expect to see with that given frequency is approximately doubled for $\beta^*_{TC|IM}$=2.

Note that $\beta^*_{TC|IM}$ =1 implies that the 84th percentile of $TC|IM$ is e^1, or 2.7 times the median. So the effect of this large variability in $TC|IM$ is damped by the large implied variability in *IM* (or hazard curve).

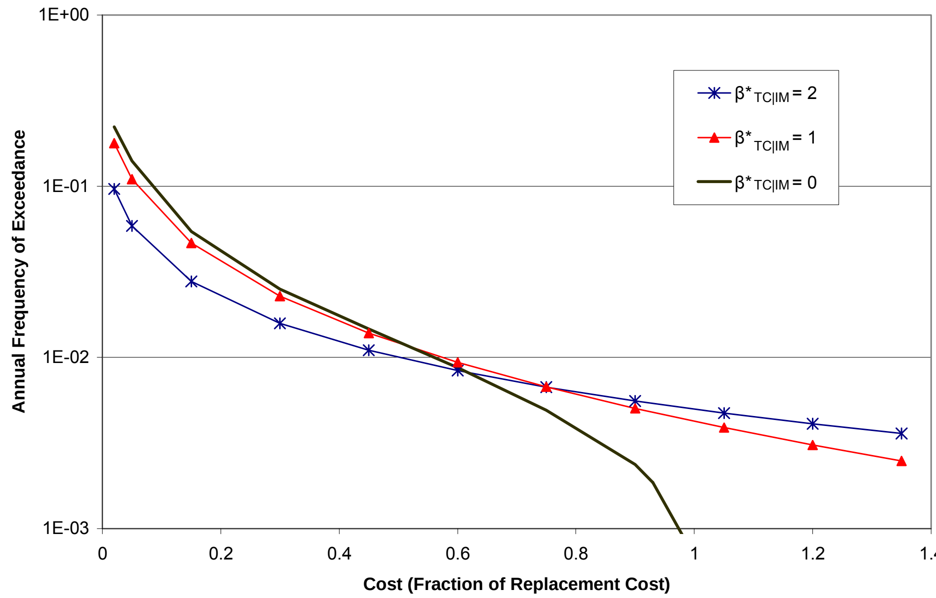

Fig. 5.1: Effect of uncertainty, $\beta_{TC|IM}$, on frequency of exceedance of Total Cost, calculated using numerical integration

To further improve our understanding of the role of uncertainty, we can look at the analytical solution of Equation 3.41. We believe it provides some insight to recognize that for a fixed mean, $E[TC|IM]$, the median TC given IM must decrease as $\beta^*_{TC|IM}$ increases (recall that the mean of a lognormal random variable is the median times $e^{\frac{1}{2}\beta^{*}_{TC|IM}{}^{2}}$). Figure 5.2 shows the shift in the $\lambda_{TC}(z)$ curve as $\beta^*_{TC|IM}$ changes from 0 to 1. Case 0 shows the result for $\beta^*_{TC|IM}$=0. Case 1 shows what the cost curve would look like if the median $TC|IM$ for $\beta^*_{TC|IM}$=1 were used in place of the mean, but with zero uncertainty ($\beta_{TC|IM}$=0). Case 2 shows what the cost curve would look like if the mean $TC|IM$ were used as the median, and with uncertainty included ($\beta^*_{TC|IM}$=1). The effects of Case 1 and Case 2 can not be separated though, so the true result is given by Case 3.

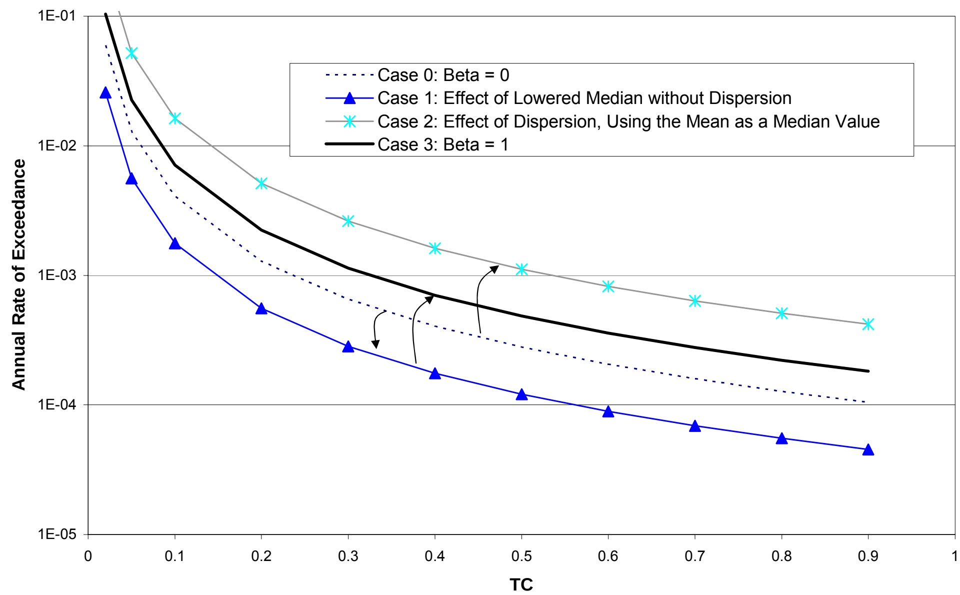

Fig. 5.2: Effect of uncertainty examined using the analytical solution for $\lambda_{TC}(z)$, when $\beta = 1$

We see from this graph that there are two competing effects as we increase $\beta^*_{TC|IM}$ for a given *mean* of $TC|IM$: the reduction due to the median decreasing, and the increase due to the dispersion. The net effect may be either an increase or a decrease depending on the slope of the hazard curve and the rate of increase of TC as IM increases (these are represented by k and b, respectively in Equation 3.41).

The important conclusion to be drawn from this section is that using expected values alone and ignoring uncertainties, although tempting because of its ease, can potentially lead to inaccurate results.

6 Guidelines for Uncertainty Estimation

The procedure outlined in this report is dependent on having values for the means, variances, and covariances of the conditional random variables present in the framing equation (Equation 2.1), which is not a trivial matter. The following may be helpful in providing guidance to those estimating these values. Section 6.1 presents potential sources of structural uncertainty. Section 6.2 presents sample results for expected values and aleatory uncertainty in the Van Nuys Testbed. Section 6.3 provides several references for estimation of uncertainty. Virtually all of the literature discussed here relates to epistemic and aleatory uncertainty in *EDP* given *IM*. The literature on uncertainty in damage and costs (e.g., *DM*|*EDP* and *DVE*|*DM*) is as yet limited.

6.1 POTENTIAL SOURCES OF STRUCTURAL UNCERTAINTY

As an aid in thinking about uncertainty, the following partial list of structural uncertainties proposed by Krawinkler (2002b) is presented:

Global Properties:

1. Period
   - Effects of nonstructural elements (cladding, partitions, infill walls, etc.)
   - Effects of not considered structural elements (staircases, floor systems, etc.)
   - Effect on scaling of Spectral Acceleration at first-mode period
2. Global strength
3. Effective damping

Element Properties

1. Effective initial loading stiffness
   - Modeling uncertainties (e.g., engineering models, using piecewise linear models fit to curves, etc.)
   - Measurement uncertainties
   - Material uncertainties
   - Construction uncertainties
   - History uncertainties (e.g., aging, previous damage, etc.)
2. Effective yield strength
   - Same sources as 1.

3. Effective strain-hardening stiffness
   - Same sources as 1.
4. Effective unloading stiffness
   - Same sources as 1.
5. Ductility capacity
   - Same sources as 1.
6. Post-cap stiffness
   - Same sources as 1.
7. Residual strength
   - Same sources as 1.
8. Cyclic Deterioration

Other Effects

1. Effect of soil-structure interaction
2. 3-D effects

Clearly, there are many sources of uncertainty to consider. These uncertainties, with the possible exceptions of material properties and history, are usually considered to be epistemic uncertainties. Work is progressing toward evaluating a subset of these sources, estimating their uncertainty, and assessing their implied effect on uncertainty in *EDP|IM* (e.g. Ibarra, 2003).

To account for these uncertainties in the structural analysis, it is necessary to vary the above parameters in accordance with the estimated distribution of possible values, and evaluate the resulting uncertainty in the structural response. This could be performed using Monte Carlo simulation or using a finite difference method (see Appendix E for an example procedure). Note that in addition to estimating uncertainty, it will also be necessary to estimate correlations, both in the structural parameters and in *EDP*s. For example, it has been proposed that the element unloading stiffness and element ductility capacity are positively (but not perfectly) correlated (Ibarra, 2003). This correlation should be accounted for in any studies evaluating uncertainty. When using this uncertain element model in an MDOF structure, it will be necessary to assume a correlation structure between, for example, the ductility capacities of each element in the MDOF structure. Here, an assumption of perfect correlation among modeling uncertainties may be valid in some cases.

6.2 ALEATORY UNCERTAINTY IN EDP|IM FOR THE VAN NUYS TESTBED

The Van Nuys Testbed is a structure currently being used by PEER to develop analysis methodologies. Results of nonlinear time history analyses are available for several different IM levels (Lowes 2002). We present results of these analyses, as an example of results that could be used in the analysis procedure outlined above. Spectral acceleration at T=1.5 seconds was used as the IM. Records were scaled to the 50% in 50 year hazard level, as well as the 10% in 50 years, and 2% in 50 year hazard (S_a equal to 0.21g, 0.53g, and 0.97g, respectively). Ten scaled records were run for each IM level. Results from these runs are presented in this section (as discussed further below 0, 5, and 7 collapses were observed at the three hazard levels, and are excluded from the analysis).

