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A Technical Framework for Probability-Based Demand and Capacity Factor Design (DCFD) Seismic Formats

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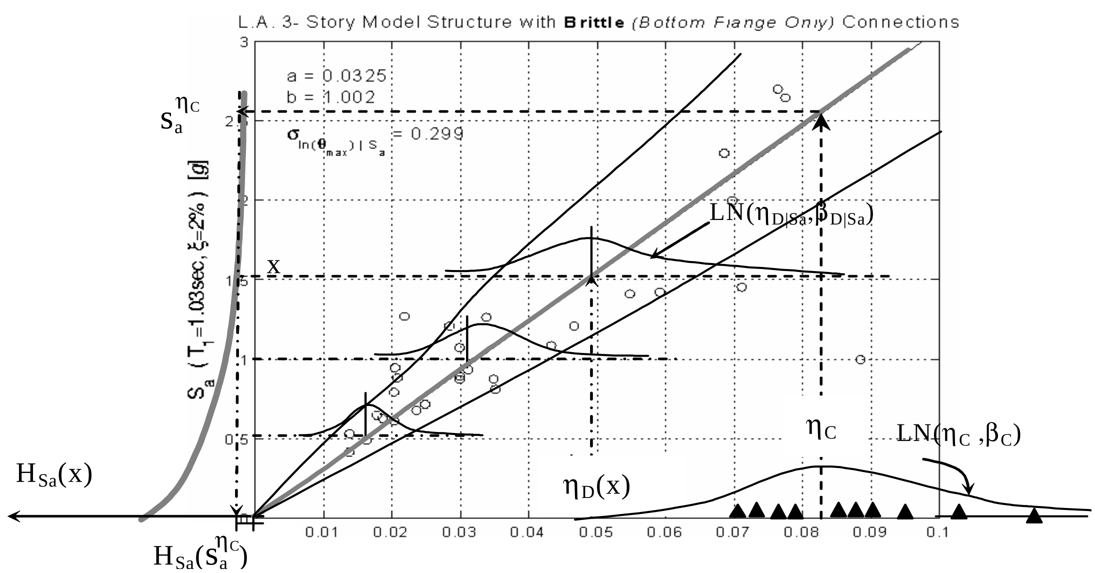
ABSTRACT

Demand and capacity factor design (DCFD) is a probability-based load and resistance factor (LRFD)-like format used for performance-based seismic design and assessment of structures. The DCFD format is based on a technical framework that provides a closed-form analytical expression for the mean annual frequency of exceeding (or not exceeding) a structural performance level, which is usually defined as specified structural parameters (e.g., ductility, strength, maximum drift ratio) reaching a structural limit state (e.g., onset of yield, collapse).

This report, which is presented in two parts, provides a step-by-step and detailed description of the development of the technical framework underlying the DCFD format, accompanied by helpful illustrations and numerical examples. In the first part, a closed-form analytic expression for the mean annual frequency of exceeding a structural limit state is derived based on certain simplifying assumptions. The expression for mean annual frequency of exceedance is derived by taking into account the aleatory uncertainty (due to inherent randomness) and the epistemic uncertainty (due to limited knowledge) in three main elements, seismic hazard, structural response (as a function of ground motion intensity) and capacity. A schematic plot of these three parameters is shown in the figure below.



Main parameters in the development of the technical framework: mean annual frequency of exceeding spectral acceleration, x, characterized by $H_{S_s}(x)$, distribution of demand variable *D* given S_a characterized by $\eta_D(x)$ and $\beta_{D|S_a}$, and distribution of capacity variable *C* characterized by η_C and β_C

In the second part of this report, the closed-form expression for the mean annual frequency of exceeding a limit state is re-arranged into alternative formats, suitable for implementation in seismic design and assessment guidelines. These formats can be used to ensure that the structural seismic design can be expected to satisfy specified probabilistic performance objectives, and perhaps (more novel) that it does so with a desired, guaranteed degree of confidence. The degree of confidence in meeting the specified performance objectives may be quantified through the upper confidence bound on the (uncertain) probability. These proposed formats are based on alternative conventional design methods such as LRFD design and fragility-hazard design. Versions of the new developments reported here are already in place in recently completed seismic guidelines such as the FEMA 350-352 documents and the ISO seismic design guidelines for offshore platforms. Numerical applications of the DCFD format and its underlying framework can be found in papers by the authors and other researchers, instances of which are outlined in this report.

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NOMENCLATURE

β_C	Standard deviation of the log of the displacement-based capacity, or fractional standard deviation of the displacement-based capacity
$\beta_{D\|S_a}(x)$ or $\beta_D(x)$	Conditional standard deviation of the log of demand for a given spectral acceleration $S_a = x$. Also referred to as the conditional fractional standard deviation of demand given spectral acceleration.
β_{RC}	Fractional standard deviation of the displacement-based capacity measuring aleatory uncertainty
β_{RD}	Fractional standard deviation of the displacement-based demand measuring aleatory uncertainty (given S_a)
$\beta_{RS_{a,C}}$	Fractional standard deviation of the spectral acceleration capacity measuring aleatory uncertainty
β_{RT}	Fractional standard deviation measuring total aleatory uncertainty in the estimation of the displacemen t-based demand and capacity
$\beta_{S_{a,C}}$	Fractional standard deviation of the spectral acceleration capacity
β_{UC}	Fractional standard deviation of the displacement-based capacity measuring epistemic uncertainty
β_{UD}	Fractional standard deviation of the displacement-based demand measuring epistemic uncertainty (given S_a)
β_{UH}	Fractional standard deviation of the spectral acceleration hazard, $\tilde{H}_{S_a}$, a dispersion measure representing epistemic uncertainty in the estimation of the spectral acceleration hazard
β_{UH_D}	Fractional standard deviation of the demand-hazard due to epistemic uncertainty in the estimation of the demand hazard
$\beta_{UH_{LS}}$	Fractional standard deviation of the frequency of exceeding a limit state due to epistemic uncertainty
$\beta_{US_{a,C}}$	Fractional standard deviation of the spectral acceleration capacity measuring epistemic uncertainty
β_{UT}	Fractional standard deviation measuring total epistemic uncertainty in the estimation of the spectral acceleration hazard, displacement-based demand and capacity
γ	Demand factor
γ_R	Demand factor representing aleatory uncertainty
γ_U	Demand factor representing epistemic uncertainty
ε	A random variable with unit median (usually assumed to be lognormal)
ε_{RC}	A unit-median random variable reflecting the aleatory uncertainty in the estimation of the displacement-based capacity

ε_{RD}	A unit-median random variable reflecting the aleatory uncertainty in the estimation of the displacement-based demand
$\varepsilon_{RS_{a,C}}$	A unit-median random variable reflecting the aleatory uncertainty in the estimation of the spectral acceleration capacity
ε_{UC}	A unit-random variable reflecting the epistemic uncertainty in the estimation of the displacement-based capacity
ε_{UD}	A unit-median random variable reflecting the epistemic uncertainty in the estimation of the displacement-based demand
ε_{UH}	A unit-median random variable reflecting the epistemic uncertainty in the estimation of the hazard function for the spectral acceleration
$\varepsilon_{US_{a,C}}$	A unit-median random variable reflecting the epistemic uncertainty in the estimation of the spectral acceleration capacity
η_C	Median of the displacement-based capacity variable
$\eta_{c\|\varepsilon_{UC}}(x)$	Conditional median for the displacement-based capacity for a given value of the deviation in capacity, ε_{UC}
$\hat{\eta}_C(x)$	Estimated median for the displacement-based capacity
$\eta_{D\|S_a}(x)$ or $\eta_D(x)$	Conditional median of displacement demand D at a given spectral acceleration value $S_a = x$
$\hat{\eta}_D(x)$	Estimated conditional median for the displacement-based demand for a given spectral acceleration value, x
$\eta_{S_{a,C}}$	Median of the spectral acceleration capacity
$\hat{\eta}_{S_{a,C}}$	Estimated median of the spectral acceleration capacity
λ_x	Confidence factor corresponding to the confidence level, x
ν	Mean annual rate of the occurrence of the events of “interest” or the seismicity rate
ϕ	Capacity factor
ϕ_R	Capacity factor representing aleatory uncertainty
ϕ_U	Capacity factor representing epistemic uncertainty
$\Phi(.)$	Standardized Gaussian CDF
$\phi(.)$	Standardized Gaussian PDF
a, b	Regression coefficients for linear regression of displacement-based demand D on intensity S_a in logarithmic space
C	Capacity variable for structural demand D
D	Generic displacement-based structural demand variable
C,D	Generic displacement-based demand and capacity variables
FC	Factored capacity
$FD(P_0)$	Factored demand for an allowable probability equal to P_0

$F_{LS}(s_a)$	Structural fragility function at $S_a = s_a$, for the limit state LS
$F_{S_a}(x)$	The cumulative distribution function (CDF) for the intensity S_a at x, or the probability that intensity S_a is smaller than or equal to x
$f_c(.)$	Probability density function of the displacement-based capacity
$f_{S_a}(x)$	The probability density function (PDF) for the intensity S_a at x
$G_{S_a}(x)$	The complementary cumulative distribution function (CCDF) for the intensity S_a at x, or the probability that intensity S_a exceeds the value x
$H_D(d)$	Mean annual frequency that displacement-based demand D exceeds a given value d. Also known as the "hazard for demand at $D = d$."
$\tilde{H}_D(d)$	Random variable representing the hazard function for the displacement-based demand at $D = d$
$\hat{H}_D(d)$	Median hazard for the displacement-based demand at $D = d$
$H_{D\|\varepsilon_{UH}}(d)$	Hazard function for the displacement-based demand for a given value of deviation in the spectral acceleration hazard, ε_{UH}
$H_{D\|\varepsilon_{UH},\varepsilon_{UD}}(d)$	Hazard function for the displacement-based demand for a given value of deviation in the spectral acceleration hazard, ε_{UH}, and a given value of deviation in the displacement-based demand, ε_{UD}
$H_{D\|\varepsilon_{UH},\varepsilon_{UD},\varepsilon_{UC}}(d)$	Hazard function for the displacement-based demand for a given value of deviation in the spectral acceleration hazard, ε_{UH}, deviation in the displacement-based demand, ε_{UD}, and deviation in the displacement-based capacity, ε_{UC}
H_{LS}	Mean annual frequency of exceeding a structural limit state; also referred to as the "limit state frequency"
$\tilde{H}_{LS}$	Random variable representing mean annual frequency of exceeding a structural limit state
$\hat{H}_{LS}$	Median estimate of the mean annual frequency of exceeding a structural limit state
$\bar{H}_D(d)$	Mean hazard for the displacement-based demand at $D = d$
$\bar{H}_{LS}$	Mean estimate of the mean annual frequency of exceeding a structural limit state
$"\hat{H}_{LS}"$	Median estimate for the limit state frequency assuming that it is obtained based on the mean estimate of the spectral acceleration hazard
H_{LS}^{x}	Limit state frequency corresponding to the confidence level, x
$H_{LS\|\varepsilon_{UH},\varepsilon_{UD},\varepsilon_{UC}}(d)$	Mean annual frequency of exceeding a structural limit state for a given value of deviation in the spectral acceleration hazard, ε_{UH}, deviation in the displacement-based demand ε_{UD}, and deviation in the displacement-based capacity, ε_{UC}
$H_{S_a}(x)$	Mean annual frequency that intensity S_a at a given site will equal or exceed x. Also known as spectral acceleration hazard at $S_a = x$

$\tilde{H}_{S_a}(x)$	Random variable representing the spectral acceleration hazard $S_a = x$
$\hat{H}_{S_a}(x)$	Median spectral acceleration hazard at $S_a = x$
K_x	Standard Gaussian variate associated with the probability x of not being exceeded.
k_0, k	Coefficients for linear regression of hazard H_{S_a} on intensity S_a
P_{LS}	Probability of exceeding a structural limit state
P_0	An allowable value for the limit state frequency
$P_{0_{S_a}}$	Spectral acceleration with a mean annual frequency of exceedance equal to P_0
S_a	Elastic spectral acceleration (measure of ground motion intensity)
$S_{a,C}$	Spectral acceleration capacity of the structure
s_a^d	Spectral acceleration corresponding to the demand value d by $\eta_{D\|S_a}{}^{-1}(d)$
$s_a^{d\|\varepsilon_{UD}}$	Spectral acceleration corresponding to the displacement-based demand value, d, for a given value of deviation in the displacement-based demand, ε_{UD}

1 A Closed-Form Analytic Foundation for Probabilistic Seismic Assessments

1.1 INTRODUCTION

The demand and capacity factor design (DCFD) format is a probability-based load and resistance factor (LRFD)-like format for the seismic design and assessment of structures. The DCFD format is based on a technical framework for probabilistic performance-based design and assessment of structures. The performance objective in this framework is stated in terms of the mean annual frequency of exceeding the desired performance level. This closed-form expression is derived by taking into account the uncertainty in the estimation of seismic hazard, structural response (as a function of the ground motion intensity level), and structural capacity (for the desired performance level) based on certain simplifying assumptions.

Recent seismic assessment guidelines such as FEMA 356 pre-standard and FEMA 273 guidelines define their rehabilitation objective as consisting of a target building performance level and an anticipated earthquake hazard level. The target performance levels are described *qualitatively* in terms of building safety before and after the earthquake, repair cost, and downtime; whereas the earthquake hazard levels can be defined on a probabilistic basis in terms of the mean probability of exceeding a certain hazard level in 50 years. In comparison to the rehabilitation objective defined in FEMA 356 and FEMA 273, the DCFD probabilistic framework states the performance objective by defining the building performance on a probabilistic basis and taking into account both the desired building performance as well as the expected seismic hazard.

Nonlinear dynamic analysis procedures can be used in order to both obtain parameter estimates for the DCFD format and/or its analytical basis and also to test the validity of the simplifying assumptions made in the derivations. Many researchers such as the authors (Jalayer and Cornell 2003a, 2003b; Medina 2002; Yun et al. 2002; and Cordova et al. 2001) have studied the implementation of nonlinear dynamic procedures both for parameter estimation and also for checking the robustness of DCFD's analytic basis. Such research efforts are mentioned in relevant sections in this report.

This report is a step-by-step and detailed guide to the construction of the probabilistic framework that underlies the DCFD format. In order for the report to be more tractable, the derivations are arranged so that each step is based on the results of the previous one(s). Whenever possible, the derivations are also accompanied by numerical examples and graphic illustrations to help the reader. The report is divided into two main parts and a summary. The first part is dedicated to the development of the technical framework underlying the DCFD format, at the core of which is the derivation of the closed-form expression for the mean annual probability of exceeding the desired performance level. The second part discusses alternative design and assessment formats that stem from the probabilistic framework developed in Chapter 1. Most of these formats are analogous to Load and Resistance Factor Design (LRFD) procedures associated with static, force-based structural engineering, e.g., the AISC LRFD Code. Due to the generalizations here to a nonlinear, dynamic displacement basis, we refer to these new formats as DCFD (Demand and Capacity Factor Design). The choice among these alternative formats must be made on grounds such as familiarity and practicality because in many cases they are technically equivalent.

1.1.1 Organization of the Report

Chapter 1 of the report, the foundation development, is intended to serve as a step-by-step derivation of a closed-form expression for the mean annual frequency of exceeding structural performance level(s) based on certain simplifying assumptions. The formulation of every piece is explained in detail in order to provide an insight into probabilistic assessments for the interested reader, including those with limited experience with such probabilistic derivations. The derivations start with hazard estimations for the intensity measure of choice, which is the first-mode spectral acceleration. The next step is to derive the mean annual frequency of exceeding the structural displacement response based on the derived expression for the spectral acceleration hazard and also on an assumed analytical form (e.g., lognormal distribution) for the (conditional) probability distribution of the displacement response given spectral acceleration. In the final step, the information about structural limit state capacity is taken into account in order to derive the expression for the mean annual frequency of exceeding a structural limit state, or limit state frequency in short, which is the primary goal of this chapter. Another layer of complexity is added by considering the uncertainty due to limited knowledge (epistemic uncertainty) in the formulation of the limit state frequency.

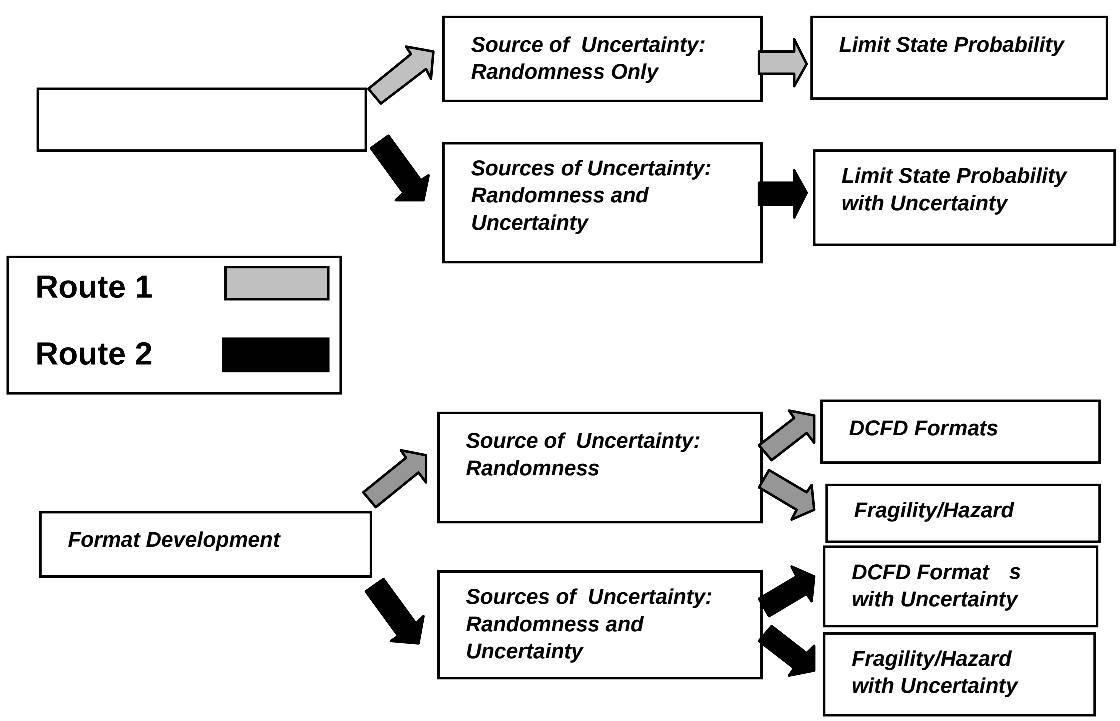
In Chapter 2, the format development discusses several of the many alternative design formats that can stem from the expression for limit state frequency. Demand and capacity factor design (DCFD) is a closed-form design and assessment format that directly results from the original formulation for the mean annual frequency of exceeding a limit state derived in Chapter 1. This format has been implemented in FEMA 350, 351, and, 352 and in an ISO offshore structure guideline (Banon et. al. 2001). The fragility-hazard design format is another way of transforming the closed-form expression for the probability of exceeding a limit state from Chapter 1 into a (potentially) graphical design format. A variation of this format has been implemented in the Department of Energy Guidelines (DOE 1020) for nuclear power plants (PRA 1983). As in Chapter 1, another level of complexity is added by including the epistemic uncertainty in the formulations. The consideration of this type of uncertainty may manifest itself in the form of a confidence statement about the performance objective being met (which may in effect modify the demand and capacity factors in the DCFD format, as adopted in FEMA 350), or in the use of the *mean* estimate for the limit state frequency (as in DOE 1020, 1994).

1.2 SOURCES OF UNCERTAINTY IN ENGINEERING PROBLEMS

Sources of uncertainty in engineering safety problems are classified into two major groups known, confusingly and unfortunately, by various pairs of words in the broader reliability community, for example, *randomness* and *statistical uncertainty*, *aleatory uncertainty* and *epistemic uncertainty*, *frequency,* and *probability*, and simply *Type I* and *Type II*. Moreover, there are ongoing discussions about the nature of uncertainty each group identifies and whether they should be distinguished in the first place. However, in the present work, the first term identifies the more familiar "natural variability" such as the times and magnitudes of future earthquakes in a region, record-to-record variability in acceleration time-history amplitudes and phases. The second term of each pair signifies the limited knowledge and data the profession currently has about, for example the modeling of structural systems in the highly nonlinear range and exact numerical values of parameters of physical and random (stochastic) models, e.g., the *median* value of the maximum interstory drift of a particular model frame under a population of future ground motions of specified intensity. This second kind of uncertainty can be reduced by more data (larger sample sizes) and/or by more research. In the following text, we shall typically use the simple pair of words "randomness" and "uncertainty." Therefore we shall be using the second word in the more restrictive sense of epistemic uncertainty and not in the broader sense as in the title of this section. Occasionally, for example when precision is imperative, we shall use the longer unambiguous terms "aleatory uncertainty" and "epistemic uncertainty," which are now quite common in seismic hazard analysis.

1.3 DOCUMENT MAP

This report contains a complete analytical background for DCFD probability-based seismic design and assessment procedure. For pedagogical reasons the development of the text follows a detailed step-wise manner that makes it somewhat long. However, it is possible to bypass some sections without losing the general picture. The document map that follows illustrates two possible routes the patient reader can follow.



Route 1 goes through the entire development of the technical framework taking into account randomness as the only source of uncertainty. Route 2 goes through a more generalized derivation that considers uncertainty also.

1.4 FOUNDATION DEVELOPMENT

The probabilistic foundation developed in this report involves the entire endeavor that leads to the derivation of a closed-form expression for the mean annual frequency of exceeding a specified limit state for a given structural system. In other words, the final product of this chapter is a closed-form solution for the mean annual frequency of exceeding a limit state calculated taking into account the uncertainty in the various parameters involved in the seismic design of the structural system.

The derivation of the limit state frequency (short for "the mean annual frequency of exceeding a specified limit state") will be presented in two parts. In the first part, the limit state frequency is derived considering only the uncertainty due to randomness. In the second part, the more generalized form of limit state probability is introduced, which accounts for both randomness and uncertainty.

1.4.1 Structural Limit States

The desired structural performance levels for the seismic design or assessment of a structure can be defined in terms of specified thresholds of structural behavior known as the "structural limit states." A structural limit state is usually defined by the structural behavior at the onset of structural demand being equal to the capacity corresponding to that limit state. Global collapse, an example of a structural limit state, is used in this report for defining the desired structural performance level. The foundation derivation represented in this text applies to virtually any limit state; however, for simplicity and clarity, this report focuses on the global collapse limit state.

1.4.2 Structural Demand Variable (State Variable)

Demand, or state variable, is normally chosen as a displacement-based structural response representative of structural dynamic and nonlinear behavior. The most common examples for buildings include: roof displacement or interstory drift.

In this report, we have chosen the maximum interstory drift ratio (MIDR) as the displacement-based structural demand variable (the maximum is obtained as the peak in response time histories over all stories in the building). MIDR is particularly relevant to global collapse predictions for moment-frame structures (FEMA 350). Maximum interstory drift values may be obtained from the results of structural analyses for various ground motion intensities.

We have chosen to refer to the maximum interstory drift variable as *D* hereafter. This will keep the future derivations general with respect to a generic demand variable *D*. It is also suggestive of the displacement-based nature of the demand variable.

1.4.3 Structural Capacity Variable (Limit State Variable)

Capacity, or the limit state variable, is as the name suggests a limit (threshold) for acceptable structural behavior. We have already introduced the demand (state) variable for describing the structural behavior. The capacity (limit state) variable describes the limiting value for the demand (state) variable. Obviously, it will be represented on the same basis as the structural demand variable, maximum interstory drift ratio in this case. The capacity can be defined as a prespecified interstory drift ratio, e.g., 2% (which FEMA 350 uses for an "onset of damage limit state"), or alternatively as capacity with respect to connection failure modeled as a random variable based on test data. In this report, we shall focus on global (dynamic) collapse limit state capacities extracted from the incremental dynamic analysis (IDA) curves, which are plotted using the nonlinear dynamic response of the structure to a suite of ground motion recordings (see Vamvatsikos and Cornell 2001).

In order to keep the derivations general, we have used the generic notation *C* for the random interstory drift capacity. This will also be consistent with the demand variable denoted as *D*.

1.4.4 Limit State Frequency H_{LS}

In the probabilistic framework discussed in this report, the performance objective is stated in terms of a target or desired mean annual frequency of exceeding a performance level. The performance levels can be designated as structural limit states defined by the condition, $D = C$. Hence, the performance objective can be stated as the mean annual frequency of exceeding a specified limit state and denoted by, H_{LS}. We will also refer to H_{LS} as limit state frequency in order to be brief. H_{LS} is defined as the product of the mean rate of occurrence of events with seismic intensity larger than a certain "minimum" level, ν, and the probability that demand D exceeds capacity C, when such an event occurs.

$$H_{LS} = \upsilon \cdot P[D > C]$$

It should be noted that since the rate parameter ν is in frequency terms (times a nondimensional probability term), the limit state frequency is also expressed as a rate of exceedance rather than the probability of exceedance. As will be seen later in the section on seismic hazard, the above definition of limit state frequency is consistent with (and related to) the seismic hazard definition.

1.4.5 General Solution Strategy

In order to determine H_{LS}, we are going to decompose the problem into more tractable pieces and then re-assemble it. First, we introduce a ground motion intensity measure *IM* (such as the spectral acceleration, S_a, at say 1 second period) because the level of ground motion is the major determinant of the demand *D* and because this permits us to separate the problem into a seismological part and a structural engineering part. To do this, we make use of a standard tool in applied probability, known as the "total probability theorem" (TPT) (see Appendix B), which permits the following decomposition of the expression for limit state frequency with respect to an interface variable (here, the spectral acceleration):

$$H_{LS} = \nu \cdot P[D > C] = \nu \cdot \sum_{\text{all x}} P[D > C \mid S_a = x] \cdot P[S_a = x] \tag{1.1}$$

where ν is the rate parameter that was defined in Section 1.4.4 as the mean annual rate of occurrence of events with seismic intensity more than a certain minimum level. In Equation 1.1 we have introduced S_a as the intensity measure. In simple terms, the problem of calculating the limit state frequency has been decomposed into two problems that we already know how to solve. The first problem is to calculate the term $P[S_a = x]$ or the likelihood that the spectral acceleration will equal a specified level, *x*. This likelihood (together with υ) is a number we can get from a probabilistic seismic hazard analysis (PSHA) of the site. The second problem is to estimate the term $P[D > C \mid S_a = x]$ or the conditional limit state probability for a given level of ground motion intensity, here represented by, $S_a = x$. Estimating the conditional limit state probability, for a given ground motion intensity, requires an understanding of, for example, response/demand variability from record-to-record of the same intensity, which is an easier and "purely" structural problem to resolve. The TPT simply tells us how to recombine these two pieces of the problem back into H_{LS}. The solution strategy outlined above, calculating the limit state probability by decomposing it with respect to spectral acceleration, shall be referred to as the "*IM*-based" solution strategy hereafter.

An alternative solution strategy (the main strategy employed in this chapter) consists of decomposing the derivation of the limit state probability in two steps, and hence it employs two interface variables. The first step is to decompose the limit state probability with respect to the displacement-based demand (the first interface variable) using TPT:

$$H_{LS} = \nu \cdot P[D > C] = \nu \cdot \sum_{\text{all d}} P[D > C \mid D = d] \cdot P[D = d]$$

The second step is to decompose the term, $P[D = d]$, or the likelihood that the displacement-based demand is equal to a value *d*, with respect to the spectral acceleration (the second interface variable):

$$H_{LS} = \nu \cdot P[D > C] = \nu \cdot \sum_{\text{all d}} \sum_{\text{all x}} P[D > C \mid D = d] \cdot P[D = d \mid S_a = x] \cdot P[S_a = x] \qquad (1.2)$$

This two-step solution strategy, which employs the displacement-based demand as one of the interface variables, shall be referred to as the "displacement-based" solution strategy. Equation 1.2 is a special case of the framework equation used by Pacific Earthquake Engineering Research (PEER) as a basis for probabilistic design and assessments.

It should be noted that the equations introduced in this section are valid for discrete interface variables. However, here they solely serve as a schematic outline of the solution strategy. Later in this chapter, we are going to present the parallel expressions for limit state frequency based on continuous interface variables.

1.4.6 Ground Motion Intensity Measure

The ground motion intensity measure, *IM,* implemented in the solution strategies outlined in the previous section, serves as an interface between the seismicity characterization and structural behavior assessment. Ideally, such a variable should contain sufficient information about the ground motion to serve as an accurate and efficient predictor of structural response, and it should preferably be a variable for which the PSHA results are available (or readily obtainable). This problem has been studied by Shome et al. (1998) and by Luco and Cornell, (2003). It has been demonstrated by Shome and Cornell (1999) that, for short- and moderate-period structures, the spectral acceleration at a period approximately equal to that of the fundamental mode of the structure satisfies the criteria mentioned above. In fact, the study of such "intensity measure" is the subject of significant current research by a variety of investigators within PEER. We shall use this variable here for specificity, but the resulting derivations will not change if spectral acceleration is replaced by any other scalar intensity measure, such as, for example, the inelastic spectral acceleration (Luco and Cornell 2003).

1.4.7 Randomness: The Only Source of Uncertainty

The probability-based seismic assessment and design procedure presented here aims to evaluate the mean annual frequency H_{LS} that the limit state variable exceeds a limit state threshold LS. Our first objective here is to derive the limit state frequency assuming that randomness is the only source of uncertainty in the demand and capacity variables.

We will follow the displacement-based solution strategy discussed in Section 1.4.5 in order to derive the limit state frequency. The derivations are presented in a step-by-step manner in order to make them easier to follow. At the end of this section we will also briefly present the *IM*-based solution strategy for deriving the limit state frequency. We start by deriving the hazard values for our adopted seismic intensity measure, which is the spectral acceleration of the "first" structural mode. Then we use common probabilistic tools (e.g., TPT as explained previously) in order to first derive the hazard values (i.e., the MAF of exceedance) for the displacement-based demand, (here, maximum interstory drift angle) and then to derive the limit state frequency H_{LS}.

1.4.7.1 Spectral acceleration hazard

The hazard corresponding to a specific value of the ground motion intensity measure (here spectral acceleration S_a) is defined as the mean annual frequency that the intensity of future ground motion events are greater than or equal to that specific value x and denoted by $H_{S_a}(x)$. We are also going to refer to $H_{S_a}(x)$ as spectral acceleration hazard, which can be defined as the product of the rate parameter ν (defined in section 1.4.4) and the probability of exceeding the spectral acceleration value, x, denoted by $G_{S_a}(x)$:

$$H_{S_a}(x) = \nu \cdot G_{S_a}(x)$$

Now that we have chosen S_a as the measure of ground motion intensity, we can be more specific in the definition of the rate ν and set it as the mean annual rate of earthquake events with spectral acceleration greater than a (designated)[1] minimum value. Also implicit in the probability term $G_{S_a}(x)$ is that the spectral acceleration value x is greater than or equal to the minimum intensity level. In other words, $G_{S_a}(.)$ is equal to unity at spectral acceleration values less than or equal to the minimum intensity level designated in the definition of the rate parameter ν. The spectral acceleration hazard values $H_{S_a}(x)$ are usually plotted against different spectral acceleration values, x; this results in a curve that is usually referred to as a spectral acceleration hazard curve.

Spectral acceleration hazard curves are normally provided by seismologists for a given site (e.g. the USGS website). Each curve provides the mean annual frequency of exceeding a particular spectral acceleration value for a given period and damping ratio. It is advantageous to approximate such a curve in the region of interest by a power-law relationship (see DOE 1994 and Luco and Cornell 1998):

$$H_{S_a}(s_a) = P[S_a \geq x] = k_0 \cdot x^{-k} \tag{1.3}$$

where k_0 and k are parameters defining the shape of the hazard curve.

Figure 1.1 shows a typical hazard curve for a Southern California site that corresponds to a period of 1.8 seconds and damping ratio of 5%. As it can be seen from the figure, a line with slope k and intercept k_0 is fit to the hazard curve (on the two-way logarithmic paper) around the region of interest (e.g., MAFs between 1/475 or 10% frequency of exceedance in 50 years, and 1/2475 or 2% frequency of exceedance). Here, k =2.73 and k_0 =0.00012.

[1] A designated minimum value so that it is generally agreed that earthquakes with spectral acceleration values lower than this certain level don't cause significant damage in the structure.

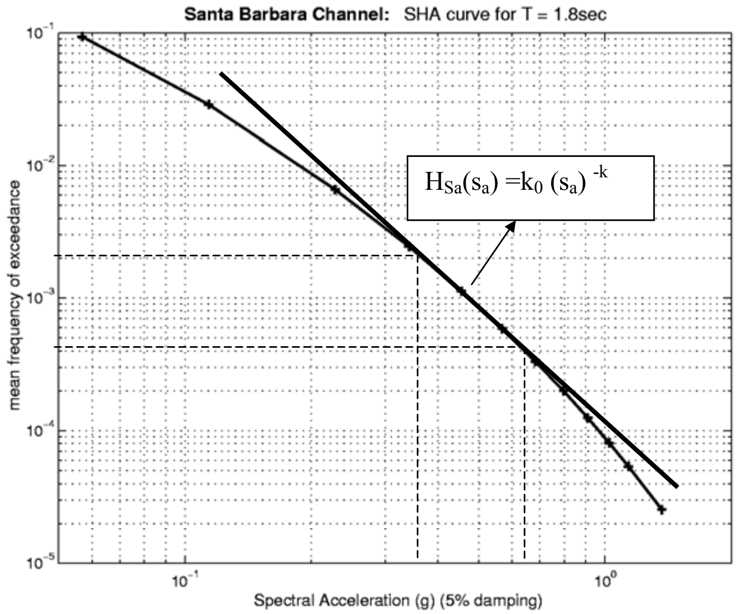


Fig. 1.1 A typical hazard curve for spectral acceleration. It corresponds to a damping ratio of 5% and a structural fundamental period of 1.8 seconds

It is important to note that the hazard values are usually provided in terms of the "mean rates" of exceedance over a certain time interval (usually a year) rather than the "probabilities" of exceedance. Therefore, it is more appropriate to refer to the hazard function as, for example, the "mean annual frequency" rather than the "annual probability" of exceeding a certain value. Nonetheless, for very small probability values, which are for example derived from a Poisson model, the average rate and the resulting probability value are almost the same. For simplicity, we are going to drop the "mean" term before the frequency. However, where epistemic uncertainty is introduced into the problem, we will need to be more precise in how we refer to the hazard function.

1.4.7.2 Median relationship between spectral acceleration and interstory drift demand

Observations of demand values are normally obtained from the result of structural time history analyses performed for various ground motion intensity levels. Figure 1.2 shows such results, e.g. maximum interstory drift, D versus S_a. For a given level of ground motion intensity, there will be variability in the displacement-based demand results over any suite of ground motion records applied to the structure. It is assumed here that this variability is a result of randomness in the seismic phenomena as discussed before (later in Section 1.4.8 we will take into account the epistemic uncertainties, such as uncertainty due to limited number of records, in addition to record-to-record variability). It is convenient to introduce a functional relationship between the ground motion intensity measure and a central value, specifically the median η_D of the demand parameter based on the data available from such time history analyses.

In general, for a spectral acceleration equal to x, the functional relationship will be:

$$\eta_D(x) = g(x)$$

This is called the conditional median of D given S_a (more formally denoted by $\eta_{D|S_a}(x)$, but we shall keep the simpler notation). We can construct a full conditional probabilistic model of the variability displayed in Figure 1.2 by writing:

$$D = \eta_D(x) \cdot \varepsilon = g(x) \cdot \varepsilon$$

where ε is a random variable with a median equal to unity and a probability distribution to be discussed below. At this point a particular functional relationship is introduced that both conforms to our perceptions of a structural performance curve and also helps simplify future analytical efforts. We have used linear regression in logarithmic space (i.e., $\ln \eta_D(x_a) = \ln a + b \ln x$) in order to fit a power-law function, $a \cdot {S_a}^b$, to our collection of maximum interstory drift ratio and "first-"mode spectral acceleration data pairs.

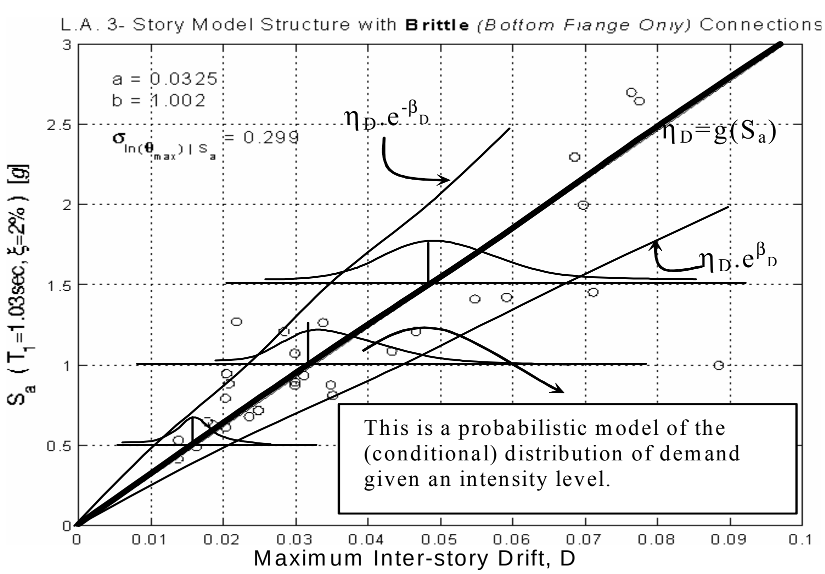


Fig. 1.2 A set of spectral acceleration and demand data pairs and the regression model fit to these data points.

It is not an objective here to describe the various ways $\eta_D(x)$ may be estimated. In design practice it is likely to come from one or more structural analysis procedures, perhaps previously calibrated to nonlinear dynamic results for similar structures (FEMA 350 2000 and Yun and Foutch 2002). In assessment practice or research it can be obtained through one or more schemes of selecting and processing records and results (Bazzurro et al. 1998; Luco and Cornell 1998; Vamvatsikos and Cornell 2001). We shall see below that the number of required time history analyses may be quite small (e.g., on the order of 5 to 10). For a set of drift demand and spectral acceleration data points, such a regression in the logarithmic scale will result in the following relationship between spectral acceleration and (median) interstory drift response:

$$\eta_D(x) = a \cdot x_a{}^b \tag{1.4}$$

Figure 1.2 illustrates a typical power-law relationship between the median maximum interstory drift demand and the spectral acceleration for a three-story steel frame building located in Los Angeles. In this case, $b \cong 1$ which is consistent with the so-called "equal displacement rule" (Veletsos and Newmark 1960).

1.4.7.3 Mean annual frequency of exceeding demand: drift hazard

We are going to break the displacement-based approach for deriving the limit state frequency in Equation 1.2 into two parts. The first part is to derive the mean annual frequency (MAF*)* that the displacement-based demand exceeds a given value *d,* also referred to as the "drift hazard*,*" and the second part is to derive the MAF that the displacement-based demand exceeds limit state capacity, also referred to as the "limit state frequency." In simple terms, the uncertainty due to randomness in demand and capacity is taken into account in two stages. This section describes the derivation of a closed-form expression for the mean annual frequency of exceeding a certain demand value *d*, also known as the "drift hazard," by taking into account the randomness in the displacement-based demand.

Recall from the last section that the median demand versus spectral acceleration relationship was introduced as:

$$\eta_D(x) = a \cdot x^{\mathrm{b}} \tag{1.5}$$

As shown above, the demand can be written in terms of the product of its median value and a lognormal random variable ε with the following characteristics:

$$D = \eta_D(x) \cdot \varepsilon \tag{1.6}$$

We assume (based on observation of data) that ε can be represented by a lognormal distribution, in which case we define its parameters, the median and standard deviation of $\ln \varepsilon$, to be:

$$\begin{aligned} \eta_\varepsilon &= e^{mean(\ln(\varepsilon))} = 1 \\ \sigma_{\ln(\varepsilon)} &= \beta_{D|S_a} \end{aligned} \tag{1.7}$$

where η_ε denotes the median value for ε. Note that what we call the "dispersion," i.e., $\beta_{D|S_a}$, will in general depend to some degree on the level of S_a. Here for analytical tractability, we assume that it is constant; the value should be chosen for S_a values in the range of primary interest. If $\eta_D(x)$ is replaced with its corresponding value from Equation 1.4, the following expression for drift demand as a function of spectral acceleration and lognormal random variable ε is obtained:

$$D = a \cdot x_a{}^b \cdot \varepsilon \tag{1.8}$$

Since we have assumed that ε is a lognormal variable, we can also conclude that the displacement-based demand *D* is also a random variable with the following statistical properties:

$$\begin{aligned} \eta_{D|S_a}(x) &= a \cdot x^b \\ \sigma_{\ln D|S_a}(x) &= \beta_{D|S_a} \end{aligned} \tag{1.9}$$

where $\eta_{D|S_a}(x)$ and $\sigma_{\ln D|S_a}(x)$ are the conditional median and standard deviation of the natural logarithm for the displacement-based demand given spectral acceleration. As mentioned above, the conditional standard deviation of the natural logarithm $\sigma_{\ln D|S_a}(x)$ or the conditional "fractional" standard deviation $\beta_{D|S_a}(x)$ of demand given spectral acceleration is assumed to be constant. The conditional median demand for a given spectral acceleration $\eta_{D|S_a}(x)$ (or more briefly $\eta_D(x)$) is approximated as a power-law function of the spectral acceleration level, *x*.

Figure 1.3 illustrates a graphical presentation of basic components of the derivation of a closed-from expression for drift hazard in which the median drift curve, the variability of the displacement-based response around it, and the conditional lognormal distribution fit to the data (at any given S_a) are all plotted together with the spectral acceleration hazard. The median drift times $\exp(\beta_{D|S_a})$ is referred to as the "mean plus one sigma" curve as it corresponds to the 84$^{\text{th}}$

percentile of the data for a lognormal variable; this is illustrated in the figure as $\eta_D(x) \cdot \exp(\beta_{D|S_a})$. In a similar manner, the median drift times $\exp(-\beta_{D|S_a})$ is referred to as the "mean minus one sigma" curve as it corresponds to the 16th percentile of the data (for a lognormal variable) that is illustrated in the figure as $\eta_D(x) \cdot \exp(-\beta_{D|S_a})$.