6.2.1 Expected value of EDPs

The EDPs selected for study are interstory drift ratios (denoted IDR_i for floor i), and peak floor accelerations (denoted ACC_i for floor i). For consistency with the procedure above, we have averaged the natural logs of the analysis results (denoted $\overline{\ln EDP_i}$), but we display the exponential of this average, (which is an estimate of the median drift or acceleration). Plots of the data are shown below in Figures 6.1 and 6.2.

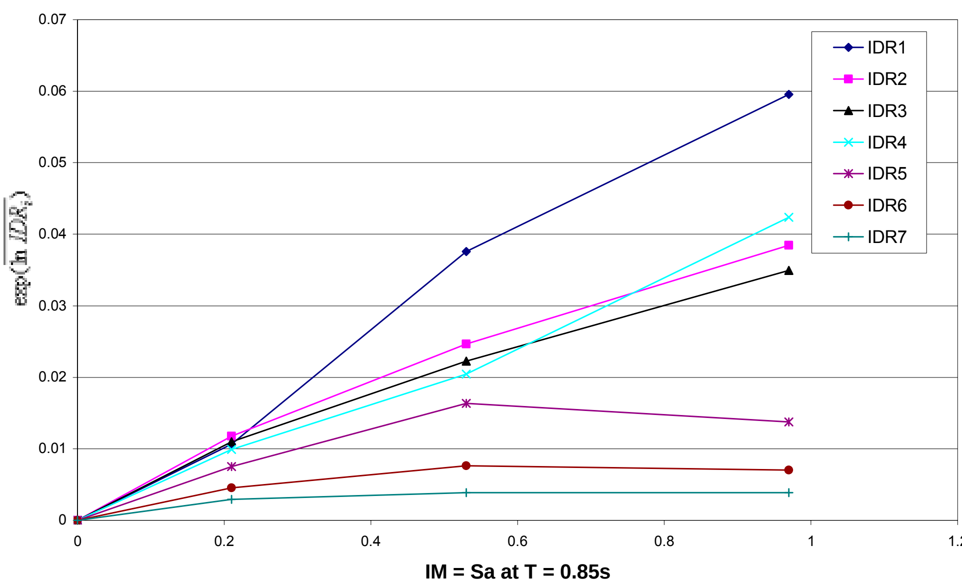

Fig. 6.1: Estimated median of IDR_i ($\exp(\overline{\ln IDR_i})$), conditioned on no collapse and S_a

This data was previously presented in Section 3.1. Data on accelerations is also presented:

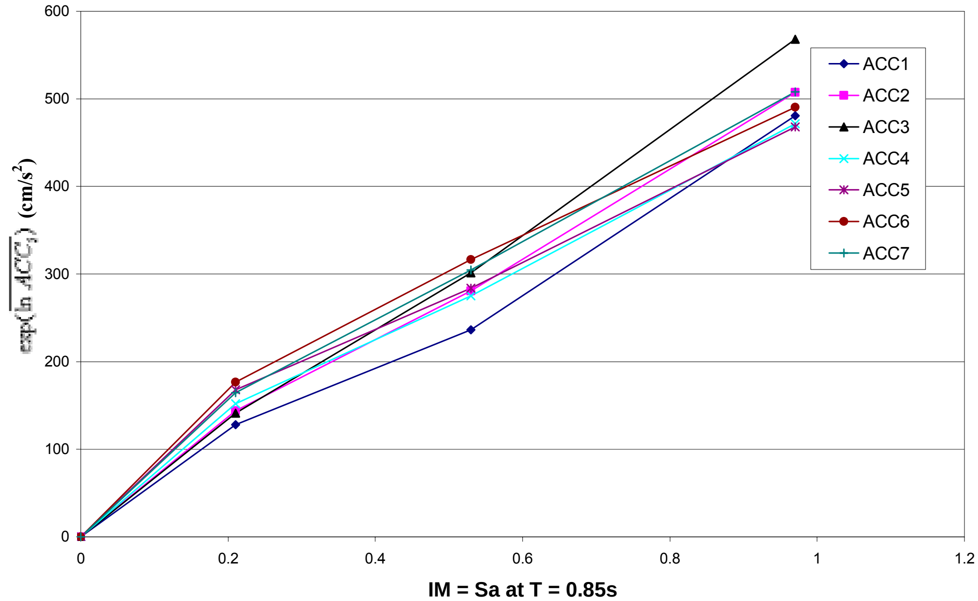

Fig. 6.2: Estimated median of ACC_i ($\exp(\overline{\ln ACC_i})$), conditioned on no collapse and S_a

We see that for the acceleration data, results are very similar for all floors, so perhaps the same function could be used for each lnACC_i when an analysis is performed.

This data on estimates of expected values of lnIDR_i and lnACC_i would be used in Equation 3.2 above.

6.2.2 Aleatory Uncertainty

Aleatory uncertainty in ln$EDP|IM$ can be computed easily by taking logs of the analysis output, and computing variances. Plots of the data are shown below in Figures 6.3 and 6.4.

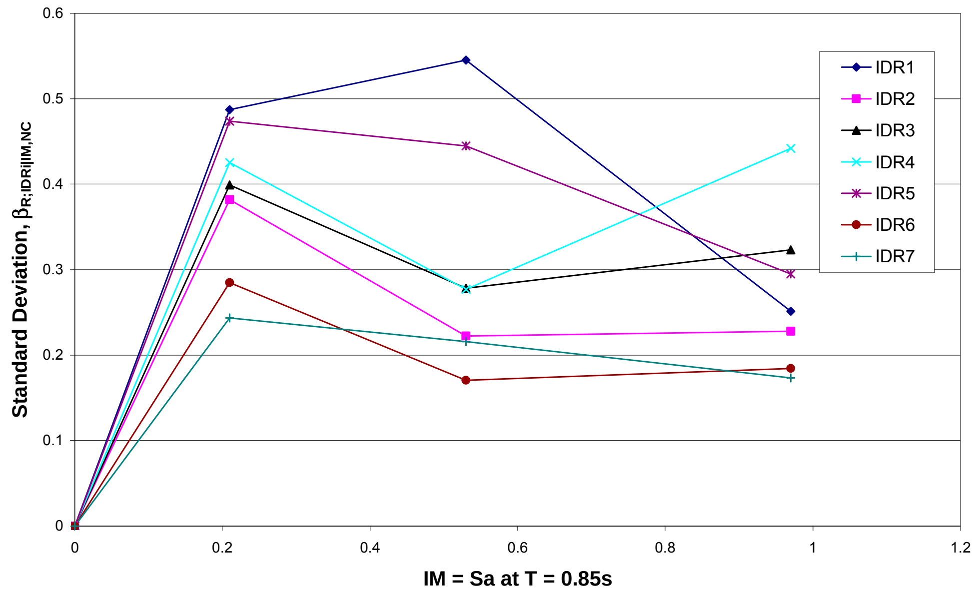

Fig. 6.3: Estimated standard deviation in ln*IDR*$_i$, conditioned on no collapse and S_a

The same results are shown for accelerations:

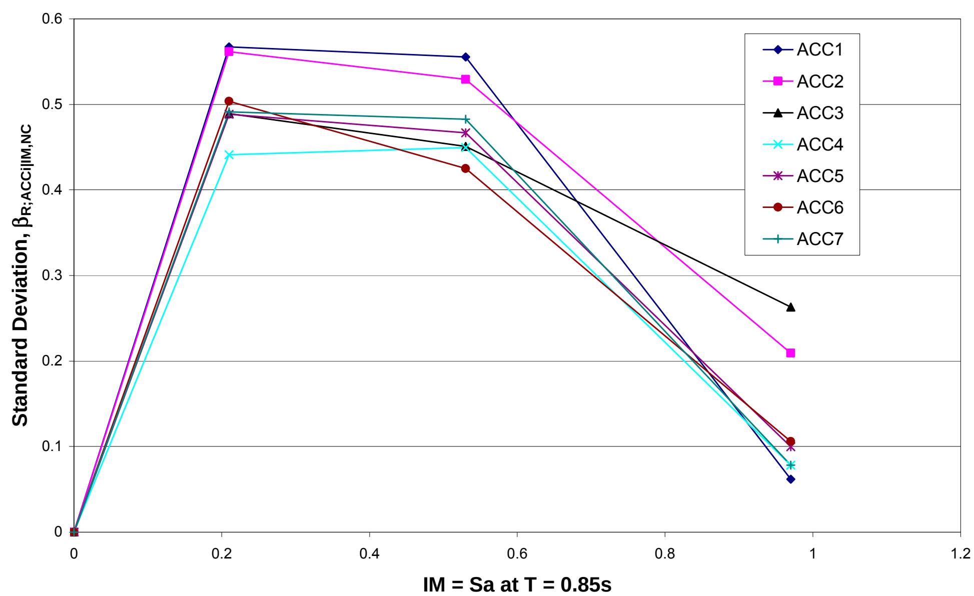

Fig. 6.4: Estimated standard deviation in ln*ACC*$_i$, conditioned on no collapse and S_a

This data represents estimates of the $\beta_{R;EDP|IM}$ values referred to in Equation 3.46. These estimates would be used in Equation 3.3 above. Note that in addition to providing an estimate of variance in the data, we can also estimate the variance in the estimate of the mean, due to the limited sample size. This variance is equal to $\sigma^2 / \sqrt{n}$ (in our case, we could state that the variance in lnEDP is equal to $\beta^2 / \sqrt{n}$). This is one part of the epistemic uncertainty in ln$EDP|IM$, and could be combined with estimates of epistemic uncertainty of other types.

6.2.3 Correlations

Correlations between lnEDPs are presented[5] from the 50/50 hazard analyses (Table 6.1). It is suggested that these correlations be assumed constant for all *IM* levels. Correlations are available from analysis at higher *IM* levels, but due to a lack of data (because many runs result in collapse and are excluded from this statistical analysis), it is more difficult to estimate correlations.

Table 6.1 Sample Correlation Coefficients for ln*EDP*s

	$\ln(IDR_1)$	$\ln(IDR_2)$	$\ln(IDR_3)$	$\ln(IDR_4)$	$\ln(IDR_5)$	$\ln(IDR_6)$	$\ln(IDR_7)$	$\ln(ACC_1)$	$\ln(ACC_2)$	$\ln(ACC_3)$	$\ln(ACC_4)$	$\ln(ACC_5)$	$\ln(ACC_6)$	$\ln(ACC_7)$
$\ln(IDR_1)$	1.00	0.93	0.81	0.59	0.51	0.40	0.20	0.34	0.28	0.17	0.10	0.17	0.37	-0.04
$\ln(IDR_2)$		1.00	0.94	0.80	0.72	0.60	0.46	0.47	0.43	0.36	0.33	0.39	0.53	0.20
$\ln(IDR_3)$			1.00	0.92	0.82	0.68	0.56	0.54	0.50	0.45	0.40	0.39	0.52	0.30
$\ln(IDR_4)$				1.00	0.95	0.79	0.70	0.67	0.67	0.62	0.54	0.52	0.61	0.51
$\ln(IDR_5)$					1.00	0.93	0.84	0.75	0.80	0.75	0.66	0.66	0.74	0.65
$\ln(IDR_6)$						1.00	0.91	0.80	0.86	0.86	0.81	0.79	0.85	0.75
$\ln(IDR_7)$							1.00	0.66	0.74	0.75	0.80	0.86	0.79	0.80
$\ln(ACC_1)$								1.00	0.94	0.92	0.85	0.77	0.91	0.83
$\ln(ACC_2)$									1.00	0.96	0.87	0.79	0.92	0.87
$\ln(ACC_3)$										1.00	0.91	0.79	0.91	0.92
$\ln(ACC_4)$											1.00	0.93	0.93	0.92
$\ln(ACC_5)$												1.00	0.93	0.87
$\ln(ACC_6)$													1.00	0.88
$\ln(ACC_7)$														1.00

Some relationships are apparent in this data. For example, the correlation between *IDR*s is higher for floors near each other than it is for floors far apart. A plot of correlation coefficients versus. number of stories of separation is shown in Figure 6.5.

[5] For low β's, the correlation coefficients between ln*EDP*s are not significantly different than the correlations between the non-log *EDP*s, but at the higher levels of β seen in this example, some correlation coefficients do differ significantly. Here we have presented the ln*EDP*s for consistency with the procedure of Section 4.

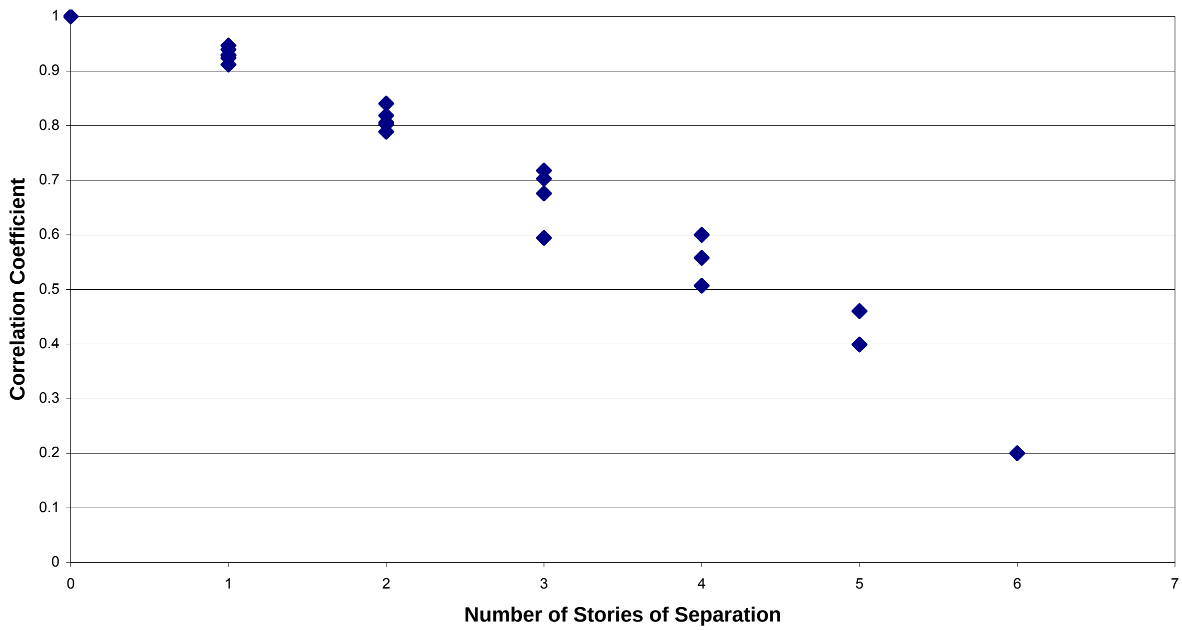

Fig. 6.5: Correlation coefficient vs. number of stories of separation (for interstory drift ratios) at 50/50 hazard level

If desired, a regression line could be fit to this data, and the predicted correlation coefficient could be used in place of the data points calculated. Similar plots can be produced for floor acceleration correlations, and correlations between *IDR*s and accelerations, but the results do not show relations as strong as that in Figure 6.5. Note that this data on correlations would be used in Equation 3.4 above.

6.2.4 Probability of Collapse

Probability of collapse was estimated by the fraction of the 10 runs that resulted in collapse for a given *IM* level. Collapse was assumed to occur if any floor experienced an *IDR* of greater than 10%. At the 50/50 hazard, no collapses occurred, while five collapses occurred at the 10/50 hazard level, and seven at the 2/50 hazard. Predictions for the probability of collapse at levels other than these three stripes could be estimated using linear interpolation, or by fitting a lognormal CDF, as shown in Fig. 6.6.

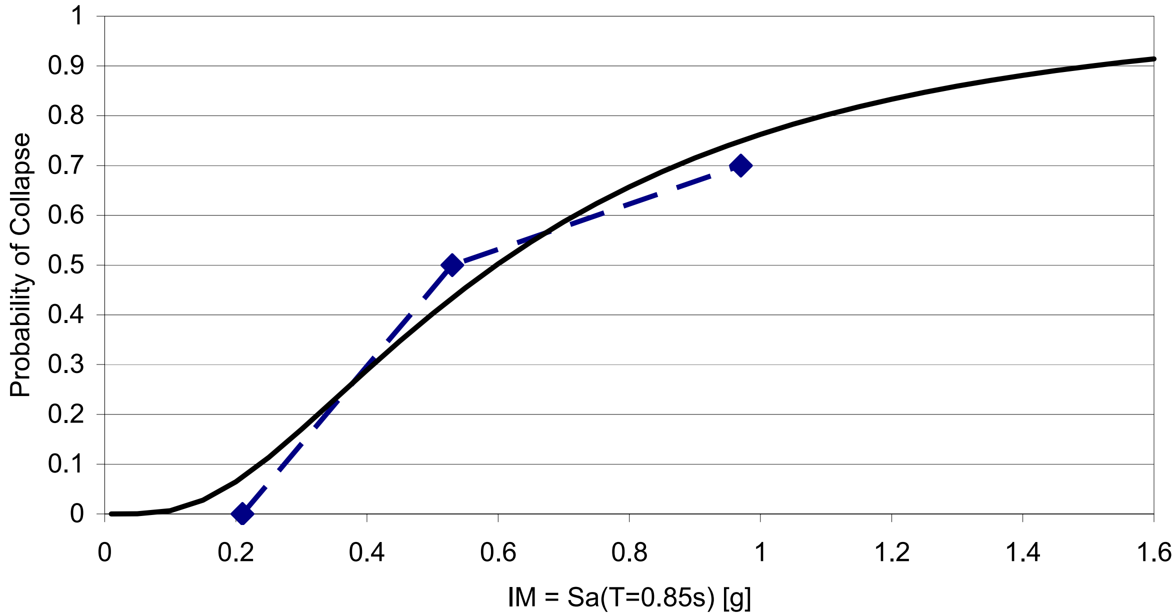

Fig. 6.6: Probability of collapse, conditioned on *IM*

This probability of collapse data is needed for the procedure presented in Section 3.6. The data from the preceding sections are all that is needed for the *EDP|IM* section of analysis with aleatory uncertainty. The additional information needed is an estimate of epistemic uncertainty.

6.3 REFERENCES FOR ESTIMATION OF EPISTEMIC UNCERTAINTY IN EDP|IM

As a resource for researchers quantifying epistemic uncertainty, several publications in this area are cited, including a brief description of their applicability to this problem. All of these publications are concerned with the uncertainty in *EDP|IM*. There is little or no literature on the uncertainty in *DVE|EDP*.

Kennedy and Ravindra (1984) published a reference on seismic fragilities for nuclear power plants (NPPs) that is a standard reference in that industry. Several tables in this document list β_R and β_U, (quantifying logarithmic aleatory and epistemic standard deviations, respectively) disaggregated by source of uncertainty (e.g., modeling effects, soil-structure interaction, damping effects, etc.). The individual entries represent the uncertainty in the effect of each parameter, i.e., the product of the uncertainty in a particular parameter (e.g., damping) *times* the sensitivity of the demand or capacity to that factor (e.g., the partial derivative of demand with respect to damping). See Appendix E for an example technique for computing this partial derivative and resulting effect on uncertainty in demand. The sources of uncertainty presented in this reference are similar to the list given in Section 6.1 above (Table 6.2 for values quoted in that reference). However, the numbers presented are for a limited class of structures, which vary greatly in structural system, quality control, etc., from most other civil structures. The numbers are based primarily on elastic analysis, they reflect comparatively small ductility levels and use peak ground acceleration (PGA) as the *IM*. For these reasons, the numbers presented are not very appropriate for general use, but they do present a good example of uncertainty quantification. If one were analyzing a structure of this class using the procedure outlined in this paper, the value of epistemic uncertainty in *EDP* given *IM* would be the value highlighted in Table 6.2, i.e., β_{UD} = 0.18 to 0.33. This value would be used for $\beta_{U;EDP|IM}$, as denoted in Equation 3.47, and used in Equation 3.3 of the procedure.