In the previous sections we have defined the spectral acceleration hazard and limit state frequency as the product of a probability of exceedance term (i.e., a complementary cumulative probability density function or CCDF for brevity) times a rate parameter. Here we are also going to define the drift hazard $H_D(d)$ as the product of the rate parameter (also encountered in sections 1.4.4. and 1.4.7.1) ν and the probability of exceeding a specific demand value, *d*:

$$H_D(d) = \nu \cdot P[D > d]$$

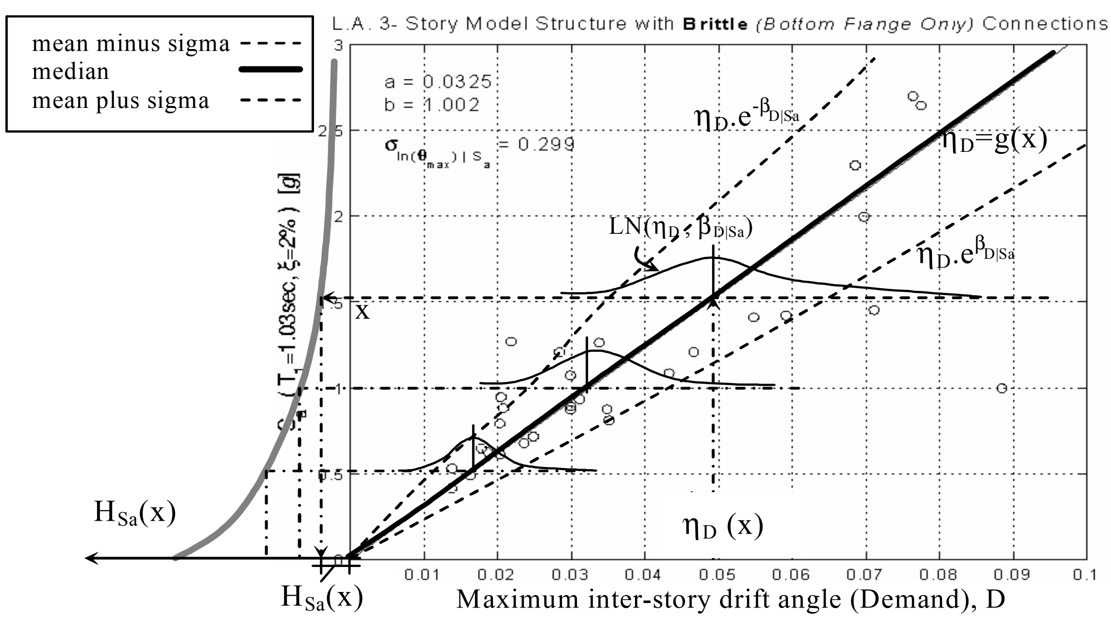


Fig. 1.3 Basic elements of the derivation of a closed-form expression for drift hazard, $H_{S_s}(x)$, and of modeling the distribution of D given S_a characterized by $\eta_D(x)$ and $\beta_{D|S_a}$.

In this section we are going to derive the drift hazard by decomposition and recomposition with respect to spectral acceleration via the total probability theorem (TPT), similar to the *IM*-based solution strategy outlined in Section 1.4.5 for deriving the limit state frequency. Applying the general solution strategy to the derivation of the drift hazard in this section, we can decompose the drift hazard into the conditional probability of exceeding drift value *d* for a given spectral acceleration value *x* and the likelihood that the spectral acceleration is equal to the value *x*:

$$H_D(d) = \nu \cdot P[D > d] = \nu \cdot \sum_{\text{all x}} P[D > d \mid S_a = x] \cdot P[S_a = x] \tag{1.10}$$

where, as mentioned before, ν represents the (mean annual) rate of the occurrence of the "events of interest," e.g., events with spectral acceleration greater than a designated minimum value. Thus, the drift hazard in Equation 1.10 is equal to $P[D > d]$ times the rate of occurrence of the earthquake events that interest us. Therefore, the drift hazard itself is expressed in terms of the "rate of exceedance," or the mean annual frequency of exceedance (MAF).

We should note that the above expression involves discrete variables. However, since we are using analytic parameter estimations, we are going to base our derivations on an equivalent expression for the drift hazard derived for continuous variables:

$$H_D(d) = \nu \cdot P[D > d] = \int_0^\infty P[D > d \mid S_a = x] \cdot \nu \cdot f_{S_a}(x) \cdot dx = \int_0^\infty P[D > d \mid S_a = x] \cdot \left| \nu \cdot dG_{S_a}(x) \right| \tag{1.11}$$

where $f_{S_a}(x)$ is the probability density function (PDF) at spectral acceleration value *x*, and, $G_{S_a}(x)$ is the complementary cumulative distribution function (CCDF) at $S_a = x$. It should be noted that the $\left| \nu \cdot dG_{S_a}(x) \right|$ term in Equation 1.11 is resulting from the following relationship between $F(\cdot)$ or the cumulative distribution function (CDF) and $f(\cdot)$ or the PDF for a random variable (e.g., spectral acceleration S_a):

$$f_{S_a}(x) = \lim_{\Delta x \to 0} \frac{P[x \le S_a \le x + \Delta x]}{\Delta x} = \lim_{\Delta x \to 0} \frac{P[S_a \le x + \Delta x] - P[S_a \le x]}{\Delta x} = \frac{dF_{S_a}(x)}{dx} = \left| \frac{dG_{S_a}(x)}{dx} \right| \tag{1.12}$$

The last equality is based on the fact that the CCDF is expressed in terms of the probability of exceedance whereas the CDF is expressed in terms of the probability of being less than or equal to exceedance. Therefore, their corresponding derivatives are equal in absolute values but will have opposite signs.

It should be noted that the spectral acceleration hazard $H_{S_a}(x)$ is equal to the spectral acceleration CCDF, $G_{S_a}(x)$ times the rate of seismicity ν:

$$H_{S_a}(x) = \nu \cdot G_{S_a}(x) \tag{1.13}$$

Therefore, we can rewrite Equation 1.11 as a function of the spectral acceleration hazard:

$$H_D(d) = \int_0^\infty P[D > d \mid S_a = x] \cdot \left|\nu \cdot dG_{S_a}(x)\right| = \int_0^\infty P[D > d \mid S_a = x] \cdot \left|dH_{S_a}(x)\right| \tag{1.14}$$

Since we have assumed that the displacement-based demand is a lognormal variable, $P[D > d \mid S_a = x]$ can be derived using the tables that provide the CDF of a standardized normal variable (Rice 1995). In order to use the normal tables, we first need to transform the random variable *D* into a standardized normal variable:

$$\begin{aligned} P[D > d \mid S_a = x] &= 1 - P[D \le d \mid S_a = x] = 1 - P[\frac{\ln D - mean \ln D}{\beta_{D|S_a}} \le \frac{\ln d - mean \ln D}{\beta_{D|S_a}} \mid S_a = x] \\ &= 1 - P[\frac{\ln D - \ln \eta_{D|Sa}(x)}{\beta_{D|S_a}} \le \frac{\ln d - \ln \eta_{D|Sa}(x)}{\beta_{D|S_a}}] = 1 - \Phi\left(\frac{\ln(\frac{d}{a \cdot x^b})}{\beta_{D|S_a}}\right) \end{aligned} \tag{1.15}$$

where $\Phi(\cdot)$ is the standardized Gaussian CDF. The above equation is derived based on the following property of a lognormal variable in which the mean of the logarithm is equal to the logarithm of the median (Benjamin and Cornell 1970):

$$mean \ln D = \ln \eta_D$$

If we substitute the standardized Gaussian representation of $P[D > d \mid S_a = x]$ in Equation 1.15 into Equation 1.14, the drift hazard will be expressed as:

$$H_D(d) = \int_0^\infty P[D > d \mid S_a = x] \cdot \left|dH_{S_a}(x)\right| = \int_0^\infty \{1 - \Phi\left(\frac{\ln(\frac{d}{a \cdot x^b})}{\beta_{D|S_a}}\right)\} \cdot \left|dH_{S_a}(x)\right| \tag{1.16}$$

We are going to use integration by parts in order to re-arrange the above equation so that we can integrate it analytically. We first need to calculate the derivative of the first term in the integrand:

$$\frac{d}{dx}\{1-\Phi\left(\frac{\ln(\frac{d}{a\cdot x^{b}})}{\beta_{D|S_a}}\right)\} = -\frac{d}{dx}\Phi\left(\frac{\ln d-\ln a\cdot x^{b}}{\beta_{D|S_a}}\right)=\frac{b}{x\cdot\beta_{D|S_a}}\cdot\phi\left(\frac{\ln d-\ln a\cdot x^{b}}{\beta_{D|S_a}}\right) \qquad (1.17)$$

where $\phi(\cdot)$ is the standardized Gaussian PDF which is equal to:

$$\phi(u)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}u^{2}} \qquad (1.18)$$

for any standardized normal variable *u*. The drift hazard in Equation 1.14 is re-arranged into the following form after applying the integration by parts assuming that the term $P[D>d\,|\,S_a=x]\cdot H_{S_a}(x)$ is close to zero for the integration limits, i.e., very small and very large S_a values. It should be noted that for a lognormal variable, the range of possible values vary from 0 to ∞.

$$H_D(d)=\int_0^{\infty}\frac{dP[D>d\,|\,S_a=x]}{dx}\cdot H_{S_a}(x)\cdot dx=\int_0^{\infty}\frac{b}{x\cdot\beta_{D|S_a}}\cdot\phi\left(\frac{\ln d-\ln a\cdot x^{b}}{\beta_{D|S_a}}\right)\cdot H_{S_a}(x)\cdot dx \qquad (1.19)$$

Now we are going to replace the hazard term $H_{S_a}(x)$ by its power-law approximation from Equation 1.3 and also replace the Gaussian PDF by its analytical form in Equation 1.17:

$$H_D(d)=\int_0^{\infty}\frac{b}{x\cdot\beta_{D|S_a}}\cdot\frac{1}{\sqrt{2\pi}}\cdot\exp(-\frac{1}{2}\left(\frac{\ln d-\ln a-b\ln x}{\beta_{D|S_a}}\right)^{2})\cdot k_0\cdot x^{-k}\cdot dx \qquad (1.20)$$

In order to calculate the above integral analytically, we are going to form a square term in the power of the exponential term inside the integral (so that we can form a Gaussian PDF). This way we can calculate the integral by using the fact that the integral of a PDF function (over all possible values of the variable) is equal to unity. We begin by some simple algebraic manipulations in order to simplify the equation a bit:

$$H_D(d) = k_0 \int_0^\infty \frac{b}{x \cdot \beta_{D|S_a}} \cdot \frac{1}{\sqrt{2\pi}} \cdot \exp(-\frac{1}{2}\left(\frac{\ln(\frac{d}{a})^{\frac{1}{b}} - \ln x}{\frac{\beta_{D|S_a}}{b}}\right)^2) \cdot \exp(-\frac{1}{2} \cdot 2k \ln x) \cdot dx \qquad (1.21)$$

The next step is to form a full squared term inside the integral and also take all the constant terms out of the integral:

$$H_D(d) =$$

$$k_0 \exp(\frac{1}{2}k^2(\frac{\beta_{D|S_a}}{b})^2) \cdot \exp(-k\ln(\frac{d}{a})^{\frac{1}{b}}) \int_0^\infty \frac{b}{x \cdot \beta_{D|S_a}} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}(\frac{\ln x - \{k(\frac{\beta_{D|S_a}}{b})^2 - \ln(\frac{d}{a})^{\frac{1}{b}}\}}{\frac{\beta_{D|S_a}}{b}})^2)dx \qquad (1.22)$$

Note that the term inside the integral is indeed the PDF for the standardized Gaussian variable u with the derivative $\frac{du}{dx}$:

$$u = \frac{\ln x - \{k(\frac{\beta_{D|S_a}}{b})^2 - \ln(\frac{d}{a})^{\frac{1}{b}}\}}{\frac{\beta_{D|S_a}}{b}}$$

$$\frac{du}{dx} = \frac{b}{x \cdot \beta_{D|S_a}}$$

Therefore the expression for drift hazard can be also written as:

$$H_D(d) = k_0 \exp(\frac{1}{2}k^2(\frac{\beta_{D|S_a}}{b})^2) \cdot \exp(-k\ln(\frac{d}{a})^{\frac{1}{b}}) \int_0^\infty \frac{d}{dx}\{\Phi\left(\frac{\ln x - \{k(\frac{\beta_{D|S_a}}{b})^2 - \ln(\frac{d}{a})^{\frac{1}{b}}\}}{\frac{\beta_{D|S_a}}{b}}\right)\} \cdot dx \qquad (1.23)$$

Noting that the integral of a normal PDF over all the possible values is equal to unity, the drift hazard can be written in the following simplified form:

$$H_D(d) = k_0 \exp(\frac{1}{2}k^2(\frac{\beta_{D|S_a}}{b})^2) \cdot \exp(-k\ln(\frac{d}{a})^{\frac{1}{b}}) = k_0 (\frac{d}{a})^{\frac{-k}{b}} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}} \qquad (1.24)$$

In order to have a more condensed formulation of the drift hazard, we introduce the notation S_a^d or *spectral acceleration "corresponding" to drift angle* d :

$$S_a^d = \left(\frac{d}{a}\right)^{\frac{1}{b}}$$

This is also the solution of Equation 1.4 for a given value of d, i.e., if we read the corresponding S_a value[2] from $d = a \cdot S_a{}^b$ curve. The graphic interpretation of S_a^d can be seen from Figure 1.4. In simple terms, this means that for a given drift demand value *d*, we find the corresponding S_a value from the median curve $\eta_D(S_a) = a \cdot S_a{}^b$.

The derived closed-form expression can be further simplified by making use of the hazard curve definition in Equation 1.3:

$$H_D(d) = k_0 \cdot \left(\frac{d}{a}\right)^{\frac{-k}{b}} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}} = k_0 \cdot \left(S_a^d\right)^{-k} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{D|S_a}}$$

$$\Rightarrow H_D(d) = H_{S_a}(S_a^d) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}} \qquad (1.25)$$

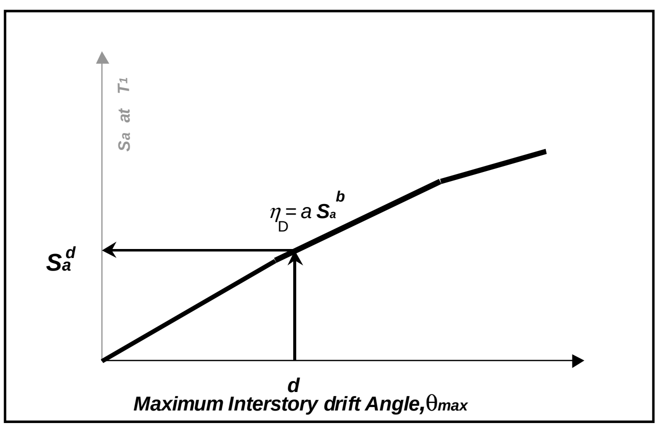


Fig. 1.4 Spectral acceleration corresponding to the demand value, d

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2 Note that S_a^d is not necessarily the median S_a for a given value of drift angle d. It is just the corresponding S_a value found from the curve. In other words, the fact that the D - S_a curve gives the median drift d for a given value of S_a does not mean that it will also provide the median S_a for a given value of drift d.

It can be seen by inspection of Equation 1.25 that the hazard curve for the drift demand $H_D(d)$ is equal to the hazard function $H_{S_a}(.)$ evaluated at the spectral acceleration corresponding to this drift demand times a (magnifying) factor related to dispersion in the drift demand for a given spectral acceleration. The first factor can be interpreted as a "first order" estimate; it is also the drift hazard if the dispersion $\beta_{D|S_a}$ is zero. Experience suggests that the second factor may typically have values in the order of 1.5 to 3. Note that in this form one can read the first factor directly from a given hazard curve without actually making the approximating fit, $k_0 \cdot x^{-k}$. The log-log slope k of the approximation is needed for the second factor, however. Numerical applications of the closed-form expression for drift hazard developed in this section can also be found in Medina (2002), where the drift hazard was derived both by using the closed-form in Equation 1.25 and also by the numerical integration of Equation 1.14 for a nine-story generic frame. Medina observed that the closed-form solution was reasonably close to the numerical integration. However, the results indicated a strong sensitivity to the estimated standard deviation that is assumed to be constant with respect to the intensity level (one of the assumptions underlying the closed-form solution). The authors (Jalayer and Cornell, 2003a,b) have plotted the drift hazard curve by incorporating local parameter-estimates (obtained from the results of nonlinear dynamic analyses) in the closed-form solution and have compared the results to that of the numerical integration for a seven-story reinforced concrete frame with degrading behavior in shear and flexure. It should be noted that by using local parameter estimates, some of the assumptions that led to the derivation of the closed-form expression (including the assumption of a constant standard deviation and of a power-law form for the median demand as a function of spectral acceleration) are overruled. Nonetheless, the results demonstrate good agreement between the numerical integration and the closed-form with local parameter estimates.

1.4.7.3.1 Numerical example

We will now derive the drift hazard curve for a three-story (model) structure with brittle connections located in Los Angeles. This structure is a typical three-story steel moment resisting frame building used in the SAC project (Luco and Cornell 1998). A set of nonlinear dynamic analyses has been conducted, and the resulting maximum interstory drift ratios have been plotted versus the first mode spectral acceleration as it is illustrated in Figure 1.5. The hazard curve represented in Figure 1.5 corresponds to oscillators with a fundamental period around 1.0 sec and located in Los Angeles, thus we have used it as the spectral acceleration hazard curve for our model structure. In approximate analytical form it is:

$$H_{S_a}(s_a) = P[S_a \geq s_a] = 0.00124 \cdot s_a^{-3.03}$$

Note that the k value is nearly equal to 3.0. Our next step is to determine the median relationship between spectral acceleration and drift. This is done in Figure 1.5, by fitting a line to the data points in a log-log scale; which gives the following information:

$$\eta_D(S_a) = 0.0325 \cdot S_a^{1.002}$$
$$\beta_{D|S_a} = 0.299 \approx 0.3$$

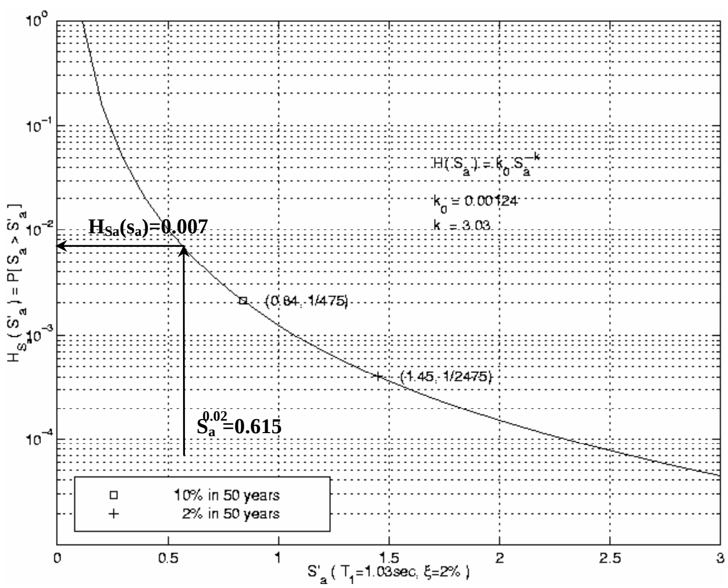

Fig. 1.5 Hazard curve for spectral acceleration at a period equal to 1.0 second and damping ratio of 2%.

Note that $b \approx 1$ for this range of data, i.e., the median drift is approximately proportional to S_a. It should be mentioned, however, that there may be a certain level of nonlinearity (material or geometric) in which b is not close to 1.0 anymore. Linear behavior is limited in this structure to interstory drifts less than about 0.01. We would like to evaluate the probability that the maximum interstory drift angle exceeds a specific value, say 2%, $H_D(0.02)$. If we substitute 0.02 for d in Equation 1.25:

$$H_D(0.02) = P[D > 0.02] = H_{S_a}(S_a^{0.02}) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}}$$

Recall that S_a^d is equal to $S_a^d = \left(\frac{d}{a}\right)^{\frac{1}{b}}$ per definition:

$$S_a^{0.02} = \left(\frac{0.02}{0.0325}\right)^1 = 0.615 \ \ [\text{g}]$$

Equivalently we could simply have read this value from median line in Figure 1.5 by entering at a drift value equal to 0.02. Now we will look up the value of $H_{S_a}(0.615)$ for the spectral acceleration hazard curve. As illustrated in Figure 1.5 it is equal to 0.007. Hence, $H_D(0.02)$ can be derived as:

$$H_D(0.02) = P[D > 0.02] = 0.007 \cdot e^{(\frac{1}{2})(\frac{3^2}{1^2})(0.3^2)} = 0.007 \times 1.5 = 0.0105$$

Note that the factor $\exp(\frac{1}{2} \cdot \frac{3^2}{1^2} \cdot 0.3^2)$ is equal to 1.50.

We can repeat the above calculations for multiple drift values in order to obtain the drift hazard curve, or we can find an analytical expression for the drift hazard. In general, we can compute the drift hazard for a specified drift value, *d*, as follows:

$$H_D(d) = P[D > d] = H_{S_a}(S_a^d) \cdot e^{\frac{1}{2} \cdot \frac{k^2}{b^2} \cdot \beta^2{}_{D|S_a}}$$

Recalling that $\exp(\frac{1}{2} \cdot \frac{3^2}{1^2} \cdot 0.3^2)$ is equal to 1.5 and $S_a^d = \left(\frac{d}{a}\right)^{\frac{1}{b}} \cong \frac{d}{0.0325}$, the above equation becomes:

$$H_D(d) = H_{S_a}(S_a^d) \cdot e^{\frac{1}{2} \cdot \frac{k^2}{b^2} \cdot \beta^2{}_{D|S_a}} = 1.5 \cdot H_{S_a}(\frac{d}{0.0325})$$

Next we need to find the expression for the spectral acceleration hazard curve evaluated at $\frac{d}{0.0325}$. This is:

$$H_{S_a}(\frac{d}{0.0325}) = 0.00124 \cdot \left(\frac{d}{0.0325}\right)^{-3.03} \approx 4.25 \cdot 10^{-8} \cdot d^{-3}$$

Finally the drift hazard for a specified value of drift, *d* is derived as:

$$H_D(d) \approx 6.375 \cdot 10^{-8} \cdot d^{-3}$$

The above relationship is plotted in Figure 1.6.