Table 6.2 Examples of β_R and β_U for NPP Structures, from Kennedy and Ravindra (1984)

	β_R	β_U
Capacity		
Ultimate strength vs. code allowable	0.06-0.12	0.12-0.18
Inelastic absorption capacity	0.08-0.14	0.18-0.26
Total capacity uncertainty	0.10-0.18	0.22-0.32
Demand		
Design response spectra	0.16-0.22	0.08-0.11
Damping effects	0.05-0.10	0.05-0.10
Modeling effects	0	0.12-0.18
Modal and component combination	0.10-0.20	0
Soil-structure interaction	0.02-0.06	0.10-0.24
Total demand uncertainty	0.20-0.32	***0.18-0.33***
Total uncertainty	0.22-0.37	0.28-0.46

FEMA-350 (2000a) incorporates uncertainty into the analysis procedure for regular steel moment-frame buildings and provides procedures for estimating this uncertainty. Appendix A of the document contains recommended values for β_{UD} (Table 6.3 for example data) for steel moment resisting frames of varying heights, at two performance levels (i.e., levels of nonlinearity), and for two connection types (pre- and post-Northridge). This table also shows variation of uncertainty in the results for several analysis methods. Note that these values are based on interstory drift demand given first-period spectral acceleration. Several other publications (e.g., FEMA, 2000b and Yun et al., 2002) show the bases for these values. These values of β_{UD} could be used for $\beta_{U;EDP|IM}$, as denoted in Equation 3.47, and used in Equation 3.3 of the procedure.

Table 6.3 Uncertainty Coefficient β_{UD} for Evaluation of Steel Moment-Frame Buildings (FEMA 2000a)

	Analysis Procedure							
	Linear Static		Linear Dynamic		Nonlinear Static		Nonlinear Dynamic	
Performance Level	IO	CP	IO	CP	IO	CP	IO	CP
Type I Connections								
Low Rise (<4 stories)	0.17	0.22	0.15	0.16	0.14	0.17	0.10	0.15
Mid Rise (4-12 stories)	0.18	0.29	0.15	0.23	0.15	0.23	0.13	0.20
High Rise (>12 stories)	0.31	0.25	0.19	0.29	0.17	0.27	0.17	0.25
Type II Connections								
Low Rise (<4 stories)	0.19	0.23	0.16	0.25	0.18	0.18	0.10	0.15
Mid Rise (4-12 stories)	0.20	0.30	0.17	0.33	0.14	0.21	0.13	0.20
High Rise (>12 stories)	0.21	0.36	0.21	0.31	0.18	0.33	0.17	0.25

The seismic assessment guidelines under development by PEER for several classes of buildings in an electrical distribution system (Bazzurro et al. 2002) provide expert-elicitation-based estimates of uncertainty, as measured in first-mode spectral acceleration terms. These values represent total epistemic uncertainty in the ground motion capacity — i.e., uncertainty in the level of ground motion that will result in the given limit state. These values give an idea of the relative uncertainties associated with different building types (e.g., newer, cleaner SMRFs versus older, irregular mill-type construction) and for different limit states (nearly linear versus collapsed). It is presumed that a nonlinear static pushover analysis has been conducted. See Table 6.4 below for examples of values quoted in that reference. The authors in this publication are estimating uncertainty in a manner similar to that used by Kennedy and Ravindra above, except that Kennedy and Ravindra use PGA as the *IM,* and Bazzurro et al. used first-mode spectral acceleration. The difference in values results primarily from the great difference in classes of structures being studied, the levels of nonlinearity, the information available on material properties, quality control, etc. Note that, at best, the values in Table 6.4 are an upper bound to the $\beta_{U;EDP|IM}$ used in the procedure above. Uncertainty in a different quantity than $EDP|IM$ is being measured, so the values cannot be simply substituted into the equations above. They are most appropriately used merely as an example of uncertainty estimation/quantification in a somewhat-related structural analysis problem.

Table 6.4 Example Uncertainty Coefficients β_U for Collapse Capacity in Different Damage States (Bazzurro et al. 2002)

	β_U
Unretrofitted tilt-up buildings	
Onset of Damage	0.7
Collapse	0.8
SMRF Buildings	
Onset of Damage	0.3
Collapse	0.5
Mill-type buildings	
Onset of Damage	0.7
Collapse	1.0

These references above may be useful in the analysis of some classes of buildings. In the absence of any other information to quantify epistemic uncertainty in *EDP* given *IM*, we suggest using $\beta_{U;EDP|IM} \cong 0.5$ (with correlations between *EDP*s equal to one) as a very rough value, assuming nonlinear time history analysis is used. It is not realistic to define a single value for a large class of structures, but this starting point will at least roughly account for epistemic uncertainty in *EDP* given *IM* when performing the calculations in this procedure.

7 Conclusion

A procedure for calculating an estimate of the uncertainty in repair costs due to earthquake damage has been proposed. This procedure works within the framework proposed by PEER for performance-based earthquake engineering. The Total Cost is defined is a function of repair costs for individual building elements, except in the collapse case, where separate cost estimation is used. The calculation procedure combines inputs of aleatory and epistemic uncertainty in ground motion hazard, building response, damage to building elements, and element repair costs to produce an estimate of the uncertainty in total repair cost.

The proposed procedure uses the approximate first-order second-moment (FOSM) method to collapse what may be high-dimensional vectors of conditional random variables into a single conditional random variable, Total Cost given *IM*. Numerical integration is then used to incorporate the ground motion hazard. This uncertainty is treated accurately because it is likely to be the dominant contributor. The quantities that can be computed are the expected value and variance of the mean annual loss, expected value and variance in mean frequency of collapse, and the expected value and variance of the mean annual rate of exceeding a given cost. It is anticipated that this procedure will be implemented in the form of a computer program. In this form, it should be easy to vary input values of means, variances, and correlations, and quickly see the end effect of these variations (i.e., a sensitivity analysis).

The report incorporates both aleatory and epistemic uncertainty in all of the analysis. While this aleatory/epistemic treatment is well developed in some areas (e.g., annual frequency of failure calculations in the nuclear industry), in cost estimation efforts to date, quantification of uncertainty is generally limited to the aleatory type. Recognizing this, we have tried to lay out consistent schemes for the incorporation of epistemic uncertainty in the future. While we appreciate the increased complexity (both in estimation and analysis) this entails, we believe it to be a necessary component in the development of this field. This is especially true because in its current data-poor early stages, cost analysis involves very large epistemic uncertainties which may have a very different correlation structure than their aleatory counterparts, affecting significantly the net variance of $TC|IM$..

The report makes suggestions for representation of correlation among the aleatory and epistemic variables, such as element repair costs, where data and information are very limited. These representations take the form of generalizations of the concept of "equi-correlations." Guidelines for estimating uncertainty in $EDP|IM$ are also presented. Values for aleatory variances, and correlations, are presented for an example structure. Several previous studies attempting to characterize epistemic uncertainty are referenced as an aid. By contrast, uncertainty in repair costs given *EDP* or *DM* is as yet a relatively undeveloped topic.

To facilitate understanding of the calculations, an example analysis has been performed. This example allows illustration of the mechanics of the calculations. The results of the example are also useful for illustrating the effect of variance on the mean annual rate of exceedance of a given Total Cost. It illustrates that variance in $TC|IM$ may or may not have a significant effect on the annual rate of exceeding a given Total Cost.

APPENDIX A: DERIVATION OF CORRELATION USING THE EQUI-CORRELATED MODEL

A general notation is used in this appendix, rather than notation for a specific case from the text. The following derivations are applicable to several situations in the text, such as Examples 1 and 2 in Section 3.2. In the case of the Section 3.2 examples, Y_1 is equivalent to $\ln DVE_k \mid \ln EDP_i$, a_1 is equivalent to $g_k(\ln EDP_i)$, B is equivalent to $\ln \varepsilon_{Struc}$, C_1 is equivalent to $\ln \varepsilon_{El_k}$, etc.

Let

$$\begin{aligned} Y_1 &= a_1 + B + C_1 \\ Y_2 &= a_2 + B + C_2 \end{aligned} \tag{A.1}$$

where a_1 and a_2 are constants, and B, C_1 and C_2 are mutually uncorrelated random variables.
Let variances of the random variables be:

$$\begin{aligned} Var[B] &= \sigma_B^{\ 2} \\ Var[C_1] &= \sigma_{C_1}^{\ 2} \\ Var[C_2] &= \sigma_{C_2}^{\ 2} \end{aligned} \tag{A.2}$$

Which implies that:

$$\begin{aligned} Var[Y_1] &= \sigma_B^{\ 2} + \sigma_{C_1}^{\ 2} \\ Var[Y_2] &= \sigma_B^{\ 2} + \sigma_{C_2}^{\ 2} \end{aligned} \tag{A.3}$$

Then

$$\begin{aligned} Cov[Y_1, Y_2] &= Cov[(a_1 + B + C_1),(a_2 + B + C_2)] \\ &= Cov[a_1, a_2] + Cov[a_1, B] + Cov[a_1, C_1] + Cov[B, a_2] + \ldots \\ &= Cov[B, B] + 0 + 0 + \ldots \\ &= \sigma_B^{\ 2} \end{aligned} \tag{A.4}$$

$$\begin{aligned} \rho_{Y_1,Y_2} &= \frac{Cov[Y_1, Y_2]}{\sqrt{Var[Y_1]Var[Y_2]}} \\ &= \frac{\sigma_B^{\ 2}}{\sqrt{\left(\sigma_B^{\ 2} + \sigma_{C_1}^{\ 2}\right)\left(\sigma_B^{\ 2} + \sigma_{C_2}^{\ 2}\right)}} \end{aligned}$$

$$= \frac{1}{\sqrt{\left(\sigma_{C_1}{}^2 / \sigma_B{}^2 + 1\right)\left(\sigma_{C_2}{}^2 / \sigma_B{}^2 + 1\right)}} \tag{A.5}$$

In the special case where $\sigma_{C_1}{}^2 = \sigma_{C_2}{}^2 = \sigma_C{}^2$,

$$\rho_{Y_1,Y_2} = \frac{\sigma_B{}^2}{\sqrt{\left(\sigma_B{}^2 + \sigma_C{}^2\right)\left(\sigma_B{}^2 + \sigma_C{}^2\right)}}$$

$$= \frac{\sigma_B{}^2}{\sigma_B{}^2 + \sigma_C{}^2} \tag{A.6}$$

For a system of n random variables $Y_1, Y_2, \ldots Y_n$, we can define a random variable B common for all elements and a unique C_i for each element. If Var[C_i] is equal for all i, then the values of the correlation matrix would be equal (Equation A.6) for all $\{Y_i, Y_j\}$ when $i \neq j$, and have value 1 for $i = j$. This is known as the *equi-correlated model*.

The equi-correlated model can be easily extended to a sum of more than two random variables, as long as all of the random variables are either perfectly correlated or uncorrelated. For example, let

$$\begin{aligned} Y_1 &= a_1 + B + D_1 + E_1 \\ Y_2 &= a_2 + B + D_1 + E_2 \end{aligned} \tag{A.7}$$

where $Var[D_1] = \sigma_{D_1}{}^2$, $Var[E_1] = \sigma_{E_1}{}^2$ and $Var[E_2] = \sigma_{E_2}{}^2$

Noting that ($B + D_1$) is equivalent to B in equations A.1, it is easily shown that

$$\rho_{Y_1,Y_2} = \frac{1}{\sqrt{\left(\sigma_{E_1}{}^2 / \left(\sigma_B{}^2 + \sigma_{D_1}{}^2\right) + 1\right)\left(\sigma_{E_2}{}^2 / \left(\sigma_B{}^2 + \sigma_{D_1}{}^2\right) + 1\right)}} \tag{A.8}$$

And when $\sigma_E{}^2 = \sigma_{E_1}{}^2 = \sigma_{E_2}{}^2$,

$$\rho_{Y_1,Y_2} = \frac{\sigma_B{}^2 + \sigma_{D_1}{}^2}{\sigma_B{}^2 + \sigma_{D_1}{}^2 + \sigma_E{}^2} \tag{A.9}$$

This formulation is used for Example 1 on page 12 of the text.

Consider next the case where the "D" random variable is not common to Y_1 and Y_2, that is:

$$
\begin{aligned}
Y_1 &= a_1 + B + D_1 + E_1 \\
Y_2 &= a_2 + B + D_2 + E_2
\end{aligned} \tag{A.10}
$$

where $Var[D_2] = \sigma_{D_2}{}^2$ and the other random variables are defined as above. Noting that $(D_1 + E_1)$ is equivalent to C_1 in equation A.1, it is easily shown that

$$
\rho_{Y_1,Y_2} = \frac{1}{\sqrt{\left(\left(\sigma_{D_1}{}^2 + \sigma_{E_1}{}^2\right)/\sigma_B{}^2 + 1\right)\left(\left(\sigma_{D_2}{}^2 + \sigma_{E_2}{}^2\right)/\sigma_B{}^2 + 1\right)}} \tag{A.11}
$$

And when $\sigma_D{}^2 = \sigma_{D_1}{}^2 = \sigma_{D_2}{}^2$ and $\sigma_E{}^2 = \sigma_{E_1}{}^2 = \sigma_{E_2}{}^2$,

$$
\rho_{Y_1,Y_2} = \frac{\sigma_B{}^2}{\sigma_B{}^2 + \sigma_D{}^2 + \sigma_E{}^2} \tag{A.12}
$$

This formulation is used for Example 2 on page 12 of the text.

For a system of random variables $Y_1, Y_2, \ldots Y_n$, we can define B common for all elements, a series of E_is unique for each element, and a series of D_ks shared between only variables of class k. If Var[E_i] is equal for all i, then we can use the simple Equations A.9 and A.12 to calculate correlation coefficients. However, now the correlation matrix will not be equal for all $\{Y_i, Y_j\}$ when $i \neq j$, but will instead depend on whether the variables share the same D_k. We shall refer to this as the *generalized equi-correlated model*.

Note that throughout this appendix, we have not explicitly shown any conditioning notion or notation. But as seen for example in Equation 3.20, where conditioning on *EDP* is displayed, in virtually all applications here these results will be conditioned upon values such as EDP_i and EDP_j, in which case the correlation coefficient (such as that in A.6) will simply be a function of EDP_i and EDP_j. In the simplest application this functional dependence will not exist (i.e., these variances and covariances will be constant with respect to *EDP* levels). This has been assumed for the development and examples in this paper.

The principles used to develop these results are taken from Ditlevsen (1981). This line of thinking is further developed in this reference.

APPENDIX B: DERIVATION OF MOMENTS OF CONDITIONAL RANDOM VARIABLES

Let X, Y, and Z be random variables, with the distributions of X and Y conditioned on Z. $E[X \mid Z=z]$ is the conditional expectation of X given Z=z:

$$E[X \mid Z=z] = \int_{-\infty}^{\infty} x \; f_{X|Z}(x,z)dx \tag{B.1}$$

We have used $E[X \mid Z=z]$ to spell out in detail what we mean by the conditional expectation, but normally we shall write $E[X \mid Z=z]$ as $E[X \mid Z]$ as a shorthand notation.

$E\big[E[X \mid Z]\big]$ is the expectation (with respect to Z) of $E[X \mid Z]$, i.e., when Z is recognized as random:

$$\begin{aligned} E\big[E[X \mid Z]\big] &= \int_{-\infty}^{\infty} E[X \mid Z=z] f_Z(z)dz \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x \; f_{X|Z}(x,z) f_Z(z)dxdz \\ &= E[X] \end{aligned} \tag{B.2}$$

Note that the same result is found by reversing the order of integration.

$$\begin{aligned} E\big[E[X \mid Z]\big] &= \int_{-\infty}^{\infty} E[X \mid Z=z] f_Z(z)dz \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x \; f_{X|Z}(x,z) f_Z(z)dxdz \\ &= \int_{-\infty}^{\infty} x \int_{-\infty}^{\infty} f_{X|Z}(x,z) f_Z(z)dzdx \\ &= \int_{-\infty}^{\infty} x \; f_X(x)dx \\ &= E[X] \end{aligned} \tag{B.3}$$

Then we see that

$$
\begin{aligned}
Var[X] &= E[X^2] - E[X]^2 \\
&= E\big[E[X^2 \mid Z]\big] - E\big[E[X \mid Z]\big]^2 \\
&= E\big[Var[X \mid Z] + E[X \mid Z]^2\big] - E\big[E[X \mid Z]\big]^2 \\
&= E\big[Var[X \mid Z]\big] + E\big[E[X \mid Z]^2\big] - E\big[E[X \mid Z]\big]^2 \\
&= E\big[Var[X \mid Z]\big] + Var\big[E[X \mid Z]\big]
\end{aligned}
\tag{B.4}
$$

This is a very important result useful in many applications. For example, if $E[X \mid Z = z] = a + bz$ and $Var[X \mid Z = z] = d + cz^2$, then

$$
\begin{aligned}
Var[X] &= d + cE[z^2] + b^2 Var[z] \\
&= d + cE[z]^2 + (c + b^2) Var[z]
\end{aligned}
\tag{B.5}
$$

Similarly, we see that:

$$
\begin{aligned}
Cov[X,Y] &= E[XY] - E[X]E[Y] \\
&= E\big[E[XY \mid Z]\big] - E\big[E[X \mid Z]\big]E\big[E[Y \mid Z]\big] \\
&= E\big[Cov[X,Y \mid Z] + E[X \mid Z]E[Y \mid Z]\big] - E\big[E[X \mid Z]\big]E\big[E[Y \mid Z]\big] \\
&= E\big[Cov[X,Y \mid Z]\big] + E\big[E[X \mid Z]E[Y \mid Z]\big] - E\big[E[X \mid Z]\big]E\big[E[Y \mid Z]\big] \\
&= E\big[Cov[X,Y \mid Z]\big] + Cov\big[E[X \mid Z], E[Y \mid Z]\big]
\end{aligned}
\tag{B.6}
$$

These formulas are of use, for instance, in Equation 3.27:

$$
\begin{aligned}
Var[\ln DVE_k \mid IM] &= E_{\ln EDP_i|IM}[Var_{\ln DVE_k|\ln EDPi}[\ln\ DVE_k \mid \ln\ EDP_i] \\
&\quad + Var_{\ln EDP_i|IM}[E_{\ln DVE_k|\ln EDP_i}[\ln\ DVE_k \mid \ln\ EDP_i] \\
&= E_{\ln EDP_i|IM}[g*_k\ (\ln EDP_i)] \\
&\quad + Var_{\ln EDP_i|IM}[g_k(\ln EDP_i)] \\
&= E_{\ln EDP_i|IM}[g*_k\ (\ln EDP_i)] \\
&\quad + E_{\ln EDP_i|IM}[(g_k(\ln EDP_i))^2] - E^2{}_{\ln EDP_i|IM}[g_k(\ln EDP_i)]
\end{aligned}
$$