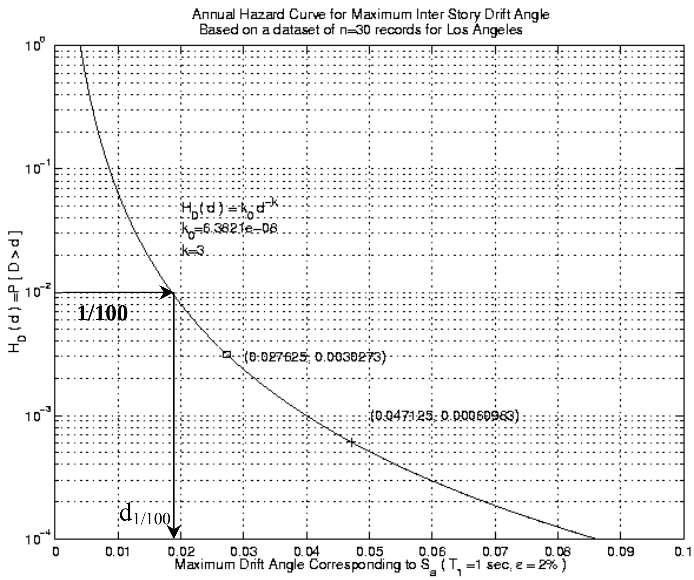

Fig. 1.6 Hazard curve derived for maximum interstory drift values

The above curve can be used to determine, for example, the 100-year return period drift, by setting $H_D(d_{1/100})$ to 1/100 and solving for $d_{1/100}$:

$$1/100 = 6.375 \cdot 10^{-8} \cdot (d_{1/100})^{-3}$$

solving for $d_{1/100}$: $d_{1/100} = 0.0185$. The same value can also be found simply from Figure 1.6.

1.4.7.4 Annual frequency of exceeding a limit state

We have already derived the mean annual frequency that the displacement response variable, *D*, exceeds a certain value. The next step is to find the probability that the response variable, *D*, exceeds a specified limit state threshold or capacity, *C*. The difference in this case is due to the fact that the limit state threshold can be a random variable itself. For example in the SAC project (FEMA 350, 2000) modern "reduced beam section" (RBS) connections were concluded to have a median capacity of $\eta_C = 0.07$ (interstory drift ratio) with a dispersion of $\beta_C = 0.2$ reflecting specimen-to-specimen variability in (hypothetical[3]) test results and even possible record-to-

[3] In fact no connections were able to be tested by the SAC project to such large drift ratios. The parameters were estimated indirectly and are based on some level of expert opinion.

record variations in the drift failure due to sequence effects in the low-cycle fatigue suffered by the connection. Beyond the drift capacity, the connection lost virtually all vertical load-carrying (shear) capacity, implying the potential collapse of the floor above. In this section we will derive the expression for the limit state frequency, H_{LS}, by introducing the variability in the limit state capacity. The basic elements involved in the derivation are illustrated in Figure 1.7. Once again we use the total probability theorem to sum up the joint probabilities that limit state variable exceeds the capacity variable for a given value of capacity, over the entire range of possible values for the capacity variable:

$$P[D \geq C] = \sum_{all\ c} P[D \geq c \mid C = c] P[C = c] \tag{1.26}$$

We next assume that demand and capacity are (statistically) independent, i.e., that:

$$P[D \geq C \mid C = c] = P[D \geq c] \tag{1.27}$$

In Appendix C, we have outlined a derivation of limit state frequency when demand and capacity are correlated.

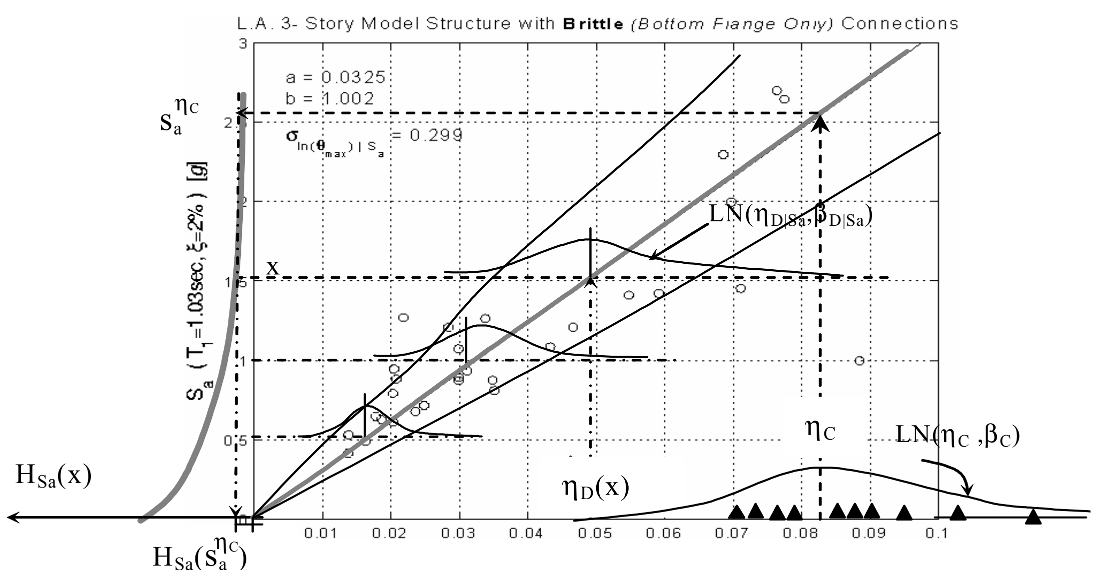

Fig. 1.7 Basic elements of the derivation of a closed-form expression for limit state frequency $H_{S_s}(x)$, distribution of drift variable *D* given S_a characterized by $\eta_D(x)$ and $\beta_{D|S_a}$, distribution of capacity variable *C* characterized by η_C and β_C

The annual frequency of exceeding the limit state, H_{LS}, can be expressed as the limit state probability $P[D \geq C]$ times the seismicity rate ν (as mentioned in Section 1.4.7.3):

$$H_{LS} = \nu \cdot P_{LS} = \nu \cdot P[D \geq C] \tag{1.28}$$

Since we are going to base our derivations in this section on the expression for drift hazard, our calculations are going to yield the mean annual frequency of exceedance (or limit state frequency in short), H_{LS}[4], as the end result. Therefore, the limit state frequency can be calculated by substituting Equations 1.26 and 1.27 into Equation 1.28:

$$H_{LS} = \nu \cdot P_{LS} = \nu \cdot P[D \geq C] = \sum_{all\ c} \nu \cdot P[D \geq c] P[C = c] \tag{1.29}$$

The probability that drift demand exceeds drift capacity for a given value of drift capacity can be readily determined from the drift hazard curve:

$$H_D(c) = \nu \cdot P[D \geq c] \tag{1.30}$$

Substituting the term $\nu \cdot P[D \geq c]$ in Equation 1.29 by the expression for $H_D(c)$ from the above equation:

$$H_{LS} = \nu \cdot P_{LS} = \sum_{all\ c} H_D(c) \cdot P[C = c] \tag{1.31}$$

However, the above equation is valid for discrete variables; in the continuous form, the summation is replaced by an integral and the probability term, $P[C = c]$, is replaced by the probability density function term, $f_C(c) \cdot dc$:

$$H_{LS} = \nu \cdot P_{LS} = \int H_D(c) \cdot f_C(c) \cdot dc \tag{1.32}$$

Substituting the drift hazard value for $H_D(c)$ from Equation 1.25 into Equation 1.32 results in:

$$H_{LS} = \int H_D(c) f_C(c) dc = \int H_{S_a}(S_a^c) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{D|S_a}} f_C(c) dc \tag{1.33}$$

---

[4] In this chapter, we have used the notation $H(.)$ in order to refer to the mean annual rate of exceedance. However, in the next chapters we may use the notation $\lambda(.)$ instead.

From Equations 1.3 and 1.4, $H_{S_a}(S_a^c)$ is equal to $k_0 \cdot \left(\frac{c}{a}\right)^{\frac{-k}{b}}$. Thus, the limit state frequency is obtained by performing the following integration:

$$H_{LS} = \int H_{S_a}(S_a^d) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{D|S_a}} \cdot f_C(c) \cdot dc = \int k_0 \cdot \left(\frac{c}{a}\right)^{\frac{-k}{b}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{D|S_a}} \cdot f_C(c) \cdot dc \qquad (1.34)$$

For the above integral to be evaluated, the probability density function of the random variable C, $f_C(c)$, has to be known. Here for tractability, it is assumed that C is a lognormal random variable with following characteristics:

$$median(C) = \eta_C$$
$$\sigma_{\ln(C)} = \beta_C$$

After some simple re-arrangements:

$$H_{LS} = \int k_0 \left(\frac{c}{a}\right)^{\frac{-k}{b}} e^{\frac{1}{2}\frac{k^2}{b^2}\cdot\beta^2_{D|S_a}} f_C(c)dc = k_0 \left(\frac{1}{a}\right)^{\frac{-k}{b}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{D|S_a}} \cdot \int c^{\frac{-k}{b}} f_C(c)dc \qquad (1.35)$$

It can be seen that the term inside the last integral equals expectation of $c^{-\frac{k}{b}}$. It has been shown in Appendix A that the expected value of lognormal random variable Y (with median η_Y and dispersion $\sigma_{\ln Y}$), to the power of α equals to:

$$E(Y^a) = E(e^{a\ln Y}) = (\eta_Y)^{a} . e^{\frac{1}{2}a^2\sigma^2_{\ln Y}}$$

Since limit state capacity *C* is assumed to be a lognormal variable, the above property can be used to solve the integral in Equation 1.35 as follows (For further details regarding the integration scheme refer to Appendix A):

$$H_{LS} = k_0 \left(\frac{1}{a}\right)^{\frac{-k}{b}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{D|S_a}} \cdot \mathrm{E}(c^{\frac{-k}{b}}) = k_0 \left(\frac{1}{a}\right)^{\frac{-k}{b}} \cdot \eta_C^{\frac{-k}{b}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{D|S_a}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta_C^2}$$

We conclude that:

$$H_{LS} = \nu \cdot P[D > C] = k_0 \left(\frac{\eta_C}{a}\right)^{\frac{-k}{b}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{D|S_a}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta_C^2}$$

We can recognize in the above expression the spectral acceleration hazard from Equation 1.3 combined with the spectral acceleration-median drift relationship in Equation 1.4, $k_0(\eta_C / a)^{-k/b}$, which equals the hazard value for the spectral acceleration corresponding to median capacity, $S_a^{\eta_C}$:

$$H_{S_a}(S_a^{\eta_C}) = k_0\left(\frac{\eta_C}{a}\right)^{\frac{-k}{b}} \tag{1.36}$$

Thus:

$$k_0 \cdot \left(\frac{\eta_C}{a}\right)^{\frac{-k}{b}} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}} = H_{S_a}(S_a^{\eta_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_C} = H_D(\eta_C) \tag{1.37}$$

where the last equality is based on the expression for drift hazard $H_D(.)$ at median capacity, η_C, from Equation 1.25, and $S_a^{\eta_C}$, as mentioned before, is the spectral acceleration "corresponding" to a drift value equal to η_C, i.e., $S_a^{\eta_C} = (\eta_C / a)^{b^{-1}}$. Finally, the limit state frequency is derived as:

$$H_{LS} = \nu \cdot P[D > C] = H_D(\eta_C) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta_C^2} = H_{S_a}(S_a^{\eta_C}) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\cdot\beta_C^2} \tag{1.38}$$

It can be observed that the limit state frequency (or the MAF of demand exceeding the limit state capacity) is equal to the hazard curve for the spectral acceleration corresponding to the median drift capacity times two coefficients accounting for the randomness in drift demand for a given spectral acceleration and the randomness in drift capacity itself. Again the first factor can be seen as a first-order approximation to the limit state frequency, H_{LS}.

1.4.7.4.1 Numerical example:

Returning to our three-story frame numerical example of the last section, we now assume that the median drift collapse capacity and its dispersion parameter are given as (the same as the SAC connections example mentioned in Section 1.4.7.4):

$$median(C) = \eta_C \cong 0.07$$
$$\sigma_{\ln(C)} = \beta_C \cong 0.20$$

We first need to find $H_{S_a}(S_a^{\eta_C})$. We can do this graphically, where $S_a^{\eta_C}$ can be calculated as the spectral acceleration corresponding to $\eta_C = 0.07$ from the median-spectral acceleration curve in Figure 1.7 resulting in $S_a^{0.07} \cong 2.15g$ (note that the capacity points in the figure are only for schematic representation). The corresponding hazard value from the hazard curve (Fig. 1.8 below) is equal to, $H_{S_a}(2.15) \cong 0.00012$.

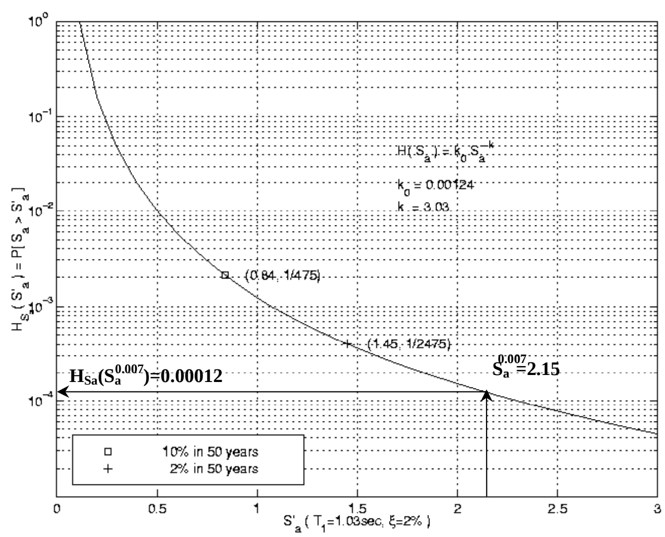

Fig. 1.8 The spectral acceleration hazard curve. The hazard value for a spectral acceleration equal to 2.15 is shown on the figure.

Alternatively, we can use the closed-form expression derived in the previous section for the limit state frequency. Using Equation 1.36, we can calculate the hazard value for the spectral acceleration corresponding to the median drift capacity as:

$$H_{S_a}(S_a^{\eta_C}) = H_{S_a}(S_a^{0.07}) = 0.00124 \cdot \left(\frac{0.07}{0.0325}\right)^{-3} = 1.2\times10^{-4}$$

Also, the capacity factor in Equation 1.38 can be calculated as follows:

$$\exp(\frac{1}{2}\frac{k^2}{b^2}\beta_C{}^2) = \exp[\frac{1}{2}\cdot\frac{3^2}{1^2}\cdot 0.2^2] = \exp[0.18] = 1.19$$

This value and the 1.50 value for the coefficient $\exp(\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{D|S_a})$ already calculated in Section 1.4.7.3.1 are used to calculate the annual frequency of exceeding the limit state from Equation 1.38 as follows:

$$H_{LS} = H_{S_a}(S_a^{\eta_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta_C^2} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta_{D|S_a}^2} = 1.2\times10^{-4} \times 1.50 \times 1.19 = 2.2\times10^{-4}$$

It can be seen that in this example that the randomness in the drift capacity and in the drift demand for a given spectral acceleration cause the limit state frequency to increase about a factor of 2 over its first order approximation of, $H_{S_a}(S_a^{\eta_C}) = 1.2\times10^{-4}$.

1.4.7.5 Annual frequency of exceeding a limit state, using the IM-based solution strategy

In this section we are going to derive the annual frequency of exceeding a limit state, H_{LS}, by following the *IM*-based solution strategy outlined in Section 1.4.5. The total probability theorem (TPT) is used to decompose the expression for the limit state frequency into (conditional) frequencies of exceeding the limit state for a given spectral acceleration (the adopted *IM*), and to compose the results by integration over all spectral acceleration values:

$$H_{LS} = \nu \cdot P[S_a \geq S_{a,C}] = \int P[S_a \geq S_{a,C} \mid S_a = x] \cdot \nu \cdot f_{S_a}(x) \cdot dx = \int P[x \geq S_{a,C}] \cdot \left| dH_{S_a}(x) \right| \qquad (1.39)$$

where S_a represents the *IM*-based demand, $S_{a,C}$ represent the limit state capacity also expressed in spectral acceleration terms, and ν represents the seismicity rate (the reason for including it in the derivations is explained before for the displacement-based derivation). We have used Equation 1.12 in order to express the PDF of spectral acceleration in terms of the increment in the spectral acceleration hazard. We assume that the spectral acceleration capacity is a lognormal variable with the following statistical parameters:

$$median(S_{a,C}) = \eta_{S_{a,C}}$$
$$\sigma_{\ln(S_{a,C})} = \beta_{S_{a,C}}$$

We can observe that the first term in the integral $P[s_a \geq S_{a,C}]$ can be also interpreted as the CDF of the spectral acceleration capacity at, $S_a = x$:

$$F_{S_{a,C}}(x) = P[x \geq S_{a,C}]$$

Since $S_{a,C}$ is assumed to be a lognormal variable, the corresponding CDF can be expressed in terms of the standardized normal CDF:

$$F_{S_{a,C}}(x) = P[x \ge S_{a,C}] = \Phi\left(\frac{\ln \frac{x}{\eta_{S_{a,C}}}}{\beta_{S_{a,C}}}\right) \tag{1.40}$$

In order to be able to integrate Equation 1.39, we use integration by parts and transform the equation into the following form:

$$H_{LS} = \nu \cdot P[S_a \ge S_{a,C}] = -\int \Phi\left(\frac{\ln \frac{x}{\eta_{S_{a,C}}}}{\beta_{S_{a,C}}}\right) \cdot dH_{S_a}(x) = \int d\{\Phi\left(\frac{\ln \frac{x}{\eta_{S_{a,C}}}}{\beta_{S_{a,C}}}\right)\} \cdot H_{S_a}(x) \tag{1.41}$$

Just as in Section 1.4.7.3, the derivative of the standard normal CDF can be calculated as:

$$\frac{d}{dx}\{\Phi\left(\frac{\ln \frac{x}{\eta_{S_{a,C}}}}{\beta_{S_{a,C}}}\right)\} = \frac{1}{x \cdot \beta_{S_{a,C}}} \cdot \phi\left(\frac{\ln x - \ln \eta_{S_{a,C}}}{\beta_{S_{a,C}}}\right) \tag{1.42}$$

After the derivative of the normal CDF in Equation 1.42 is substituted into Equation 1.41, and the hazard term is replaced by the power-law approximation from Equation 1.3:

$$H_{LS} = \nu \cdot P[S_a \ge S_{a,C}] = \int \frac{k_0}{x \cdot \beta_{S_{a,C}}} \cdot \phi\left(\frac{\ln x - \ln \eta_{S_{a,C}}}{\beta_{S_{a,C}}}\right) \cdot x^{-k} \cdot dx \tag{1.43}$$

If we substitute the expression for the normal PDF in Equation 1.18 into the above equation:

$$H_{LS} = \nu \cdot P[S_a \ge S_{a,C}] = \int \frac{k_0}{\sqrt{2\pi} \cdot x \cdot \beta_{S_{a,C}}} \cdot \exp\frac{-1}{2}\left(\frac{\ln x - \ln \eta_{S_{a,C}}}{\beta_{S_{a,C}}}\right)^2 \cdot x^{-k} \cdot dx \tag{1.44}$$

Similar to the derivation in Section 1.4.7.3, we transform the integrand into a complete square term and take all the constant terms outside of the integrand:

$$H_{LS} = \nu \cdot P[S_a \ge S_{a,C}] =$$

$$k_0 \exp(\frac{1}{2}k^2 \cdot \beta_{S_{a,C}}{}^2) \cdot \exp(-k \cdot \eta_{S_{a,C}}) \int_0^\infty \frac{1}{x \cdot \beta_{S_{a,C}}} \cdot \frac{1}{\sqrt{2\pi}} \cdot \exp(-\frac{1}{2}\left(\frac{\ln x - \{k\beta_{S_{a,C}}{}^2 - \ln \eta_{S_{a,C}}\}}{\beta_{S_{a,C}}}\right)^2) \cdot dx \tag{1.45}$$

The term inside the integral is itself the derivative for a standard normal CDF:

$$H_{LS} = \nu \cdot P[S_a \geq S_{a,C}] =$$

$$k_0 \exp(\frac{1}{2}k^2 \cdot \beta_{S_{a,C}}{}^2) \cdot \exp(-k \cdot \eta_{S_{a,C}}) \int_0^\infty \frac{d}{dx} \{\Phi(\frac{\ln x - \ln \eta_{S_{a,C}} \cdot e^{k \cdot \beta_{S_{a,C}}{}^2}}{\beta_{S_{a,C}}})\} \cdot dx \qquad (1.46)$$

Noting that the integral is equal to unity, the limit state probability can be derived as:

$$H_{LS} = P[S_a \geq S_{a,C}] = k_0 \cdot \exp(\frac{1}{2}k^2 \cdot \beta_{S_{a,C}}{}^2) \cdot \exp(-k \cdot \eta_{S_{a,C}}) = k_0 \cdot \eta_{S_{a,C}}{}^{-k} \cdot \exp(\frac{1}{2}k^2 \cdot \beta_{S_{a,C}}{}^2) \qquad (1.47)$$

We can observe the power-law term outside the exponential is equal to the frequency of exceeding (i.e., hazard) a spectral acceleration equal to the median spectral acceleration capacity:

$$H_{LS} = P[S_a \geq S_{a,C}] = H_{S_a}(\eta_{S_{a,C}}) \cdot \exp(\frac{1}{2}k^2 \cdot \beta_{S_{a,C}}{}^2) \qquad (1.48)$$

We can argue that $H_{S_a}(\eta_{S_{a,C}})$ is a first-order approximation to the limit state frequency and the exponential term $\exp(k^2 \cdot \beta_{S_{a,C}}{}^2 / 2)$ is a magnifying factor that accounts for the sensitivity of the limit state probability to the randomness in the spectral acceleration capacity. If we compare the *IM*-based expression for the limit state frequency in Equation 1.48 to the displacement-based one in Equation 1.38, we can observe that the exponential term accounting for the dispersion in displacement-demand is missing. Also the slope parameter *b* that measures the gradient of the displacement-based demand with regard to spectral acceleration is absent. This is because the *IM*-based solution strategy, when applying TPT to derive the limit state frequency, does not employ the displacement-based demand as (one of the) an intermediate variable(s).