Expanding a Taylor series about the mean of $\ln EDP_i$:

$$
\begin{aligned}
Var[\ln DVE_k \mid IM] &= E_{\ln EDP_i|IM}\left[g*_k\ (E[\ln EDP_i]) + \left.\frac{\partial g*_k\ (\ln EDP_i)}{\partial \ln EDP_i}\right|_{E[\ln EDP_i]} (\ln EDP - E[\ln EDP_i]) + \ldots \right] \\
&\quad + E_{\ln EDP_i|IM}\left[\left(g_k(E[\ln EDP_i]) + \left.\frac{\partial g_k(\ln EDP_i)}{\partial \ln EDP_i}\right|_{E[\ln EDP_i]} (\ln EDP - E[\ln EDP_i]) + \ldots \right)^2 \right] \\
&\quad - E^2{}_{\ln EDP_i|IM}\left[g_k(E[\ln EDP_i]) + \left.\frac{\partial g_k(\ln EDP_i)}{\partial \ln EDP_i}\right|_{E[\ln EDP_i]} (\ln EDP - E[\ln EDP_i]) + \ldots \right]
\end{aligned}
$$

Taking a first-order approximation yields:

$$
\begin{aligned}
Var[\ln DVE_k \mid IM] &= g*_k\,(h_i(IM) \\
&\quad + \left(g_k(h_i(IM)\right)^2 + \left(\frac{\partial g_k\left(\ln EDP_i\right)}{\partial \ln EDP_i}\right)^2\Bigg|_{h_i(IM)} E\left[\left(\ln EDP_i - E[\ln EDP_i]\right)^2\right] \\
&\quad - \left(g_k(h_i(IM)\right)^2 \\
&= g*_k\,(h_i(IM) + \left(\frac{\partial g_k\left(\ln EDP_i\right)}{\partial \ln EDP_i}\right)^2\Bigg|_{h_i(IM)} h*_i\,(IM)
\end{aligned}
\tag{B.7}
$$

In this equation, the functions $h_i(IM)$, $h*_i\,(IM)$, $g_k(\ln EDP_i)$, and $g*_k\,(\ln EDP_i)$ were substituted, as defined in Equations 3.2, 3.3, 3.18, and 3.19, respectively. Equation 3.28 can be derived using the same procedure.

APPENDIX C: DERIVATION OF THE ANALYTICAL SOLUTION FOR $\lambda_{TC}(z)$

The derivation of the analytical solution for $\lambda_{TC}(z)$ is performed under the assumptions of Section 4.6. A similar solution, under the same assumptions, has been previously derived by Cornell et al. (2002). The solution of this paper is presented below.

The assumptions used in this solution are repeated for clarity. Namely, the distribution of $TC \mid IM$ is lognormal with median

$$\hat{TC} = aIM^{b} \tag{C.1}$$

where a and b are constants. This is equivalent to fitting the mean of $TC \mid IM$ by $a'IM^b$, where $a = a'e^{-\frac{1}{2}\beta*_{TC|IM}{}^{2}}$. $\beta_{TC|IM}$ is approximated as a constant $\beta*_{TC|IM}$. Finally, the ground motion hazard is approximated by

$$\lambda_{IM}(x) = k_0 x^{-k} \tag{C.2}$$

where k_0 and k are constants.

We evaluate $\lambda_{TC}(z)$ using the formulation of Equation 3.39. Namely,

$$\lambda_{TC}(z) = \int_{IM} \left(1 - F_{TC|IM}(z,x)\right) \left| d\lambda_{IM}(x) \right| \tag{C.3}$$

With the lognormality assumption of Equation C.1, the first term of Equation C.3 becomes

$$\left(1 - F_{TC|IM}(z,x)\right) = P(TC > z \mid IM = x) = 1 - \Phi\left(\ln\left[z / ax^{b}\right] / \beta*_{TC|IM}\right) \tag{C.4}$$

where $\Phi(\bullet)$ is the Standard Normal distribution function. Using this result and Equation C.2, Equation C.3 becomes, on integration,

$$\lambda_{TC}(z) = k_0 \left(\frac{z}{a}\right)^{-k/b} \exp\left(\frac{1}{2}\frac{k^2}{b^2}\beta*_{TC|IM}{}^{2}\right) \tag{C.5}$$

We now make the substitution:

$$a = a'e^{-\frac{1}{2}\beta*_{TC|IM}{}^{2}} \tag{C.6}$$

Factoring the $e^{-\frac{1}{2}\beta^*{}_{TC|IM}{}^2}$ term outside of the first set of parentheses:

$$\lambda_{TC}(z) = k_0 \left(\frac{z}{a'}\right)^{-k/b} \exp\left(-\frac{1}{2}\frac{k}{b}\beta^*{}_{TC|IM}{}^2\right)\exp\left(\frac{1}{2}\frac{k^2}{b^2}\beta^*{}_{TC|IM}{}^2\right) \tag{C.7}$$

Finally, combining exponential terms:

$$\lambda_{TC}(z) = k_0 \left(\frac{z}{a'}\right)^{-k/b} \exp\left(\frac{1}{2}\frac{k}{b}\left(\frac{k}{b}-1\right)\beta^*{}_{TC|IM}{}^2\right) \tag{C.8}$$

This is the result presented in Equation 3.41.

APPENDIX D: Use of the Bootstrap to Compute Sample Variance and Correlation

The bootstrap is a simple but versatile tool that allows us to compute, for instance, variances and correlations of sample averages, using the same set of data that we used to compute the sample average itself (Efron and Tibshirani, 1998).

This idea is applicable to several problems in the body of this report. For instance, we estimate the two mean values of a particular *EDP* at two *IM* levels by scaling a set of records to the two *IM* values and then taking the average values of the observed *EDP*s. The estimates of the mean values are uncertain (random) and correlated. This correlation can be estimated using the bootstrap, as outlined below:

1. For the set of N records, scale the records to two *IM* levels, im_1 and im_2. Run all of the records through the model at both *IM* levels to compute *N* realizations of *EDP* at im_1 and *N* realizations at im_2.
2. Compute estimates of $\mu_{\ln EDP}(im_1) = E[\ln EDP \mid IM = im_1]$ and $\mu_{\ln EDP}(im_2) = E[\ln EDP \mid IM = im_2]$ as before
3. For b = 1, 2, … B, generate bootstrap replicates:
   a. Sample *N* records from the original set of records, *with replacement*.
   b. Select the *N* realizations of *EDP* from im_1 and im_2 corresponding to the *N* records selected in step 3a (there will be duplicate record realizations in these sets). We denote these sets as $EDP^{(b)}(im_1)$ and $EDP^{(b)}(im_2)$. Compute the geometric means of these sets, $y^{(b)} = mean[\ln EDP^{(b)} \mid IM = im_1]$ and $z^{(b)} = mean[\ln EDP^{(b)} \mid IM = im_2]$
4. Estimate the correlation between $\mu_{\ln EDP}(im_1)$ and $\mu_{\ln EDP}(im_2)$ by computing the correlation coefficient between the B bootstrap replicates $y^{(b)}$ and $z^{(b)}$:

$$\hat{p}_{\mu_{\ln EDP}(im_1),\mu_{\ln EDP}(im_2)} = \frac{\sum_{b=1}^{B}\left(y^{(b)}-\hat{\mu}_{y^{(b)}}\right)\left(z^{(b)}-\hat{\mu}_{z^{(b)}}\right)}{\left[\sum_{b=1}^{B}\left(y^{(b)}-\hat{\mu}_{y^{(b)}}\right)^2\sum_{b=1}^{B}\left(z^{(b)}-\hat{\mu}_{z^{(b)}}\right)^2\right]^{1/2}} \tag{D.1}$$

Where $\hat{\mu}_{y^{(b)}}$ and $\hat{\mu}_{z^{(b)}}$ are the sample averages of $y^{(b)}$ and $z^{(b)}$, respectively.

This technique is very useful for computing statistics such as these, where our only estimate of the underlying distribution is based on samples that we have (e.g., we are estimating the response of our building under the population of all ground motions by averaging the response of a few representative ground motions).

We can also compute several other statistics using this same method. For instance, computing the variance of the $y^{(b)}$ replicates above gives an estimate of the variance due to estimation uncertainty in $E[\ln EDP \mid IM = im_1]$ (i.e. $Var[\boldsymbol{\varepsilon}_{U_{estimate};EDP|IM}] \equiv \beta^2_{U_{estimate};EDP|IM}$):

$$Var[\boldsymbol{\varepsilon}_{U_{estimate};EDP|IM}] \equiv \beta^2_{U_{estimate};EDP|IM} = \frac{1}{B}\sum_{b=1}^{B}\left(y^{(b)}-\hat{\mu}_{y^{(b)}}\right)^2 \tag{D.2}$$

In addition, this technique can be easily used to compute the correlation between two *EDP*'s at one *IM* level. For this case, we would define $y^{(b)} = mean[\ln EDP_1^{(b)} \mid IM = im]$ and $z^{(b)} = mean[\ln EDP_2^{(b)} \mid IM = im]$. The procedure is then the same as outlined above. Or we could compute the correlation between two *EDP*'s at two *IM* levels. In this case, we would define $y^{(b)} = mean[\ln EDP_1^{(b)} \mid IM = im_2]$ and $z^{(b)} = mean[\ln EDP_2^{(b)} \mid IM = im_2]$.

APPENDIX E: ESTIMATING THE ROLE OF SUPPLEMENTARY VARIABLES IN UNCERTAINTY

To estimate the effect of uncertainties in supplementary variables, it is necessary to measure the sensitivity of *EDP* to changes in these variables. As noted in the body, this uncertainty can be measured in several ways. For instance Monte Carlo or Latin Hypercube simulation could be used. This appendix outlines a technique known as the finite difference method.

We use a model assuming that for a given level of *IM*, $EDP = g(X1, X2) + X3$, where *X1* and *X2* are parameters to be studied, and *X3* is a zero-mean random residual. *X3* represents the record-to-record variability that we have already captured. *X1* and *X2* are the supplementary variables that we now wish to study. They may represent either random or uncertain parameters in the nonlinear model. (Note that we may want to think of *EDP* here as being the natural log of the drift, in which case *Var*[*X3*] is the β^2 we have been using in the text.) This model is easily expanded to more than *n* uncertain variables, but two are shown here for simplicity.