A numerical application of the *IM*-based closed-form expression for the limit state frequency in Equation 1.48 can be found in a paper by Cordova et al. (2000), where the closed-form solution is used for the seismic assessment of a composite frame, using spectral acceleration and a proposed *IM* that also carries spectral shape information as intensity measures.

1.4.8 Randomness and Uncertainty as the Sources of Uncertainty

In previous sections, a closed-form expression for the mean annual frequency of exceeding a limit state (here, the collapse limit state) was derived. We observed that the hazard value for the load intensity measure corresponding to median drift capacity (i.e., the MAF of exceeding the load intensity measure corresponding to median drift capacity) is a first-order approximation to the limit state frequency. This first-order approximation is multiplied by two coefficients accounting for the randomness in drift demand for a given spectral acceleration and the randomness in drift capacity itself.

Our objective here is to derive the limit state frequency when there is both randomness and uncertainty in the design variables such as spectral acceleration hazard, drift demand given spectral acceleration, and drift capacity. Our derivations are going to be based on the assumption that, to a first approximation, we can represent all the epistemic uncertainty in variable X by the uncertainty in its median. The model becomes:

$$X = \hat{\eta}_X \cdot \varepsilon_\eta \cdot \varepsilon_X \tag{1.49}$$

where $\hat{\eta}_X$ is the current point estimate of the median of X, the unit-median random variable ε_η represents the epistemic uncertainty in the estimation of the median of X, and the unit-median random variable ε_X represents the aleatory randomness of X. We are also going to assume that the deviation from median, ε_η, can be properly modeled by a lognormal distribution. In general, of course, the epistemic uncertainty in β_X should also be taken into account. Also, the shape of the distribution of X may not be properly described by a lognormal distribution.

As in the previous section, we start by deriving the hazard values for the load intensity variable, spectral acceleration of the "first" structural mode. We use some probabilistic tools (e.g., TPT as explained previously) to derive the hazard values for the limit state variable, maximum interstory drift, and then complete the derivation by obtaining the limit state probability P_{LS}. Whenever possible the results obtained in the previous part are used and generalized to the case where there is both randomness and uncertainty in the design variables.

1.4.8.1 Spectral acceleration hazard

The concept of hazard curves for the load intensity measure was introduced in the previous section. Our focus was on the spectral acceleration hazard curves which are normally provided by seismologists for a given site condition and its location with respect to a fault. The hazard curve estimation involves many scientific assumptions (Kramer 1996). In other words there is uncertainty in the estimation of a hazard curve. That is why spectral acceleration hazard curves are normally provided as mean and 84th percentile hazard curves (Fig. 1.9). Here we are going to take into account the uncertainty in the estimation of the spectral acceleration hazard.

In the previous sections, we found it advantageous to approximate the hazard curve by a power-law relationship as proposed by Kennedy and Short (1994) and Luco and Cornell (1998):

$$H_{S_a}(s_a) = k_0 \cdot x^{-k}$$

where k_0 and k are parameters defining the shape of the hazard curve. We are going to let an equation of the same form as the one above represent the median estimate of the uncertain hazard curve:

$$\hat{H}_{S_a}(x) = k_0 \cdot x^{-k} \tag{1.50}$$

Further, we introduce the random variable ε_{UH} that represents the uncertainty in the spectral acceleration hazard, so that we have:

$$H_{S_a}(x) = \hat{H}_{S_a}(x) \cdot \varepsilon_{UH} \tag{1.51}$$

Here we have assumed that ε_{UH} is a lognormal random variable whose statistical parameters have the following characteristics:

$$\begin{aligned} &median(\varepsilon_{UH}) = \eta_{\varepsilon_{UH}} = e^{mean(\ln(\varepsilon))} = 1 \\ &\sigma_{\ln(\varepsilon_{UH})} = \beta_{UH} \end{aligned} \tag{1.52}$$

where β_{UH} reflects the degree of uncertainty in the PSHA estimation. We recognize the spectral acceleration hazard itself as an uncertain (random) variable, $\tilde{H}_{S_a}(x)$, which can be represented as the median ("best") estimate times this uncertain deviation, $\tilde{\varepsilon}_{UH}$:

$$\tilde{H}_{S_a}(x) = \hat{H}_{S_a}(x) \cdot \tilde{\varepsilon}_{UH} \tag{1.53}$$

Note the use of a tilde to denote a random variable when clarity is needed. Considering our assumption about $\tilde{\varepsilon}_{UH}$ being lognormal, we can observe from the above equation that the hazard value for any value of s_a can also be treated as a lognormal random variable (i.e., instead of having a single deterministic value assigned to it, it has a probability distribution). The spectral acceleration hazard can be written as:

$$\tilde{H}_{S_a}(x) = \hat{H}_{S_a}(x) \cdot \varepsilon_{UH} = k_0 \cdot x^{-k} \cdot \varepsilon_{UH} \tag{1.54}$$

where $\widetilde{H}_{S_a}(x)$ is a lognormal random variable with its median equal to $\widehat{H}_{S_a}(x)$ from Equation 1.50 and its dispersion measure (i.e., the standard deviation of the natural logarithm or fractional standard deviation) equal to β_{UH}. The mean hazard curve can be written as:

$$\overline{H}_{S_a}(x) = \widehat{H}_{S_a}(x) \cdot mean(\varepsilon_{UH}) = \widehat{H}_{S_a}(x) \cdot e^{\frac{1}{2}\beta_{UH}^2} \qquad (1.55)$$

This equation is based on a property of the lognormal variables, where the expected value of a lognormal variable is equal to its median times the exponential of half of the squared standard deviation (Appendix A). Figure 1.9 shows the 16th percentile, median, mean, and 84th percentile hazard curves for a California site that corresponds to a period of 1.8 seconds and damping ratio of 5%. Note that the median curve in the figure is the same hazard curve used in the previous section. The 84th percentile is given by: $\hat{H}_{S_a}(x) \cdot e^{\beta_{UH}}$.

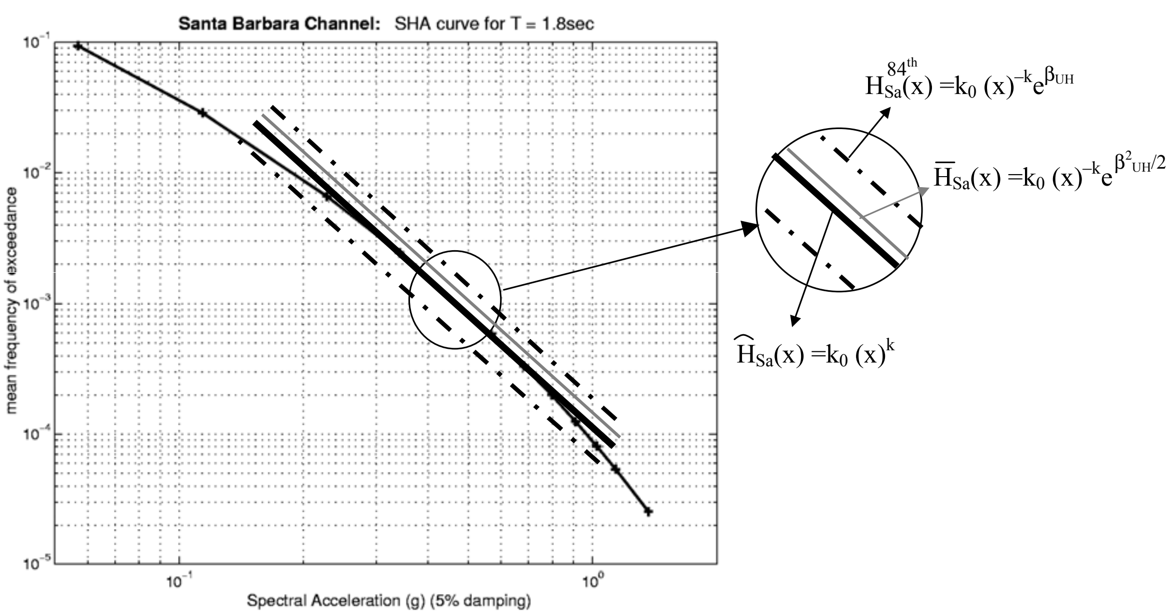

Fig. 1.9 16th, 50th (median), and 84th percentile spectral acceleration hazard corresponding to a damping ratio equal to 5% and a structural fundamental period of 1.8 seconds

Figure 1.10 shows the basic components of drift hazard estimation when there is uncertainty (due to limited knowledge and data) in the estimation of the spectral acceleration hazard, $\widetilde{H}_{S_a}(s_a)$. The probability density for uncertainty in hazard is plotted with solid black lines. The two hazard curves on the graph correspond to the median estimate of hazard, $\hat{H}_{S_a}(s_a)$, and hazard curve for a given value of deviation, ε_{UH}, in the estimation of hazard curve, $\hat{H}_{S_a}(s_a)\cdot\varepsilon_{UH}$, respectively.

1.4.8.2 Mean annual frequency of exceeding a drift demand value: drift hazard

Recall from the previous section that the drift demand variable (given a specified s_a level) was introduced as the median demand value times a random variable ε representing the random variation (e.g., record-to-record) around the median value. We assumed that ε has a lognormal distribution:

$$D = \eta_D(x)\cdot\varepsilon \tag{1.56}$$

Randomness is assumed to be the only source of variability in the above expression. In general, the median drift demand is also an uncertain quantity. The uncertainty in the median drift demand is caused by the limited knowledge and data about modeling and analysis of the structural system especially in the highly nonlinear range and/or exact numerical values of the parameters of the structural model.

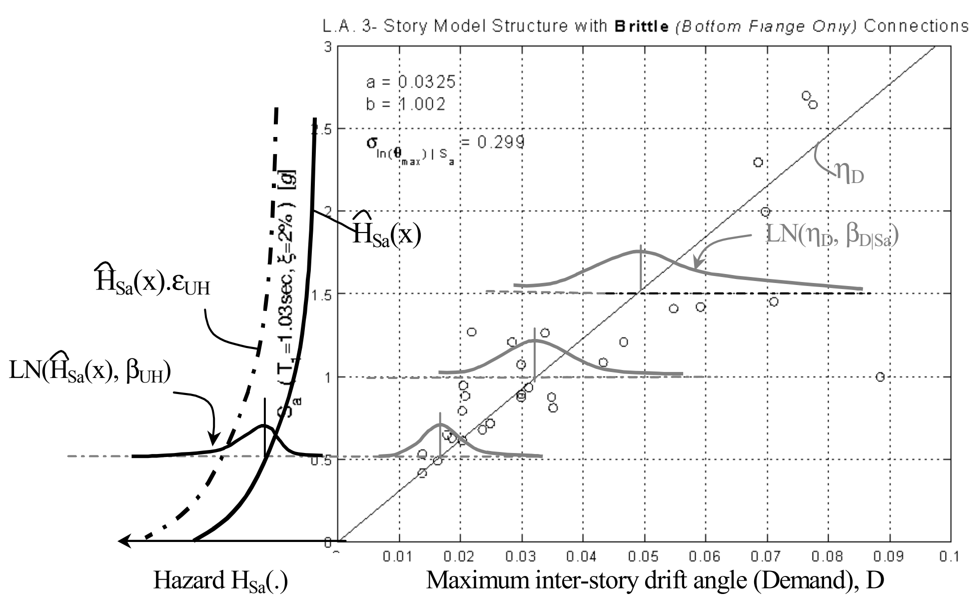

Fig. 1.10 Basic components for the evaluation of drift hazard taking into account the uncertainty in the estimation of spectral acceleration hazard, $\tilde{H}_{S_a}(x)$

The uncertainty is also caused by using a finite number of nonlinear analyses to estimate the median value. The scatter of the displacement-based response in Figure 1.2 indicating record-to-record variability implies that the estimate of the median, $\hat{\eta}_D(x)$, can depend on the particular sample of records used and its size. In order to distinguish this type of uncertainty from the one considered in the previous section, we refer to it as *epistemic* uncertainty. The median interstory drift can be expressed as the product of its median estimate, $\hat{\eta}_D(x)$ and a random variable ε_{UD} (*UD* stands for the uncertainty in evaluation of *D*) representing the uncertainty involved in the evaluation of $\eta_D(x)$:

$$\eta_D(x) = \hat{\eta}_D(x) \cdot \varepsilon_{UD} \tag{1.57}$$

Replacing $\eta_D(x)$ in Equation 1.56 with its representation in Equation 1.57, the drift demand can be written as:

$$D = \hat{\eta}_D(x) \cdot \varepsilon_{UD} \cdot \varepsilon \tag{1.58}$$

In order to be consistent with the notation, ε_{UD}, we now subscript ε with *RD*, standing for the randomness (aleatory uncertainty) in drift demand evaluation. Finally the drift demand is represented as:

$$D = \eta_D(x) \cdot \varepsilon_{UD} = \hat{\eta}_D(x) \cdot \varepsilon_{UD} \cdot \varepsilon_{RD} \tag{1.59}$$

where ε_{RD} and ε_{UD} are assumed to be independent and to have lognormal distributions with the following characteristics:

$$\begin{aligned} &\eta_{\varepsilon_{RD}} = \eta_{\varepsilon_{UD}} = e^{mean(\ln(\varepsilon))} = 1 \\ &\sigma_{\ln(\varepsilon_{RD})} = \beta_{RD} \\ &\sigma_{\ln(\varepsilon_{UD})} = \beta_{UD} \end{aligned} \tag{1.60}$$

Our objective in this section is to derive the probability that the drift demand *D* exceeds a specific value *d*. In order to minimize the calculation efforts, we will make use of the drift demand hazard that was derived in the previous section assuming that there was no epistemic uncertainty. The drift hazard or the mean annual frequency that the drift demand exceeds a specific value was derived from Equation 1.25 as:

$$H_D(d) = \nu \cdot P[D > d] = k_0 \cdot \left(s_a^d\right)^{-k} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RD}} = H_{S_a}(s_a^d) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{RD}}$$

The spectral acceleration hazard for a given value of deviation (from the median) in its estimation, ε_{UH}, can be found from Equation 1.54 as:

$$H_{S_a|\varepsilon_{UH}}(x) = \hat{H}_{S_a}(x) \cdot \varepsilon_{UH} = k_0 \cdot x^{-k} \cdot \varepsilon_{UH}$$

Replacing the above value for spectral acceleration hazard in Equation 1.25, we obtain the drift demand hazard for a given value of deviation in the estimation of the spectral acceleration hazard, ε_{UH} :

$$H_{D|\varepsilon_{UH}}(d) = \nu \cdot P[D > d] = \widetilde{H}_{S_a|\varepsilon_{UH}}(s_a^d) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{RD}} = \hat{H}_{S_a}(s_a^d) \cdot \varepsilon_{UH} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{RD}}$$

In the next step, we derive the drift hazard function for a given value of deviation in the estimation of spectral acceleration hazard, ε_{UH}, and a given value of deviation in the estimation of the median drift demand, ε_{UD} :

$$H_{D|\varepsilon_{UH},\varepsilon_{UD}}(d) = \nu \cdot P[D > d \mid \varepsilon_{UD}, \varepsilon_{UH}] = \hat{H}_{S_a|\varepsilon_{UD}}(s_a^d) \cdot \varepsilon_{UH} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{RD}} \tag{1.61}$$

The term $\hat{H}_{S_a|\varepsilon_{UD}}(s_a^d)$ can be interpreted as the median spectral acceleration hazard for the spectral acceleration that corresponds to drift *d* for a *given value* of deviation ε_{UD} in the median drift estimation:

$$\hat{H}_{S_a|\varepsilon_{UD}}(s_a^d) = \hat{H}_{S_a}(s_a^{d|\varepsilon_{UD}}) \tag{1.62}$$

In order to be able to calculate the above value, we need to find $s_a^{d|\varepsilon_{UD}}$ or the spectral acceleration corresponding to drift *d* for a given value of deviation ε_{UD} in drift estimation. The median drift demand for a given value of deviation ε_{UD} can be found from Equation 1.57 as:

$$\eta_D(x) = \hat{\eta}_D(x) \cdot \varepsilon_{UD}$$

At this stage we assume that $\hat{\eta}_D(x)$ has the same functional form as the one $\eta_D(x)$ had in the previous part, stated in Equation 1.4, namely:

$$\hat{\eta}_D(x) = a \cdot x^b \tag{1.63}$$

Substituting the value for $\hat{\eta}_D(x)$ from Equation 1.63 in Equation 1.57, the median drift demand can be derived as:

$$\eta_D(x) = a \cdot x^b \cdot \varepsilon_{UD} \tag{1.64}$$

$s_a^{d|\varepsilon_{UD}}$ or the spectral acceleration corresponding to drift *d* for a given value of deviation ε_{UD} in drift estimation can be calculated by setting $\eta_D(x)$ in Equation 1.64 equal to *d* and solving for $s_a^{d|\varepsilon_{UD}}$. Hence, $s_a^{d|\varepsilon_{UD}}$ can be defined as[5]:

$$s_a^{d|\varepsilon_{UD}} = \left(\frac{d}{a \cdot \varepsilon_{UD}}\right)^{\frac{1}{b}} = \left(\frac{d/\varepsilon_{UD}}{a}\right)^{\frac{1}{b}} = s_a^{d/\varepsilon_{UD}} \tag{1.65}$$

A graphic interpretation of $s_a^{d|\varepsilon_{UD}}$ can be seen in Figure 1.11. As can be observed from the figure, $s_a^{d|\varepsilon_{UD}}$ is the S_a corresponding to drift value d from the curve $a \cdot x^b \cdot \varepsilon_{UD}$:

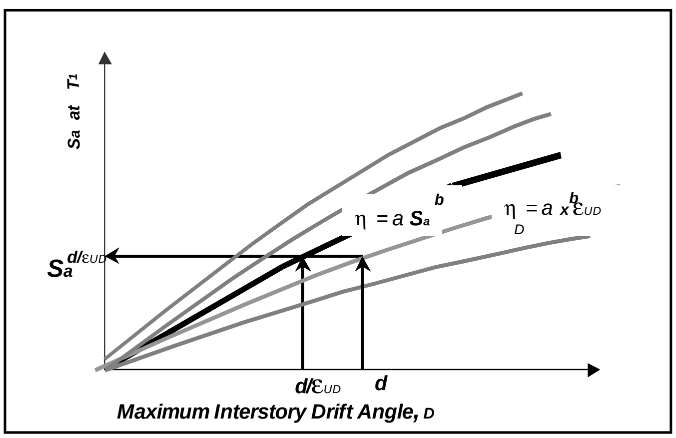

Fig. 1.11 Spectral acceleration corresponding to the interstory drift ratio value *d* for a given value of deviation in the estimation of median drift, ε_{UD}

[5] In fact $s_a^{d|\varepsilon_{UD}}$ is nothing but the spectral acceleration corresponding to drift demand *d* which is being calculated from the curve $a \cdot x^b \cdot \varepsilon_{UD}$ instead of $a \cdot x^b$.