The first-order approximation of the mean and variance (using mean-centered Taylor expansions) are given by:

$$\mu_{EDP} \cong g(\mu_{X1}, \mu_{X2}) \tag{E.3}$$

$$\begin{aligned} Var[EDP] \cong & \left(\frac{\partial g(X1,X2)}{\partial X1}\right)^2 \Bigg|_{\substack{X1=\mu_{X1} \\ X2=\mu_{X2}}} Var[X1] + \left(\frac{\partial g(X1,X2)}{\partial X2}\right)^2 \Bigg|_{\substack{X1=\mu_{X1} \\ X2=\mu_{X2}}} Var[X2] \\ & + 2\left(\frac{\partial g(X1,X2)}{\partial X1}\right)\left(\frac{\partial g(X1,X2)}{\partial X2}\right)\Bigg|_{\substack{X1=\mu_{X1} \\ X2=\mu_{X2}}} \rho_{X1,X2}\sqrt{Var[X1]}\sqrt{Var[X2]} \\ & + Var[X3] \end{aligned} \tag{E.4}$$

(For *n* parameters, $\mu_{EDP} \cong g(\mathbf{M_x})$ and $Var[EDP] \cong \sum_{i=1}^{n}\sum_{j=1}^{n}\left[\frac{\partial g(\boldsymbol{X})}{\partial x_i}\frac{\partial g(\boldsymbol{X})}{\partial x_j}\right]_{\boldsymbol{X}=\boldsymbol{M_x}} \rho_{i,j}\sigma_i\sigma_j$, where $\mathbf{X}$ is the vector of all *n* parameters, and $\mathbf{M_x}$ is the vector of mean values of $\mathbf{X}$)

In our case, we do not know the function $g(X1, X2)$ explicitly, but using NLTH analysis and the finite difference procedure below, we can estimate the derivatives needed for these first-order approximations.

Procedure:

1. Select an *IM* level
2. Estimate by data and/or judgment the means, variances, and correlation of *X1* and *X2*
3. Set $X1 = \mu_{X1}$, and $X2 = \mu_{X2}$, and make several (say *n*=10) runs with records scaled to the selected *IM* level
4. Estimate μ_{EDP} by $\frac{1}{n}\sum_{i=1}^{n} EDP_i$ (the sample average of the set of runs)
5. Estimate $Var[X3]$ by $\frac{1}{(n-1)}\sum_{i=1}^{n}(EDP_i - \mu_{EDP})^2$ (the sample variance of the runs)
6. Rerun the analyses with $X1' = (1+y)\mu_{X1}$ (the best multiplier here is $y \cong C.O.V._{X1}$) and average the results of these runs—call this $E[EDP']$
7. Calculate $\left.\frac{\partial g(X1, X2)}{\partial X1}\right|_{\substack{X1=\mu_{X1} \\ X2=\mu_{X2}}} = \frac{E[EDP'] - \mu_{EDP}}{X1' - \mu_{X1}}$. (Note this is numerically equal to estimating the derivative for each record and then averaging.)
8. Repeat steps 6 and 7 for *X2* (and for all other parameters if more than two are involved)
9. Evaluate $Var[EDP]$ using Equation E.4
10. Repeat for several values of *IM*, to estimate the change in $Var[EDP]$ with *IM*. This is $Var[EDP \mid IM]$

The value of $Var[EDP \mid IM]$ estimated using this procedure can now be used in Equation 3.3 of the text.

In addition, this idea can be extended to compute covariances. To compute $Cov[EDP \mid IM_1, EDP \mid IM_2]$, one would use the above procedure, but simultaneously compute μ_{EDP} and $E[EDP']$ at two *IM* levels simultaneously. Computed once for *X1* and once for *X2*. The resulting information can be used to compute the covariance:

$$Cov[EDP\,|\,IM_1, EDP\,|\,IM_2] \cong \left(\frac{\partial g_1(X1,X2)}{\partial X1}\right)\Bigg|_{\substack{X1=\mu_{X1}\\ X2=\mu_{X2}}} \left(\frac{\partial g_2(X1,X2)}{\partial X1}\right)\Bigg|_{\substack{X1=\mu_{X1}\\ X2=\mu_{X2}}} Var[X1]$$
$$+\left(\frac{\partial g_1(X1,X2)}{\partial X2}\right)\Bigg|_{\substack{X1=\mu_{X1}\\ X2=\mu_{X2}}} \left(\frac{\partial g_2(X1,X2)}{\partial X2}\right)\Bigg|_{\substack{X1=\mu_{X1}\\ X2=\mu_{X2}}} Var[X2]$$
$$+2\left(\frac{\partial g_1(X1,X2)}{\partial X1}\right)\left(\frac{\partial g_2(X1,X2)}{\partial X2}\right)\Bigg|_{\substack{X1=\mu_{X1}\\ X2=\mu_{X2}}} \rho_{X1,X2}\sqrt{Var[X1]}\sqrt{Var[X2]}$$
$$+Var[X3] \tag{E.5}$$

In this equation, $g_1(X1, X2)$ represents $EDP\,|\,IM_1$ and $g_2(X1, X2)$ represents $EDP\,|\,IM_2$.

The same covariance relation could be used for $Cov[EDP_1\,|\,IM, EDP_2\,|\,IM]$ as well (e.g., two *EDP*'s at the same *IM* level, rather than the same *EDP* at two *IM* levels). Before proceeding with these computations, however, it is worth considering whether a simple covariance structure such as perfect correlation is appropriate. This would simplify the computational complexity, and may not result in a significant change in the final results.

Example of Procedure Using Hypothetical Results:

We study a hypothetical EDP, assuming a function of the form: $EDP = g(X1, X2) + X3$

1. We select $IM = 0.2$
2. Our best judgments for the moments of *X1* and *X2* are:

   $\mu_{X1} = 2 \quad Var[X1] = 0.5$
   $\mu_{X2} = 1.5 \quad Var[X2] = 0.1 \qquad \rho_{X1,X2} = 0.5$

   (Note: $C.O.V._{X1} = \dfrac{\sqrt{Var[X1]}}{\mu_{X1}} = 0.25$ and $C.O.V._{X2} = 0.21$)

3. We Set $X1 = 2$, and $X2 = 1.5$, and make 10 analysis runs with records scaled to *IM*=0.2. Our results are:

8.0755	7.1321
6.7431	5.9607
5.6429	7.1268
6.7663	7.8334
7.1667	6.6033

4. $\mu_{EDP} \cong 6.91$

5. $Var[X3] = 0.56$

6. Rerunning the analyses with $X1' = (1+0.25)\mu_{X1} = 2.5$ gives results:

```
 9.2464        9.2410
 9.8014       11.0976
10.4101       10.0271
10.4688        9.6521
11.4150        9.1090
```

$E[EDP'] = 7.23$

7. Calculate $\left.\dfrac{\partial g(X1,X2)}{\partial X1}\right|_{\substack{X1=\mu_{X1}\\ X2=\mu_{X2}}} = \dfrac{7.23-6.91}{2.5-2} = 0.65$

8. Repeating steps 6 and 7 with $X1 = 2,\ X2' = (1+0.21)\mu_{X2} = 1.815$ gives:

```
 9.0824       10.7085
12.6917       11.6775
10.0145       10.6867
11.0657        8.5867
11.0814        9.5392
```

$E[EDP'] = 10.04$

$$\left.\frac{\partial g(X1,X2)}{\partial X2}\right|_{\substack{X1=\mu_{X1}\\ X2=\mu_{X2}}} = \frac{10.04-6.91}{1.815-1.5} = 9.93$$

9. Using the formula for the variance of $g(X1,X2)$:

$$\begin{aligned} Var[EDP] &\cong (0.65)^2(0.5) + (9.93)^2(0.1) + 2(0.65)(9.93)0.5\sqrt{0.5}\sqrt{0.1} + 0.56 \\ &= 0.423 + 9.880 + 2.043 + 0.56 \\ &= 12.91 \end{aligned}$$

$$C.O.V._{EDP} = \frac{6.91}{\sqrt{12.91}} = 0.52$$

Results: $\mu_{EDP} \cong 6.91$, $Var[EDP] \cong 12.91$, and $\sigma_{EDP} \cong 3.59$, for $IM = 0.2$

Looking at the four terms from the variance calculation, we see that the second term dominates the result. This is the term corresponding to uncertainty in *X2*. We also see that the C.O.V. of *EDP* is about 0.5, while the C.O.V.'s of *X1* and *X2* were 0.25 and 0.21, respectively. This is because of a nonlinearity in the function used to generate these results (the function is given at the end of this example.

Example of Procedure, Part 2:

Repeat the procedure from the previous page, but with a new *IM* level

1. We select *IM* = 1.0
2. Our judgments for the moments of *X1* and *X2* remain the same
3. Make 10 runs using mean values with records scaled to *IM*=1.0. Our results are:

8.1356	10.0192
8.4307	9.2481
8.8036	8.5052
8.5007	7.6260
8.4565	8.6631

4. $\mu_{EDP} \cong 8.64$
5. $Var[X3] = 0.41$
6. Rerunning the analyses with $X1' = 2.5$ gives results:

10.1096	8.9187
10.0504	9.5999
8.8515	8.6263
10.0022	9.3211
9.3404	9.3973

$E[EDP'] = 9.42$

7. Calculate $\left.\dfrac{\partial g(X1, X2)}{\partial X1}\right|_{\substack{X1=\mu_{X1} \\ X2=\mu_{X2}}} = \dfrac{9.42 - 8.64}{2.5 - 2} = 1.57$
8. Repeating steps 6 and 7 with $X1 = 2, X2' = (1 + 0.21)\mu_{X2} = 1.815$ gives:

11.7027	11.9354
12.5720	11.6243
12.4860	13.0486
11.9975	10.8452
12.5987	11.9870

$E[EDP'] = 12.08$

$$\left.\frac{\partial g(X1, X2)}{\partial X2}\right|_{\substack{X1=\mu_{X1} \\ X2=\mu_{X2}}} = \frac{12.08 - 8.64}{1.815 - 1.5} = 10.92$$

9. Using the formula for the variance of g:

$$Var[EDP] \cong (1.57)^2(0.5) + (10.92)^2(0.1) + 2(1.57)(10.92)0.5\sqrt{0.5}\sqrt{0.1} + 0.41 = 20.22$$

$$C.O.V._{EDP} = \frac{8.64}{\sqrt{20.22}} = 0.52$$

Results: $\mu_{EDP} \cong 8.64$, $Var[EDP] \cong 20.22$, and $\sigma_{EDP} \cong 4.50$, for $IM = 1.0$

Note: the function used to generate the results for this example is $EDP = X1 * IM + 3X2^2 + X3$, where *X3* is normal with Mean = 0 and Variance = 0.5. With a structural model, the function could be determined by a regression analysis on NLTH results, but it is not needed for this procedure.