Replacing the value for $s_a^{d|\varepsilon_{UD}}$ from Equation 1.65 in Equation 1.61:

$$H_{D|\varepsilon_{UH},\varepsilon_{UD}}(d) = \hat{H}_{S_a|\varepsilon_{UD}}(s_a^d) \cdot \varepsilon_{UH} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{RD}} = \hat{H}_{S_a}(s_a^{d/\varepsilon_{UD}}) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{RD}} \cdot \varepsilon_{UH} \quad (1.66)$$

where $\hat{H}(s_a^{d/\varepsilon_{UH}})$ can be calculated from Equations 1.65 and 1.50 as follows:

$$\hat{H}_{S_a}(s_a^{d/\varepsilon_{UD}}) = k_0 \cdot \left(\frac{d}{a \cdot \varepsilon_{UD}}\right)^{\frac{-k}{b}} \quad (1.67)$$

Substituting the value of $\hat{H}(s_a^{d/\varepsilon_{UH}})$ from Equation 1.67 in Equation 1.66 results in:

$$H_{D|\varepsilon_{UH},\varepsilon_{UD}}(d) = \hat{H}_{S_a}(s_a^{d/\varepsilon_{UD}}) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}} \cdot \varepsilon_{UH} = k_0 \cdot \left(\frac{d}{a \cdot \varepsilon_{UD}}\right)^{\frac{-k}{b}} \cdot \varepsilon_{UH} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}}$$

$$= k_0 \cdot \left(\frac{d}{a}\right)^{\frac{-k}{b}} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}} \cdot \varepsilon_{UH} \cdot \varepsilon_{UD}{}^{\frac{k}{b}} \quad (1.68)$$

In short, we have an expression for the drift hazard conditioned on given values of deviations in the estimation of spectral acceleration hazard, ε_{UH}, and drift demand, ε_{UD}, due to epistemic uncertainty. Recalling from the last section, the spectral acceleration hazard could be interpreted as an uncertain variable,

$$\widetilde{H}_{S_a}(s_a^d) = k_0 \cdot \left(\frac{d}{a}\right)^{\frac{-k}{b}} \cdot \varepsilon_{UH}$$

In the same manner, the drift hazard can also be interpreted as an uncertain (random) variable $\widetilde{H}_D(d)$, which is a function of the uncertain spectral acceleration hazard, $\widetilde{H}_{S_a}(s_a)$, and the uncertain variable representing the epistemic uncertainty in drift prediction, ε_{UD}:

$$\widetilde{H}_D(d) = \nu \cdot P[D > d] = k_0 \cdot \left(\frac{d}{a}\right)^{\frac{-k}{b}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RD}} \cdot \widetilde{\varepsilon}_{UH} \cdot \widetilde{\varepsilon}^{\frac{k}{b}}{}_{UD} = \widetilde{H}_{S_a}(s_a^d) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RD}} \cdot \widetilde{\varepsilon}^{\frac{k}{b}}{}_{UD} \quad (1.69)$$

The product of independent lognormal random variables raised to powers, such as k/b, is again a lognormal random variable (Benjamin and Cornell 1970). Therefore, we can conclude that the drift hazard can also be represented by a lognormal random variable whose distribution parameters can be calculated based on the information about the distribution characteristics of ε_{UH} and ε_{UD} from Equations 1.52 and 1.60:

$$median(\tilde{H}_D(d)) = \hat{H}_D(d) = median(\hat{H}_{S_a}(s_a^d) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{RD}} \varepsilon_{UH} \cdot \varepsilon^{\frac{k}{b}}{}_{UD}) = \hat{H}_{S_a}(s_a^d) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{RD}}$$

$$\beta_{UH_D} = \sqrt{\beta_{UH}{}^2 + \frac{k^2}{b^2}\beta_{UD}{}^2}$$

Therefore, the drift hazard $\tilde{H}_D(d)$ is an uncertain quantity with median:

$$\hat{H}_D(d) = \hat{H}_{S_a}(s_a^d) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{RD}} \tag{1.70}$$

and fractional standard deviation:

$$\beta_{UH_D} = \sqrt{\beta_{UH}{}^2 + \frac{k^2}{b^2}\beta_{UD}{}^2} \tag{1.71}$$

Also, the mean drift hazard, $\overline{H}_D(d)$, is equal to:

$$\overline{H}_D(d) = \hat{H}_D(d) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{UH_D}} = \hat{H}_{S_a}(s_a^d) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{RD}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{UH_D}} \tag{1.72}$$

After substituting β_{UH_D} from Equation 1.69:

$$\overline{H}_D(d) = \hat{H}_{S_a}(s_a^d) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{RD}} \cdot e^{\frac{1}{2}\beta^2_{UH}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{UD}} = \overline{H}_{S_a}(s_a^d) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{RD}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{UD}} \tag{1.73}$$

Note that the uncertainty in the hazard curve can be dealt with simply by using the mean estimate of the hazard curve. Similar to Figure 1.3, Figure 1.12 illustrates a graphical presentation of basic components for the derivation of drift hazard, but in this case there is uncertainty both in the estimation of median drift curve $\tilde{\eta}_D$ and spectral acceleration hazard, $\tilde{H}_{S_a}(s_a)$. Figure 1.12 is a plot of the median estimate, $\hat{\eta}_D$, of the median drift curve (that is treated as an uncertain quantity itself), the probability density reflecting the uncertainty in $\hat{\eta}_D$ about that estimate, with a fractional standard deviation equal to, β_{UD}, a realization of median drift curve $\hat{\eta}_D \cdot \varepsilon_{UD}$ for a given value of deviation ε_{UD}, and the fractional standard deviation β_{RD} due to record to record variability in the results of dynamic analyses around it. The median estimate for spectral acceleration hazard $\hat{H}_{S_a}(s_a)$ and the spectral acceleration hazard for a given value of deviation in hazard estimation, $\hat{H}_{S_a}(s_a)\cdot\varepsilon_{UH}$, are shown in the same manner as in Figure 1.10.

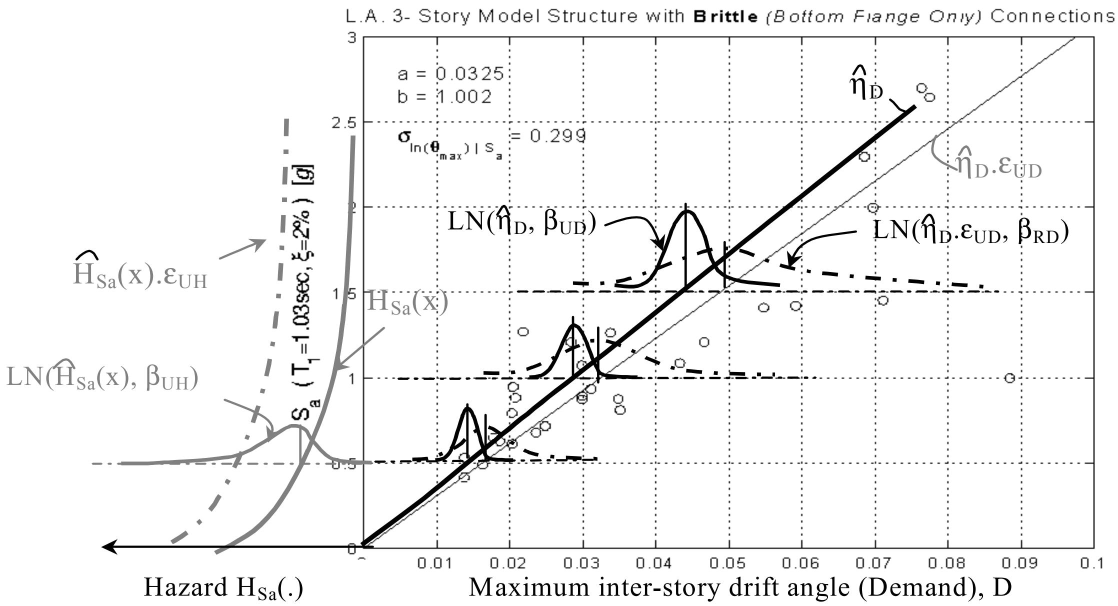

Fig. 1.12 Basic components for the derivation of drift hazard taking into account the epistemic uncertainty in the estimation of spectral acceleration hazard $\tilde{H}_{S_a}(s_a)$ and median drift $\tilde{\eta}_D$

1.4.8.2.1 Numerical example

Returning to our numerical example, we are going to calculate the mean drift hazard in the case where there is epistemic uncertainty in the evaluation of the drift demand. We have the maximum interstory drift values resulting from 30 different nonlinear time history runs plotted in Figure 1.2. Fitting a line in log-log space to the data points gives us the following information about the median interstory drift and the dispersion around it:

$$\eta_D(x) = 0.0325 s_a^{\ 1.002}$$
$$\beta_{RD} = \beta_{D|S_a} = 0.299 \approx 0.3$$

But strictly, this is just the median estimate, $\hat{\eta}_D(x)$, of the median drift curve. The error in the estimation of the median interstory drift can be due to modeling errors and other approximations involved in the analysis procedure. Here we limit the consideration to the statistical uncertainty in the median due to the finite sample size (*n_{sample}=30*). The statistical properties of the median interstory drift can be calculated as (see Rice 1995 for the statistical parameters for the mean estimate):

$$\hat{\eta}_D(x) = 0.0325 \cdot x^{1.002}$$
$$\beta_{UD} = \frac{\beta_{D|S_a}}{\sqrt{\mathrm{n}_{\mathrm{sample}}}} \approx \frac{0.3}{5.48} = 0.055$$

Analogous to the previous section, we would like to evaluate the mean estimate of the MAF that the maximum interstory drift angle exceeds a specific value, say 2%, $\overline{H}_D(0.02)$. If we substitute 0.02 for *d* in Equation 1.73:

$$\overline{H}_D(0.02) = \hat{H}_D(0.02) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{UHD}} = \overline{H}_{S_a}(s_a^{0.02}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RD}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{UD}}$$

Recall that s_a^d is equal to $s_a^d = \left(\frac{d}{a}\right)^{\frac{1}{b}}$ per definition:

$$s_a^{0.02} = \left(\frac{0.02}{0.0325}\right)^1 = 0.615 \;\; [\mathrm{g}]$$

The mean estimate for the spectral acceleration hazard can be calculated from:

$$\overline{H}_{S_a}(s_a) = \hat{H}_{S_a}(s_a) \cdot e^{\frac{1}{2}\beta^2{}_{UH}}$$

Here, it is assumed that β_{UH} is equal 0.50. With this assumption we can look up the value for $\hat{H}_{S_a}(0.615)$ from the spectral acceleration hazard curve in Figure 1.5, which is equal to *0.007*. Hence,

$$\overline{H}_{S_a}(0.615) = \hat{H}_{S_a}(0.615) \cdot e^{\frac{1}{2}(0.5)^2} = 0.007 \times 1.13 = 0.079$$

$\overline{H}_D(0.02)$ can be calculated as:

$$\overline{H}_D(0.02) = \nu \cdot P[D > 0.02] = 0.0079 \cdot e^{\frac{1}{2}x\frac{3^2}{1^2}x0.3^2} \cdot e^{\frac{1}{2}x\frac{3^2}{1^2}x0.055^2} = 0.0079 \times 1.5 \times 1.014 = 0.012$$

Note that in the previous section, where (it was assumed that) there was no epistemic uncertainty in the estimation of interstory drift demand, $H_D(0.02)$ was equal to 0.0105. The net uncertainty here in the estimation of $H_D(0.02)$ can be represented by the fractional standard deviation of the drift hazard:

$$\beta_{UH_D} = \sqrt{(0.5)^2 + \frac{3^2}{1^2} 0.055} = 0.526$$

It can be observed that the primary contribution to β_{UH_D} comes from the epistemic uncertainty in probabilistic seismic hazard estimation (PSHA). However, this can change if β_{UD} increases to as large as 0.15. If nonlinear dynamic modeling errors are considered, this value is likely to be considerably larger than 0.15.

1.4.8.3 Annual frequency of exceeding a limit state

Next, we will derive a closed-form expression for the frequency that the drift demand exceeds the drift capacity or the limit state frequency taking into account the epistemic uncertainty. In the last section a closed-form expression was derived for the mean annual frequency that the drift demand exceeds a specified value of drift (also known as drift hazard), in which epistemic uncertainty in the estimation of spectral acceleration hazard and drift demand were taken into account. Now, we are interested in calculating the probability that the drift demand exceeds drift capacity, which is an uncertain quantity itself. In the previous section (1.4.7), the capacity was assumed to be an uncertain (random) variable due to aleatory uncertainty in its estimation, e.g., connection-to-connection variability in a to-be-built design and record-to-record variability.

The drift capacity variable was introduced above as a median capacity value times a random variable ε_C representing the deviations from the median value. We assumed that ε_C is represented by a lognormal distribution:

$$C = \eta_C \cdot \varepsilon_C \tag{1.74}$$

In general, median capacity can also be treated as an uncertain (random) variable. The epistemic uncertainty in the median capacity is caused by the limited knowledge and data about for example, untested connection designs or nonlinear structural modeling and/or structural analysis for global stability prediction. The median capacity variable can be expressed as the product of its median value, $\hat{\eta}_C$, and a random variable ε_{UC} (*UC* stands for the uncertainty in evaluation of capacity, C) representing the uncertainty involved in the estimation of η_C:

$$\eta_C = \hat{\eta}_C \cdot \varepsilon_{UC} \tag{1.75}$$

Finally, we subscript ε with *RD,* standing for the randomness in drift demand evaluation:

$$C = \eta_C \cdot \varepsilon_{UC} = \hat{\eta}_C \cdot \varepsilon_{UC} \cdot \varepsilon_{RC} \tag{1.76}$$

ε_{RC} and ε_{RC} are assumed to be independent and to have lognormal distributions with the following characteristics:

$$\begin{aligned} &median(\varepsilon_{RC}) = median(\varepsilon_{UC}) = e^{mean(\ln(\varepsilon))} = 1 \\ &\sigma_{\ln(\varepsilon_{RC})} = \beta_{RC} \\ &\sigma_{\ln(\varepsilon_{UC})} = \beta_{UC} \end{aligned} \tag{1.77}$$

Our objective in this section is to derive the probability that drift demand, *D,* exceeds drift capacity, *C*, recognizing the uncertainty in spectral acceleration hazard, structural demand, and structural capacity. Based on the expression derived for the limit state frequency in the previous part, Equation 1.38:

$$H_{LS} = \nu \cdot P[D > C] = H_D(\eta_C) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta_{RC}^2} \tag{1.78}$$

We are going to use the above equation and combine it with the closed-form expression derived for the drift hazard (taking into account epistemic uncertainty) in order to derive the limit state frequency, H_{LS}. Assuming that capacity is a specific deterministic value, *c*, (i.e., there is neither randomness nor uncertainty associated with the evaluation of capacity), the drift hazard function for drift level *c* (for a given value of uncertainty in spectral acceleration hazard, ε_{UH}, and uncertainty in drift demand, ε_{UD}) can be derived based on the results of the previous section from Equation 1.61, substituting *c* for *d*:

$$H_{D|\varepsilon_{UH},\varepsilon_{UD}}(c) = P[D > c \mid \varepsilon_{UH}, \varepsilon_{UD}] = k_0 \cdot \left(\frac{c}{a}\right)^{\frac{-k}{b}} \cdot \varepsilon_{UH} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RD}} \cdot \varepsilon_{UD}^{\frac{k}{b}} \tag{1.79}$$

We are going to first find the limit state frequency conditioned on the uncertainties in spectral acceleration hazard, drift demand, and drift capacity. The median capacity for a given deviation, ε_{UC} , of the estimated median drift capacity from median drift capacity is written below based on Equation 1.75:

$$\eta_{C|\varepsilon_{UC}} = \hat{\eta}_C \cdot \varepsilon_{UC} \tag{1.80}$$

If we substitute the median capacity associated with this given deviation, $\eta_{C|\varepsilon_{UC}}$, from Equation 1.80 for c in Equation 1.79, the drift hazard conditioned on the uncertainties in spectral acceleration hazard, drift demand and drift capacity will be found as:

$$H_{D|\varepsilon_{UH},\varepsilon_{UD},\varepsilon_{UC}}(\eta_{C|\varepsilon_{UC}}) = k_0 \cdot \left(\frac{\eta_{C|\varepsilon_{UC}}}{a}\right)^{\frac{-k}{b}} \cdot \varepsilon_{UH} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta_{RD}^2} \cdot \varepsilon^{\frac{k}{b}}{}_{UD} \cdot \tag{1.81}$$

Substituting the conditional drift hazard term, calculated at $\eta_{C|\varepsilon_{UC}}$ into Equation 1.78, the limit state frequency for a given value of uncertainty in spectral acceleration hazard, drift demand and drift capacity can be found:

$$H_{LS|\varepsilon_{UH},\varepsilon_{UD},\varepsilon_{UC}} = \nu \cdot P[D > C \mid \varepsilon_{UH}, \varepsilon_{UD}, \varepsilon_{UC}] = H_{D|\varepsilon_{UH},\varepsilon_{UD},\varepsilon_{UC}}(\eta_{C|\varepsilon_{UC}}) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta_{RC}^2}$$

$$= k_0 \cdot \left(\frac{\hat{\eta}_C \cdot \varepsilon_{UC}}{a}\right)^{\frac{-k}{b}} \cdot \varepsilon_{UH} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RD}} \cdot \varepsilon^{\frac{k}{b}}{}_{UD} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta_{RC}^2}$$

$$\Rightarrow H_{LS|\varepsilon_{UH},\varepsilon_{UD},\varepsilon_{UC}} = k_0 \cdot \left(\frac{\hat{\eta}_C}{a}\right)^{\frac{-k}{b}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta_{RD}^2} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta_{RC}^2} \cdot \varepsilon_{UH} \cdot \varepsilon_{UD}^{\frac{k}{b}} \cdot \varepsilon_{UC}^{\frac{-k}{b}} \tag{1.82}$$

The above expression gives the limit state frequency conditioned on the uncertainty in the estimation of spectral acceleration hazard, drift demand, and drift capacity as an analytical function of $\varepsilon_{UH}, \varepsilon_{UD}$, and ε_{UC}, the random variables representing the above-mentioned uncertainties. Similar to what we did in the previous section for the derivation of drift hazard, here we can interpret the limit state frequency itself as an uncertain (random) variable, $\tilde{H}_{LS}$. Recall from Equation 1.54 that the term

$$k_0 \cdot \left(\frac{\hat{\eta}_C}{a}\right)^{\frac{-k}{b}} \cdot \varepsilon_{UH}$$

is equal to $\widetilde{H}_{S_a}(S_a^{\hat{\eta}_C})$. Hence, the limit state frequency can be introduced as an uncertain quantity that is a function of the spectral acceleration hazard, $\widetilde{H}_{S_a}(\hat{\eta}_C)$, the deviation in drift demand prediction, ε_{UD} and the deviation in drift capacity prediction, ε_{UC}:

$$\widetilde{H}_{LS} = \widetilde{H}_{S_A}(S_a^{\hat{\eta}_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RD}} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta\ {}_{RC}} \cdot \varepsilon^{\frac{k}{b}}{}_{UD} \cdot \varepsilon^{\frac{-k}{b}}{}_{UC} \tag{1.83}$$

It can be observed that the limit state frequency is also a lognormal random variable whose distribution parameters can be calculated based on the information about the distribution characteristics of ε_{UH}, ε_{UD}, and ε_{UC} from Equations 2.52, 2.60, and 2.77:

$$median(\widetilde{H}_{LS}) = \hat{H}_{LS} = median(\widetilde{H}_{S_A}(s_a^{\hat{\eta}_C}) \cdot \varepsilon^{\frac{k}{b}}{}_{UD} \cdot \varepsilon^{\frac{-k}{b}}{}_{UC} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RD}} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta_C^2}) = \hat{H}_{S_a}(\hat{\eta}_C) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RD}} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta_C^2}$$

$$\beta_{\widetilde{H}_{LS}} = \sqrt{\beta_{UH}{}^2 + \frac{k^2}{b^2}\beta_{UD}{}^2 + \frac{k^2}{b^2}\beta_{UC}{}^2}$$

Therefore, the uncertain limit state frequency $\widetilde{H}_{LS}$ is an uncertain quantity with median,

$$\hat{H}_{LS} = \hat{H}_{S_a}(s_a^{\hat{\eta}_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RD}} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta_C^2} \tag{1.84}$$

and, fractional standard deviation equal to:

$$\beta_{\widetilde{H}_{LS}} = \sqrt{\beta_{UH}{}^2 + \frac{k^2}{b^2}\beta_{UD}{}^2 + \frac{k^2}{b^2}\beta_{UC}{}^2} \tag{1.85}$$

Also, the mean estimate of limit state frequency denoted by, $\overline{H}_{LS}$, is equal to:

$$\begin{aligned}\overline{H}_{LS} &= \hat{H}_{LS} \cdot e^{\frac{1}{2}\beta^2{}_{H_{LS}}} = \hat{H}_{LS} \cdot e^{\frac{1}{2}\beta^2{}_{UH}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{UD}} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\beta^2{}_{UC}} \\ &= \overline{H}_{S_a}(s_a^{\hat{\eta}_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}(\beta^2{}_{RD}+\beta^2{}_{UD})} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot(\beta^2{}_{RC}+\beta^2{}_{UC})}\end{aligned} \tag{1.86}$$

Note that in this final form, the mean estimate of $\widetilde{H}_{LS}$ looks like H_{LS} without uncertainty (Equation 1.38) but now based on the mean estimate of $\widetilde{H}_{S_a}(s_a)$, and with increased β^2 in capacity and demand (given s_a) exponential terms. Figure 1.13 illustrates a graphical presentation of basic components for the derivation of the limit state frequency; where there is (epistemic) uncertainty in the estimation of spectral acceleration hazard, $\tilde{H}_{S_a}(s_a)$, median drift demand curve, $\tilde{\eta}_D$, and median drift capacity, $\tilde{\eta}_C$. In Figure 1.13, we plot together the median estimate, $\hat{\eta}_C$, of the uncertain drift capacity, the probability density reflecting the uncertainty in η_C about that estimate, with dispersion, β_{UC}, median drift capacity for a given deviation, ε_{UC}, in the estimation of median drift capacity $\hat{\eta}_C \cdot \varepsilon_{UC}$, and the probability density reflecting the randomness type of uncertainty (e.g., specimen-to-specimen variability in the estimation of capacity) in capacity C about the median drift capacity $\hat{\eta}_C \cdot \varepsilon_{UC}$, with dispersion, β_{RC}.

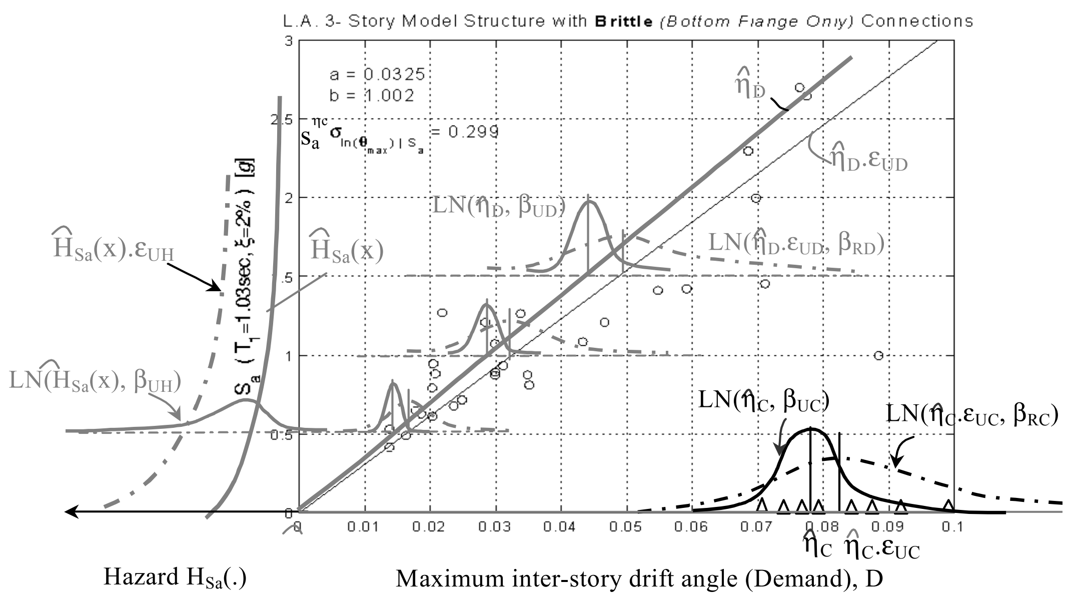

Fig. 1.13 Basic components for the derivation of the limit state frequency when there is uncertainty in the estimation of the spectral acceleration hazard, $\tilde{H}_{S_a}(s_a)$, median drift demand, η_D, and median drift capacity, η_C.