APPENDIX F: ESTIMATING THE VARIANCE AND COVARIANCE STRUCTURE OF THE GROUND MOTION HAZARD

When performing a PSHA analysis, many combinations of models of fault structure, recurrence relationship, ground motion attenuation, etc., are used. The full set of models is described by a logic tree, composed of all possible models, and weights associated with each model. Each one of these individual models results in a hazard curve, representing the annual frequencies of exceeding a range of ground motion levels, given the particular model. The collection of weighted hazard curves associated with all models is then a stochastic description of the ground motion hazard. For a given ground motion level, the mean and variance of the ground motion hazard can be easily calculated by taking the mean and variance of the population of hazard curves at that given level. However, there also exists a covariance structure in the ground motion hazard between two different ground motion levels. If there are many hazard curves, this may be somewhat more expensive to compute. The *Final Report of the Diablo Canyon Long Term Seismic Program,* as constructed by Robin McGuire (Pacific Gas & Electric, 1988), provides a procedure for collapsing a large number of hazard curves into a few while retaining most of the variance and covariance structure of the original full set of curves. The procedure of this document is reprinted below:

> To derive hazard results appropriate for a probabilistic risk assessment, an aggregation process is employed that reduces the large number of hazard curves (20,700) to a few (typically 8 to 12), using a procedure that optimally determines how to combine pairs of curves sequentially so that the character of the original curves will be maintained, and the set of aggregate curves will represent as much of the original uncertainty in hazard as possible for each ground-motion amplitude. The procedure uses the following steps:
>
> 1. A contribution to variance analysis is used to select nodes on the logic tree that do not contribute significantly to uncertainty in hazard. The logic tree is then restructured to reduce the number of end branches by combining hazard results for end branches by combining hazard results for end branches at nodes that contribute little to the uncertainty in hazard. By this mechanism the family of hazard curves is reduced to several hundred in number.

These hazard curves typically represent greater than 96 percent of the total uncertainty in hazard.

2. The hazard curves are characterized by the frequency of exceedance at three ground-motion amplitudes, chosen as those most critical to the determination of Plant response and system states. The total variance in frequency of exceedance at these three amplitudes is calculated.
3. A small number of possible aggregate curves (for example, 64) is estimated by dividing the ranges of frequencies of exceedance into intervals and constructing a first set of aggregates at the centers of these intervals.
4. Each of the hazard curves is assigned to a tentative aggregate hazard curve, based on its proximity in frequency-of-exceedance for the three amplitudes.
5. The tentative aggregate curves are recomputed as the conditional mean of the assigned curves.
6. Steps 4 and 5 are repeated, because step 5 may change the assignments based on proximity, until the tentative aggregate curves are stable (that is, until there are no more changes in assignments). A weight for each tentative aggregate curve is calculated as the sum of the weights of the assigned curves.
7. All possible pairs of tentative aggregate curves are examined as candidates for combination; the pair that, when combined, will result in the minimum reduction of variance is selected and combined by computing the weighted average frequency of exceedance for all three amplitudes. The combined curve is assigned a weight equal to the sum of the weights of the two curves used to calculate it.
8. Steps 4 through 7 are repeated to reduce sequentially the number of tentative aggregate curves. The process ends when the desired number of aggregate curves is reached.
9. The curve assignments are used to calculate aggregate hazard curves for all ground-motion amplitudes; the weight given to each aggregate is the sum of the weights of the assigned curves.

There are no general solution techniques for aggregating a discrete, multidimensional distribution, but the above algorithm has been tested for a number of seismic hazard problems and works well. It is efficient up to several hundred initial hazard curves (which is the reason for Step 1). Typically 8 to 12 aggregate curves can be constructed with this algorithm that replicate about 90 percent of the total variance of the original data set, for all ground-motion amplitudes (that is, the standard deviation of frequency of exceedance is 95 percent of the original). Figure 6-4 [Fig. F.1 below] illustrates how this procedure would work for the case of reducing nine hazard curves. Three aggregate curves adequately represent the amplitude and slope of the original nine curves.

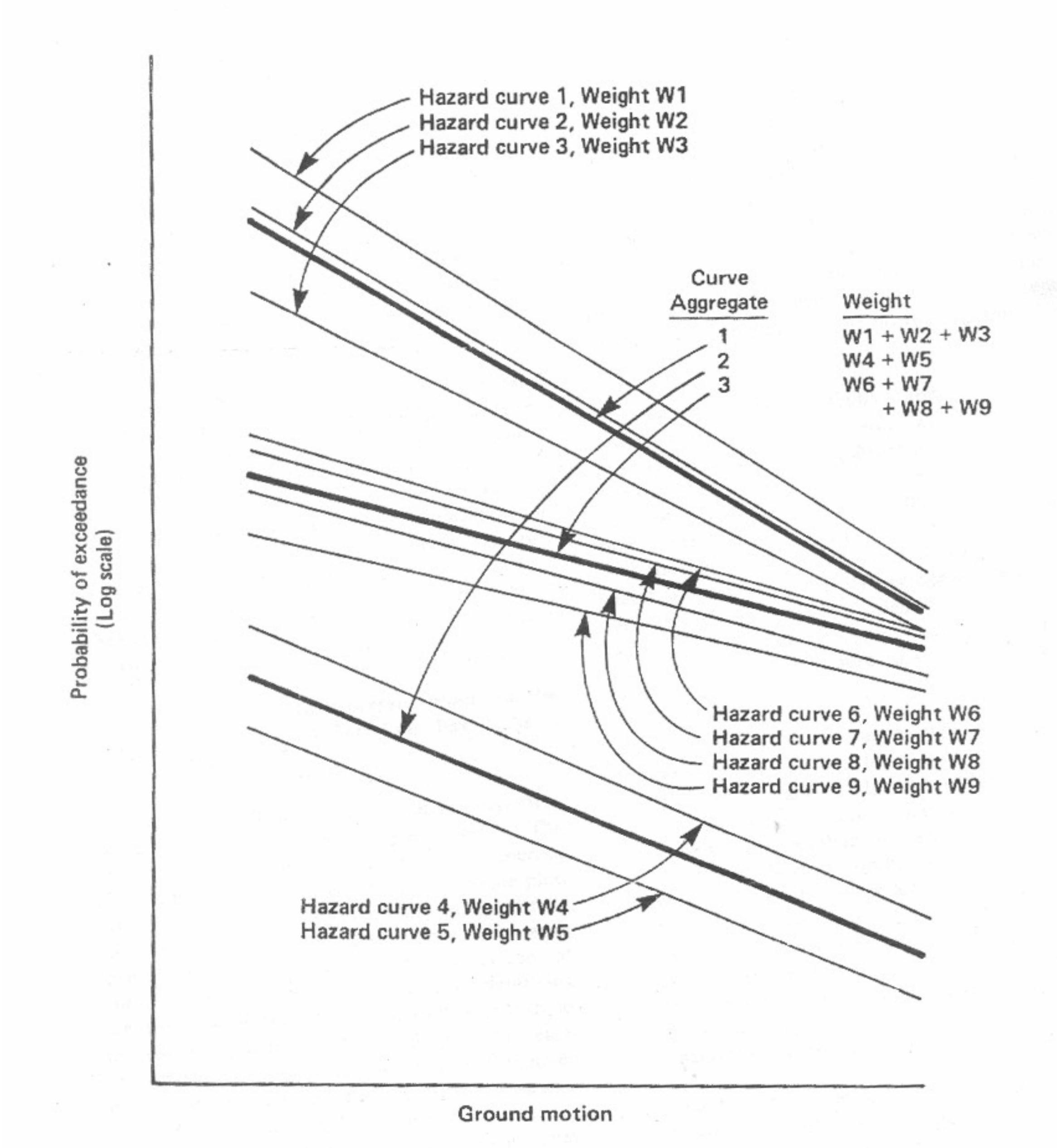

Fig. F.1: Example of aggregation of nine hazard curves to obtain three curves (Figure 6-4 taken from Pacific Gas & Electric, 1988)

This procedure is a more detailed implementation of the following idea, as communicated by Veneziano (2003):

1. Read the hazard curves for a discrete number of values, generating vectors from the functions
2. Cluster the vectors by minimizing a distance function such as squared errors (where the errors can more generally be weighted to reflect greater emphasis on a region of interest).

Veneziano suggests that a K-means algorithm could be used, for a varying number of clusters K.

3. Choose K to retain a desired level of the original variance, and then collapse the curves into their clusters-means, ignoring the within-cluster variance (recognizing that the clusters were selected to keep within-cluster variance to a minimum).

An additional reference by Veneziano et al. (1984) provides an algorithm this calculation.

Once this collapse to a few representative hazard curves has been performed, the covariance of the hazard at two ground motion levels can be inexpensively calculated by calculating the covariance of the representative hazard curves.

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