1.4.8.3.1 Numerical example

For our three-story frame numerical example, we would like to calculate the mean estimate of the limit state frequency when there is uncertainty both in the estimation of median drift demand and median drift capacity. Recall from the last section that the median drift and the dispersion of drift for a given level of spectral acceleration was estimated (by fitting a line in the log-log space to the data points obtained by performing 30 different nonlinear time history analyses) as:

$$\eta_D(x) = 0.0325 \cdot s_a^{\,1.002}$$
$$\beta_{RD} = \beta_{D|S_a} = 0.299 \approx 0.3$$

We have also estimated the statistical properties of the uncertain median drift demand as:

$$\hat{\eta}_D(x) = \eta_D(x) = 0.0325 \cdot x^{1.002}$$
$$\beta_{UD} \cong \frac{\beta_{D|S_a}}{\sqrt{\mathrm{n}_{\mathrm{sample}}}} = \frac{0.3}{5.48} = 0.055$$

The median and dispersion for drift capacity were given before:

$$\eta_C = 0.07$$
$$\beta_{RC} = \sigma_{\ln(C)} = \beta_C = 0.20$$

Note that the dispersion parameter β_C represents the aleatory (randomness) type of uncertainty in drift capacity. We have estimated the statistical properties of the (now uncertain) median drift capacity as follows (for an assumed sample of size 4 as the number of tests upon which the estimate of the median connection capacity is based):

$$\hat{\eta}_C \cong 0.07$$
$$\beta_{UC} \cong \frac{\beta_C}{\sqrt{4}} = \frac{0.2}{2} = 0.1$$

Equation 1.86 gives the mean limit state frequency as:

$$\overline{H}_{LS} = \hat{H}_{LS} \cdot e^{\frac{1}{2}\beta^2_{UH}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{UD}} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\beta^2_{UC}} = \overline{H}_{S_a}(s_a^{\hat{\eta}_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}(\beta^2_{RD}+\beta^2_{UD})} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot(\beta^2_{RC}+\beta^2_{UC})}$$

We have defined s_a^d equal to $\left(\frac{d}{a}\right)^{\frac{1}{b}}$; therefore $s_a^{\hat{\eta}_C}$ can be calculated as:

$$\left(\frac{\hat{\eta}_C}{a}\right)^{\frac{1}{b}} : s_a^{\hat{\eta}_C} = s_a^{0.07} = \left(\frac{0.07}{0.0325}\right)^1 = 2.15 \ \ [\mathrm{g}]$$

Our next step is to calculate the median spectral acceleration hazard at a spectral acceleration corresponding to median drift capacity:

$$\overline{H}_{S_a}(s_a^{\hat{\eta}_C}) = \hat{H}_{S_a}(s_a^{\hat{\eta}_C}) \cdot e^{\frac{1}{2}\beta^2_{UH}} = k_0 \cdot \left(s_a^{\hat{\eta}_C}\right)^{-k} \cdot e^{\frac{1}{2}\beta^2_{UH}} = 0.00124 \times 2.15^{-3.0} \times e^{\frac{1}{2}(0.5^2)}$$

$$= 0.000124 \times 1.13 = 0.00014$$

assuming that $\beta_{UH} = 0.5$, the same as in the previous sections. We can also look up the value for $\hat{H}_{S_a}$ from the spectral acceleration hazard curve, which is equal to 0.00012 (Fig. 1.14 below). from Figure 1.5,. Hence, the mean estimate of the limit state frequency $\overline{H}_{LS}$ can be derived as:

$$\overline{H}_{LS} = 0.00014 \cdot \mathrm{e}^{\frac{1}{2}(\frac{3^2}{1^2})(0.3^2)} \cdot \mathrm{e}^{\frac{1}{2}(\frac{3^2}{1^2})(0.2^2)} \cdot \mathrm{e}^{\frac{1}{2}(\frac{3^2}{1^2})(0.1^2)} \cdot \mathrm{e}^{\frac{1}{2}(\frac{3^2}{1^2})(0.055^2)}$$
$$= 0.00014 \times 1.5 \times 1.2 \times 1.014 \times 1.05 = 2.68 \times 10^{-4}$$

Note that in Section 1.4.7 we calculated the limit state frequency when the epistemic uncertainty in the estimation of median demand and capacity was not taken into account. The limit state frequency in that case was equal to 2.2×10^{-4}, whereas the mean estimate of the limit state frequency calculated in the presence of uncertainty in the estimation of hazard, demand and capacity is 2.68×10^{-4}

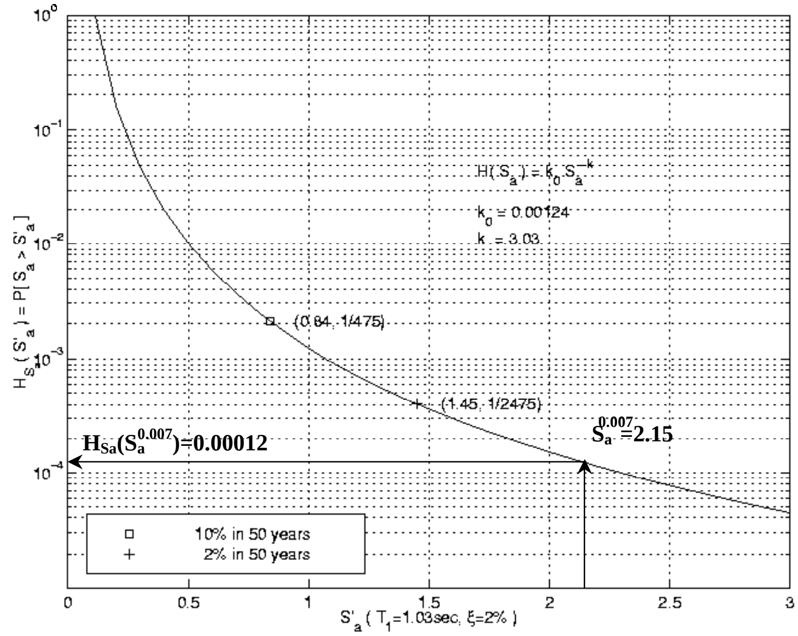

Fig. 1.14 Median estimate for the spectral acceleration hazard curve. The hazard value for a spectral acceleration equal to 2.15 is shown.

1.4.8.4 Annual frequency of exceeding a limit state: the IM-based approach

The MAF of exceeding a limit state following the *IM*-based solution strategy was derived in Section 1.4.7.5, considering only the aleatory uncertainty (due to record-to-record variability) in demand and capacity. This section follows the same approach in order to derive the limit state frequency considering also the epistemic uncertainty. Similar to the previous sections, it is assumed that the median capacity variable can be expressed as the product of its median value, $\hat{\eta}_{S_{a,C}}$ and a random variable $\varepsilon_{US_{a,C}}$ representing the uncertainty in the prediction of $\eta_{S_{a,C}}$:

$$\eta_{S_{a,C}} = \hat{\eta}_{S_{a,C}} \cdot \varepsilon_{US_{a,C}} \tag{1.87}$$

Similar to the previous sections, we can represent the spectral acceleration capacity as:

$$S_{a,C} = \eta_{S_{a,C}} \cdot \varepsilon_{RS_{a,C}} = \hat{\eta}_{S_{a,C}} \cdot \varepsilon_{RS_{a,C}} \cdot \varepsilon_{US_{a,C}} \tag{1.88}$$

where $\varepsilon_{RS_{a,C}}$ and $\varepsilon_{US_{a,C}}$ are assumed to be independent and to have lognormal distributions with the following characteristics:

$$\begin{aligned} &median(\varepsilon_{RS_{a,C}}) = median(\varepsilon_{US_{a,C}}) = 1 \\ &\sigma_{\ln(\varepsilon_{RS_{a,C}})} = \beta_{RS_{a,C}} \\ &\sigma_{\ln(\varepsilon_{US_{a,C}})} = \beta_{US_{a,C}} \end{aligned} \tag{1.89}$$

where $\beta_{RS_{a,C}}$ and $\beta_{US_{a,C}}$ are fractional standard deviations representing the randomness and uncertainty in the spectral acceleration capacity, respectively. It can be shown (the procedure is similar to the one described for the displacement-based approach in detail) that the limit state frequency $\tilde{H}_{LS}$ is an uncertain quantity with its median equal to,

$$\hat{H}_{LS} = \hat{H}_{S_a}(\hat{\eta}_{S_{a,C}}) \cdot \exp(\frac{1}{2} k^2 \beta_{RS_{a,C}}{}^2) \tag{1.90}$$

and the fractional standard deviation equal to:

$$\beta_{\tilde{H}_{LS}} = \sqrt{\beta_{UH}{}^2 + k^2 \cdot \beta_{US_{a,C}}{}^2} \tag{1.91}$$

and the mean limit state frequency, $\overline{H}_{LS}$, is equal to:

$$\overline{H}_{LS} = \hat{H}_{LS} \cdot e^{\frac{1}{2}\beta^2{}_{P_{LS}}} = \hat{H}_{LS} \cdot e^{\frac{1}{2}\beta^2{}_{UH}} \cdot e^{\frac{1}{2}k^2\beta^2{}_{US_{a,C}}} = \overline{H}_{S_a}(s_a^{\hat{\eta}_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}(\beta^2{}_{RS_{a,C}} + \beta^2{}_{US_{a,C}})} \tag{1.92}$$

As with the limit state frequency derived following the displacement-based case, the mean estimate of $\tilde{H}_{LS}$ looks like H_{LS} without uncertainty (Equation 1.48) but based on the mean estimate of $\tilde{H}_{S_a}(s_a)$, and with increased β^2 in the spectral acceleration capacity due to the consideration of epistemic uncertainty.

1.5.1 Summary

A closed-form analytic foundation for the probabilistic seismic assessment of structures has been developed, taking into account the randomness (aleatory uncertainty) and uncertainty (epistemic uncertainty) in the seismic hazard, demand, and capacity parameters. This foundation is based on a closed-form analytical expression for the mean annual frequency of exceeding a limit state (limit state frequency in short). Two different solution strategies were presented for deriving the limit state frequency, namely, displacement-based and *IM*-based. Both approaches are based on simplifying assumptions regarding the shape of the hazard curve and the probabilistic models representing demand and capacity. This technical foundation forms an analytic basis upon which alternative design and assessment formats can be developed. These formats are discussed in the next chapter.

1.5.1 The Developed Technical Framework in the Context of PEER

A probabilistic foundation for seismic performance assessment of structures can be based on the acceptable probability of exceeding specific performance levels (Cornell and Krawinkler 2001). The performance levels can be described and quantified as different levels of acceptable structural behavior. The Pacific Earthquake Engineering Center (PEER) employs the notion of decision variable vector ($\underline{DV}$) to quantify various performance levels, in which $\underline{DV}$ may include a discrete (e.g., a binary variable indicating collapse) and/or a continuous (e.g., amount of loss in dollar terms) indicator variable(s) marking the exceedance of one or more limit states. Hence the probability of exceeding a specific performance level can be expressed as the mean annual frequency (MAF) that the corresponding DV indicator variables exceeds zero. A practical way of estimating the MAF for the decision variable vector consists of expanding it with respect to structural demand vector, $\underline{D}$, and the vector of ground motion intensity measures, $\underline{IM}$

$$H(\underline{DV}) = \iint G(\underline{DV} \mid \underline{D}, \underline{IM}) \cdot dG(\underline{D} \mid \underline{IM}) \cdot dH(\underline{IM})$$

where $H(\underline{DV})$ is the MAF of exceeding the vector of decision variables $\underline{DV}$, and $G(\underline{DV} \mid \underline{D}, \underline{IM})$ is the conditional probability of exceeding $\underline{DV}$ given the demand vector $\underline{D}$ and the vector of ground motion intensity measures $\underline{IM}$, $G(\underline{D} \mid \underline{IM})$ is the probability of exceeding the structural demand vector $\underline{D}$ given $\underline{IM}$, and, $H(\underline{IM})$ is the MAF of exceeding $\underline{IM}$.

In this report we outlined a step-by-step procedure for evaluating $H(\underline{DV})$ from the above integral for a special case where (a) the decision variable is defined as a (scalar) binary indicator variable that assumes the value of 1 when the capacity for a specified limit state is exceeded and 0 otherwise, (b) the structural demand vector is a scalar displacement-based demand variable (e.g., maximum interstory drift ratio), (c) the ground motion intensity measure is the scalar spectral acceleration at the first mode frequency of the structure, and (d) given $\underline{D}$, $\underline{DV}$ is conditionally independent of $\underline{IM}$. Therefore, the MAF of exceeding the decision variable is written as:

$$H_{LS} = H(DV = 1) = \iint G[D > C_{LS} \mid D] \cdot dG(D \mid S_a) \cdot dH(S_a)$$

where $DV = 1$ when the demand variable D exceeds C_{LS}, the capacity for limit state *LS*.

2 Probability-Based Design (DCFD) Seismic Formats

2.1 INTRODUCTION: FORMAT DEVELOPMENT

Chapter 1 of this report was dedicated to developing an analytical foundation for the probability-based seismic assessment of structures. The final product of this foundation development was the mean annual frequency of exceeding a limit state or the "limit state frequency" in short. Limit state frequency H_{LS} was calculated taking into account the uncertainty in various elements involved in the seismic assessment and design of the structural system. An analytical framework for calculating the limit state frequency is helpful for seismic assessment of the structures, e.g., calculating the limit state frequency for an existing structure and checking to see if its design falls within the acceptable region. However, in a design problem, the actual structural members and connections are not known beforehand. They are, rather, the end product of the design process. Conversely, the performance objective for the design is usually set beforehand and can be expressed in terms of the limit state frequency that is in turn a function of the structural design properties. Therefore, a design process has an iterative nature and consists of assessing a proposed design against a specified performance objective(s) and modifying the proposed design if it does not meet the performance objective(s).

This part of the report addresses problems similar to the following: how to assess a proposed (or existing) design for a structure with respect to a specified collapse limit state frequency of, e.g.,0.04% per annum or how to address the uncertainty (due to limited knowledge) involved in the estimation of the design parameters. This uncertainty is usually stated through questions such as how to design a structure for a known mean annual collapse limit state frequency of 0.04% *with a confidence interval level of 95%.*

We shall discuss various alternative design formats that stem from the probability-based foundation developed in the first part of this report. These formats are in general suitable for guidelines and code implementation. A major class of these formats, which is analogous in form to (linear, static, force-based) load and resistance factor design (LRFD) procedures (AISC LRFD code), is discussed in this chapter. However, these formats are based on generic, random (usually) displacement-based, nonlinear dynamic response variables: “demand” and “capacity”, and hence are referred to as “demand and capacity factor design” (DCFD). The DCFD format can also be formulated in terms of *IM*-based generic demand and capacity variables. The fragility-hazard format, also discussed in this chapter, is another *IM*-based format that is useful for designing/assessing the global behavior of a structure or a class of structures.

Unlike the foundation, which is unique (with respect to the set of assumptions made and the solution strategy used for the derivation of the limit state probability), the formats are numerous. They are just various representations of a common foundation. Hence, the choice among these alternative formats is subjective. It is to be made on grounds such as familiarity and practicality.

2.2 RANDOMNESS: THE ONLY SOURCE OF UNCERTAINTY

Similar to the foundation development in Chapter 1, the alternative design formats discussed in this chapter are also presented in two parts. The probability-based formats developed in this section are based on the assumption that randomness is the only source of uncertainty, and hence, they are based on the expression for limit state frequency derived in the Section 1.4.7 (Equations 1.38 and 1.48).

Recall from Equation 1.38 that the mean annual frequency of exceeding a limit state that is expressed in displacement-based terms, was derived as:

$$H_{LS} = \nu \cdot P[D > C] = H_D(\eta_C) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta_C^2} = H_{S_a}(S_a^{\eta_C}) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\cdot\beta_C^2} \tag{2.1}$$

where $H(\cdot)$ denotes the mean annual frequency of exceedance in general. We are going to re-arrange the above equation into alternative forms, also known as “DCFD design formats.” The purpose for this re-arrangement is to present this probability-based formulation in a way that is easy to be implemented in the design practice. The basic components of demand and capacity factor design format (DCFD) are outlined in the following equation,

$$\eta_{D|^{P_0}s_a} \cdot e^{\frac{1}{2}\cdot\cdot\frac{k}{b}\cdot\beta_{D|S_a}^2} = \eta_C \cdot e^{-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2} \tag{2.2}$$

where $\eta_{D|^{P_0}s_a}$ is the median drift *demand* for a given spectral acceleration, ${}^{P_0}s_a$, corresponding to hazard levels in the proximity of an acceptable limit state probability, P_0. η_C is the median drift *capacity*, $\exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$ is the *demand factor,* and $\exp(-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_C)$ is the *capacity factor*. Equation 2.2 offers an alternative presentation of the formal foundation equation (Equation 2.1), and is obtained by re-arranging Equation 2.1. We shall go through the re-arrangement step-by-step later in this chapter.

The fragility-hazard format is another format discussed in this chapter. This format is derived by re-arranging the closed-form derivation of the limit state frequency following the *IM*-based solution strategy in Chapter 1 (Equation 1.48):

$$ {}^{P_0}s_a \cdot e^{\frac{1}{2}\cdot\frac{k}{b^2}\cdot\beta^2_{D|S_a}} = s_a^{\eta_C} \cdot e^{-\frac{1}{2}\cdot\frac{k}{b^2}\cdot\beta^2_C} \tag{2.3} $$

where ${}^{P_0}s_a$ is the spectral acceleration with a *hazard* value equal to the acceptable limit state probability, P_0, and $s_a^{\eta_C}$ is the spectral acceleration with a *fragility* of 50%. Each format will be developed and discussed in detail in the corresponding section. Before proceeding to the details of the derivations, we are going to outline a few parameters that are going to be helpful in our future format derivations.

2.2.1 Spectral Acceleration s_a^d Corresponding to a Displacement-Based Demand Equal to *d*

s_a^d, the spectral acceleration corresponding to displacement-based demand value, d, is defined as the spectral acceleration corresponding to the value, *d*, from the median displacement-based curve as a function of the spectral acceleration, in fact as the inverse of this function:

$$ s_a^d = \eta^{-1}_{D|S_a}(d) $$

Recalling from the previous chapter, the median displacement-based demand was approximated by a power-law function of the spectral acceleration, $\eta_{D|S_a}(x) = a \cdot x^b$. Based on this power-law approximation, s_a^d, or the spectral acceleration corresponding to the displacement-based response *d,* can be expressed as:

$$s_a^d = \eta_{D|S_a}^{-1}(d) = \left(\frac{d}{a}\right)^{\frac{1}{b}} \qquad (2.4)$$

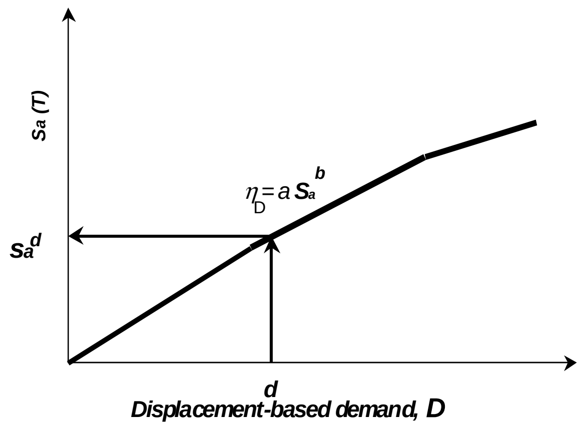

Fig. 2.1 Spectral acceleration corresponding to a displacement-based demand equal to *d*

s_a^d is illustrated graphically in Figure 2.1. In simple terms, s_a^d represents the spectral acceleration value corresponding to a given demand value *d* from the median demand curve approximated by $a \cdot S_a^{\ b}$[1].

2.2.2. Spectral Acceleration ${}^{P_0}s_a$ for a Hazard Level Equal to P_0

${}^{P_0}s_a$ is the spectral acceleration with a mean annual frequency of being exceeded (also known as the hazard, defined in Chapter 1) equal to P_0:

$${}^{P_0}s_a = \lambda_{S_a}^{-1}(P_0) \cong \left(\frac{P_0}{k_0}\right)^{\frac{-1}{k}} \qquad (2.5)$$

in which we make use of the fact that (in Chapter 1) the mean annual frequency of exceeding a given spectral acceleration value (also known as the spectral acceleration hazard curve) can be approximated (at least locally) by the power-law function, $\lambda_{S_a}(x) = k_0 \cdot x^{-k}$. Figure 2.2 illustrates

[1] parameters a and b can be determined by performing linear regression analysis on a sample of spectral acceleration and demand pairs obtained from nonlinear dynamic analyses (see Jalayer and Cornell, 2003a).

the graphical presentation of ${}^{P_0}s_a$. The spectral acceleration s_a^d corresponding to a drift demand equal to d is also plotted on the same figure.

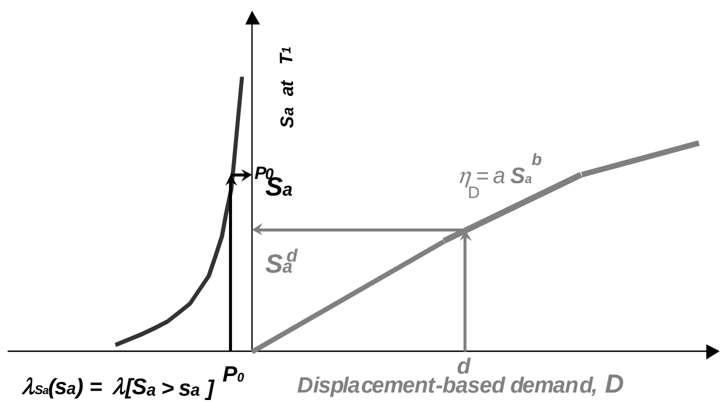

Fig. 2.2 Spectral acceleration for a hazard level equal to P_0

2.2.3 DCFD Format

The DCFD format is analogous in form to the load and resistant factor design (LRFD) procedures (see AISC design procedures, 1994). As the name suggests, this format is constituted of demand and capacity multiplied by their respective factors. As with LRFD procedures, the DCFD format can be used to design a building against a certain factored demand by finding a factored capacity. The probabilistic demand and capacity factors for the DCFD format are very similar in concept to the partial safety factors applied to the load and resistance in LRFD design procedures.

This format stems directly from the expression for limit state frequency (Equation 2.1) after some re-arrangements. It should be noted that the same simplifying assumptions that led to the derivation of the closed-form foundation equation in the previous chapter are implicit here in the derivation of the DCFD format. In order to develop a design format, we first need to set a design criterion. A criterion can be stated as designing a structure so that the mean annual frequency of exceeding a certain limit state (limit state frequency in short) is less than or equal to the allowable annual probability of exceedance, P_0[2]:

---

[2] Note the mixing of the usage of the terms "mean annual frequency" and "annual probability." Although the more precise term to be used in these derivations is "mean annual frequency," for the type of rare events that we are interested, the corresponding values are virtually numerically identical.

$$H_{LS} \leq P_0 \tag{2.6}$$

where the equality holds at the onset of the limit state. Recalling from the previous chapter, the limit state frequency can be expressed through a closed-form relationship (Equation 1.38 or 2.1). This closed-form expression can be substituted for H_{LS} in Equation 2.6:

$$H_{S_a}(s_a^{\eta_C}) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\cdot\beta_C^2} \leq P_0 \tag{2.7}$$

where $H_{S_a}(s_a^{\eta_C})$ is the hazard value (mean annual frequency of exceedance) for a spectral acceleration equal to $s_a^{\eta_C}$ (i.e., the spectral acceleration corresponding to the median capacity η_C), and it can be derived from Equation 1.36 as:

$$H_{S_a}(s_a^{\eta_C}) = k_0 \left(\frac{\eta_C}{a}\right)^{\frac{-k}{b}} \tag{2.8}$$

If $H_{S_a}(s_a^{\eta_C})$ from the Equation 2.8 is replaced in Equation 2.7:

$$k_0 \left(\frac{\eta_C}{a}\right)^{\frac{-k}{b}} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\cdot\beta_C^2} \leq \mathrm{P}_0 \tag{2.9}$$

After re-arranging the above equation in order to solve for median capacity, η_C, we get the following expression for the median capacity required so that the limit state frequency H_{LS} is less than or equal to the allowable probability, P_0:

$$\eta_C \leq a \cdot \left(\left(\frac{P_0}{k_0}\right)^{-\frac{1}{k}}\right)^b \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a}} \cdot e^{\frac{1}{2}\frac{k}{b}\cdot\beta_C^2} \tag{2.10}$$

The expression in the parentheses, $(P_0/k_0)^{-1/k}$, is nothing but the spectral acceleration, ${}^{P_0}s_a$, having a hazard value equal to the allowable probability P_0 as given in Equation (2.5). Substituting $(P_0/k_0)^{-1/k}$ with ${}^{P_0}s_a$ in Equation 2.10 will make it look simpler:

$$\eta_C \leq a \cdot \left({}^{P_0}s_a\right)^b \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a}} \cdot e^{\frac{1}{2}\frac{k}{b}\cdot\beta_C^2} \tag{2.11}$$

where $a\left({}^{P_0}s_a\right)^b$ is in turn equal to the median drift demand $\eta_{D|^{P_0}S_a}$ for a given spectral acceleration value of ${}^{P_0}s_a$ (Equation 2.4). Thus, Equation 2.11 can be further simplified by replacing $a\left({}^{P_0}s_a\right)^b$ with $\eta_{D|^{P_0}S_a}$:

$$\eta_{D|^{P_0}S_a} \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a}} \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2} \leq \eta_C$$

Finally we transfer the capacity-related exponential term, $\exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_C)$, to the other side of the equation changing the sign of the exponent:

$$\eta_{D|^{P_0}S_a} \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a}} \leq \eta_C \cdot e^{-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2} \tag{2.12}$$

Equation 2.12 represents the DCFD format in its final form. The right-hand, or "capacity," side of the equation is called the "factored capacity," parallel to LRFD's factored resistance. In a similar manner, the left-hand, or the "demand" side" of the equation is called the "factored demand for the allowable probability P_0", parallel to LRFD's factored load. It should be noted that the factored demand (the equivalent to LRFD's factored load) is a function of the allowable probability level, P_0, whereas the factored capacity does not depend on P_0 in contrast to the two factors in the AISC LRFD, where neither the demand factor nor the capacity factor depends on P_0. The DCFD format offers an alternative and equivalent statement for the design criterion, according to which the factored demand for the allowable probability P_0 should be less than or equal to the factored capacity. This implies that at the onset of the limit state, the factored demand for the allowable probability P_0 is equal to the factored capacity. One of the main advantages of the DCFD design format is that the probabilistic design criteria can be stated in terms of familiar displacement-based response parameters. This makes the DCFD format compatible with existing (deterministic) design procedures.

The following sections will discuss in more detail the components of the DCFD format (Equation 2.12).

2.2.3.1 Displacement-based demand, $\eta_{D|^{P_0}S_a}$

$\eta_{D|^{P_0}S_a}$ is the median displacement-based demand for a spectral acceleration equal to spectral acceleration, ${}^{P_0}s_a$, (i.e., spectral acceleration with a mean annual frequency of exceedance equal to the allowable probability, P_0). We may also refer to it as the median demand for a given ground motion intensity, ${}^{P_0}s_a$, in short. Adopting the analytical definitions outlined in Sections 2.2.1 and 2.2.2, the median demand can be calculated from the following expression:

$$\eta_{D|^{P_0}s_a} = a\left({}^{P_0}s_a\right)^b = a\left(\frac{P_0}{k_0}\right)^{-\frac{b}{k}} \tag{2.13}$$

But Figure 2.3 illustrates a graphical presentation of $\eta_{D|^{P_0}s_a}$ that demonstrates its more general applicability. Looking at the figure, we can see that $\eta_{D|^{P_0}s_a}$ can be calculated in two simple steps. Step 1 is to find the spectral acceleration ${}^{P_0}s_a$ that has a mean annual frequency of exceedance (i.e., hazard) equal to the allowable probability, P_0, from the hazard curve for the spectral acceleration. Step 2 is to find the displacement-based demand $\eta_{D|^{P_0}s_a}$ that corresponds to a spectral acceleration equal to ${}^{P_0}s_a$ from the median demand curve. Note that in application neither the hazard curve nor the median demand curve need to be in analytical form to evaluate ${}^{P_0}s_a$ and $\eta_{D|^{P_0}s_a}$. This fact can be exploited in seismic assessments of structures implementing nonlinear dynamic procedures (e.g., see Jalayer and Cornell, 2003a).

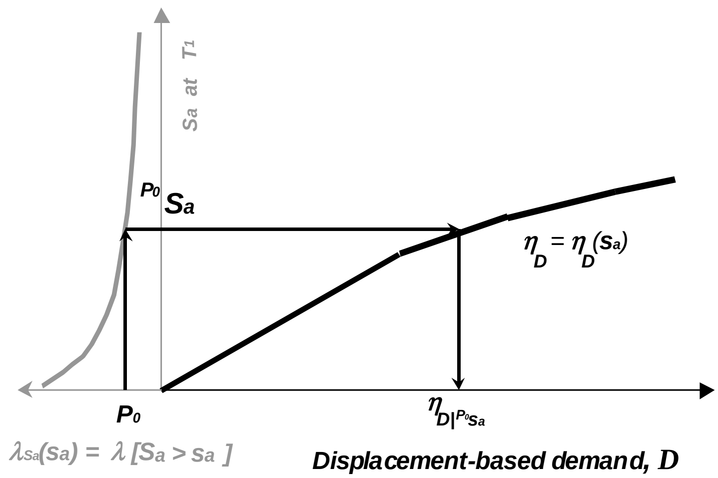

Fig. 2.3 Graphical presentation of median demand $\eta_{D|^{P_0}s_a}$ for a spectral acceleration equal to $^{P_0}s_a$

2.2.3.2 Displacement-based capacity, η_C

The median displacement-based capacity for the structure is denoted by η_C. Figure 2.4 illustrates the median drift demand, $\eta_{D|^{P_0}s_a}$, and capacity, η_C, on the same graph.

2.2.3.3 Demand factor $\exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$

The displacement-based demand factor denoted by $\exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$ is a magnifying factor that takes into account the randomness in the displacement-based demand. The randomness represented by the demand factor is usually due to record-to-record variability. The dispersion measure for the displacement-based demand denoted by $\beta_{D|S_a}$**,** is equal to the standard deviation of the (natural) logarithm of displacement-based demand for a given spectral acceleration. Typical values for $\beta_{D|S_a}$**,** in the nonlinear range, are about 0.30 to 0.60. In the special case (e.g., a linear SDOF oscillator) where there is no dispersion in demand (given S_a), the demand factor will be equal to unity. k/b can be interpreted as the sensitivity of the probability of exceedance to a unit change in the displacement-based demand; which means that a factor of x change on the displacement scale will cause a factor of $x^{-k/b}$ change on the probability scale.

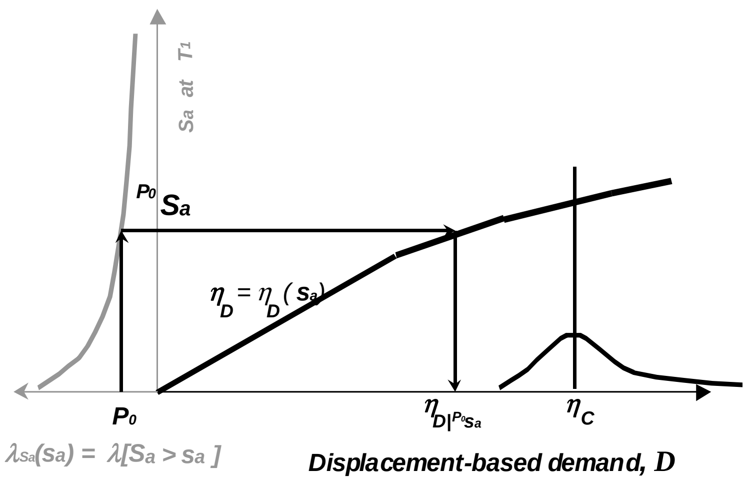

Fig. 2.4 Graphical presentation of median drift capacity η_C

As in the LRFD design procedures, the demand factor $\exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$ is also denoted by γ. Clearly, γ is always greater than or equal unity (an exponential raised to a non-negative power). Thus, the "design" displacement-based demand is always greater than or equal to the median demand due to the randomness-type of uncertainty in displacement-based demand.

2.2.3.4 Capacity factor $\exp(-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2)$

The capacity factor, $\exp(-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2)$, is a reduction factor that takes into account the randomness type of uncertainty in the displacement-based capacity. It is an exponential term raised to a non-positive power and hence is always smaller than one. Therefore, the design capacity is always less than or equal to the median capacity due to the randomness-type of uncertainty. The dispersion term in the exponential power, β_C**,** is the standard deviation of the (natural) logarithm of the displacement-based capacity. Also k/b is a factor reflecting the sensitivity of the probability of exceedance to a unit change in displacement-based capacity. Similar to the LRFD design procedures, the capacity factor $\exp(-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2)$ is also denoted by ϕ.

2.2.3.5 Factored demand and demand hazard

The following presents an alternative interpretation of factored demand. This alternative interpretation relates the factored demand to the demand hazard (mean frequency of exceedance).In DCFD format, the factored demand (FD) was derived as:

$$FD = \eta_{D|^{P_0}s_a} \cdot \exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$$

Replacing the analytic expression for $\eta_{D|^{P_0}s_a}$ from Equation 2.13:

$$FD = a\left(\frac{P_0}{k_0}\right)^{-\frac{b}{k}} \cdot \exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$$

Now we can solve the above equation for P_0:

$$P_0 = k_0\left(\frac{FD}{a}\right)^{-\frac{k}{b}} \cdot \exp(\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}) \qquad (2.14)$$

Realizing that (according to Equation 2.8) the term $k_0\cdot(FD/a)^{-k/b}$ is equal to the hazard value for a spectral acceleration corresponding to a (median) demand value equal to FD:

$$H_{S_a}(s_a^{FD}) = k_0\left(\frac{FD}{a}\right)^{-\frac{k}{b}} \qquad (2.15)$$

Replacing the term $k_0\cdot(FD/a)^{-k/b}$ in Equation 2.14 with its equivalent from Equation 2.15:

$$P_0 = H_{S_a}(s_a^{FD})\cdot\exp(\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}) \qquad (2.16)$$

We can observe that the right side of the above equation is equal to the (demand) hazard for a demand value equal to *FD* (Equation 2-25):

$$P_0 = H_D(FD) \qquad (2.17)$$

Conversely, the factored demand can be written as the inverse of the hazard function at value P_0:

$$FD = H_D^{-1}(P_0) \qquad (2.18)$$

The above equation states that the factored demand for an allowable probability P_0 is equal to the (displacement-based) demand with a mean annual frequency of exceedance (hazard) equal to P_0. This alternative interpretation for the factored demand is going to be helpful when we need to estimate the factored demand for more general cases (i.e., when the analytic assumptions underlying the derivation of DCFD may not be valid). Numerical examples related to the interpretation of factored demand as the inverse of the hazard function for demand can be found in Jalayer and Cornell 2003 a, b. In both papers, this property is used to estimate the factored demand using numerical integration and comparing it to the one calculated from the left-hand side of Equation 2.12. This is quite helpful since the numerical integration can be used to check the robustness of the closed-form solution in providing accurate estimates for the factored demand.

2.2.3.6 General form for the DCFD design format

We have already discussed the derivation of a closed-form for the DCFD format (Equation 2.12), which resulted from re-arranging the expression for limit state probability in Equation 2.1:

$$\eta_{D|^{P_0}S_a} \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a}} \leq \eta_C \cdot e^{-\frac{1}{2}\frac{k}{b}\cdot\beta_C^2}$$

However, it should be kept in mind that this format is based on the same simplifying assumptions that were made in the foundation derivations in Chapter 1. The general form for the DCFD design can be introduced based on the format we derived in Equation 2.12, but replacing $\eta_{D|^{P_0}S_a}$ with[3] D, η_C with C, $\exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$ with γ**,** and $\exp(-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2)$ with ϕ**:**

$$D \cdot \gamma \leq C \ \cdot \phi \qquad (2.19)$$

where D and C refer to demand and capacity displacement-based parameters, and γ and ϕ are their corresponding factors. It can be noted that the DCFD format presented in its general form as in Equation 2.19 looks similar to the LRFD format presentation. Another alternative general

[3] Despite their capital letter designation, *D* and *C* do not represent stochastic variables in this DCFD context. (They do typically represent stochastic variables in Chapters 2and 3). Here, they are just referring to some generic displacement-based demand and capacity parameter.

way to present the DCFD format is by simply comparing the factored demand to factored capacity:

$$F.D. \leq F.C. \tag{2.20}$$

The benefit of this alternative representation is that factored demand and factored capacity can be defined in a different manner from the DCFD format. A generalized definition for factored demand is already discussed in the previous section. According to this definition the factored demand is the demand value that has a mean annual frequency of exceedance equal to the allowable probability, P_0. The authors (Jalayer and Cornell 2003 a,b) implement alternative nonlinear dynamic analysis procedures to assess the performance of an existing seven-story reinforced concrete frame for the global instability performance level using the DCFD format. These (nonlinear dynamic) procedures are used to locally estimate the parameters, $\eta_{D|^{P_0}S_a}$, $\beta_{D|S_a}$, and, b in the closed-form expression for the factored demand.

2.2.3.7 Numerical example: Performance evaluation for an existing building

Returning to the numerical example presented in Chapter 1, now we can assess the performance of an existing three-story frame for the collapse limit state for an allowable probability of $P_0 = 4\times10^{-4}$ (2% in 50 years). Based on the DCFD design format, we are going evaluate and compare factored demand for the allowable probability $P_0 = 4\times10^{-4}$ and factored capacity for the collapse limit state.

2.2.3.7.1 Factored demand $D \cdot \gamma$

Evaluation of the factored demand consists of two parts: (a) calculating the median drift demand $\eta_{D|^{0.0004}S_a}$ for a spectral acceleration with a hazard equal to 4×10^{-4} and (b) calculating the demand factor. The median demand $\eta_{D|^{0.0004}S_a}$ itself can be calculated in two steps. The first step is to calculate the spectral acceleration $^{0.0004}s_a$ with a MAF of exceedance (i.e., hazard) equal to 4×10^{-4}. This can be done either by using Equation 2.5 or more generally by simply finding the spectral acceleration corresponding to $P_0 = 4\times10^{-4}$ from the hazard curve. The advantage of the second approach is that the hazard curve does not necessarily need to have a power-law form.

Here, we are going find ${}^{0.0004}s_a$ both analytically and graphically. ${}^{0.0004}s_a$ can be calculated from Equation 2.5 for $P_0 = 4\times10^{-4}$:

$${}^{0.0004}s_a = (\frac{0.0004}{k_0})^{\frac{-1}{k}}$$

Recalling from the first part of the numerical example in the previous chapter, the parameter estimates for k_0 and k were equal to:

$$k_0 = 0.00124$$
$$k = 3.03$$

Finally, ${}^{0.0004}s_a$ can be calculated as:

$${}^{0.0004}s_a = (\frac{0.0004}{0.00124})^{\frac{-1}{3.03}} = 1.458 \text{ [g]}$$

Graphically speaking, ${}^{0.0004}s_a$ is the spectral acceleration corresponding to the hazard value equal to 0.0004 from the spectral acceleration hazard curve. The hazard curve with parameters k_0 and k (listed above) is plotted in Figure 2.5. It can be observed that a hazard value 0.0004 corresponds to ${}^{0.0004}s_a$ equal to 1.45. After ${}^{0.0004}s_a$ is calculated, the next step is to find the median displacement-based demand that corresponds to this spectral acceleration. Again, the median demand can be either calculated from the power-law approximation, $\eta_{D|S_a}(x) = a \cdot x^b$, or estimated graphically from the median displacement-based demand curve that is plotted versus spectral acceleration.

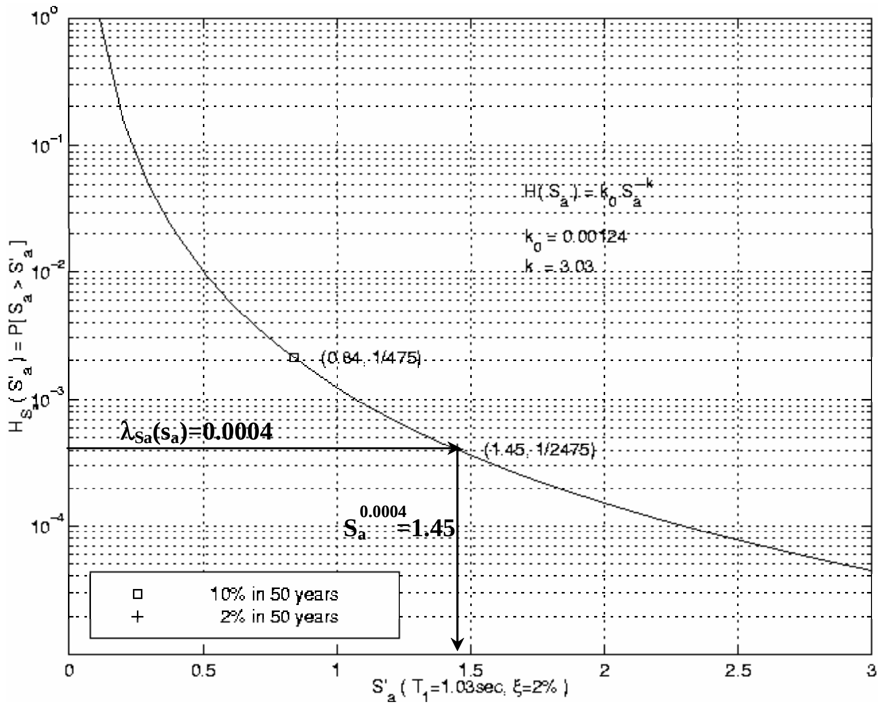

Fig. 2.5 The hazard curve for $S_a(T=1, \xi = 2\%)$

The median demand corresponding to a spectral acceleration equal to ${}^{0.0004}s_a = 1.458[g]$ can be calculated from the following power-law relation:

$$\eta_{D|S_a}({}^{0.0004}s_a) = a \cdot \left({}^{0.0004}s_a\right)^b$$

Recalling from the previous chapter, the parameter estimates for *a* and *b* are equal to:

$$a = 0.0325$$
$$b = 1.002$$

Finally, $\eta_{D|S_a}({}^{0.0004}s_a)$ can be calculated as:

$$\eta_{D|S_a}({}^{0.0004}s_a) = 0.0325 \cdot \left({}^{0.0004}s_a\right)^{1.002} \quad = 0.0325 \cdot (1.458)^{1.002} = 0.047$$

We can also obtain $\eta_D({}^{0.0004}s_a)$ graphically by finding the median demand value corresponding to a given spectral acceleration of ${}^{0.0004}s_a$ or 1.45 [g] from the median demand curve. In this example, we have chosen the maximum interstory drift angle (the "absolute" maximum of the response time-history over all the stories in the structure) as the displacement-based demand parameter. The maximum interstory drift angle is plotted versus spectral acceleration in Figure 2.6 below.

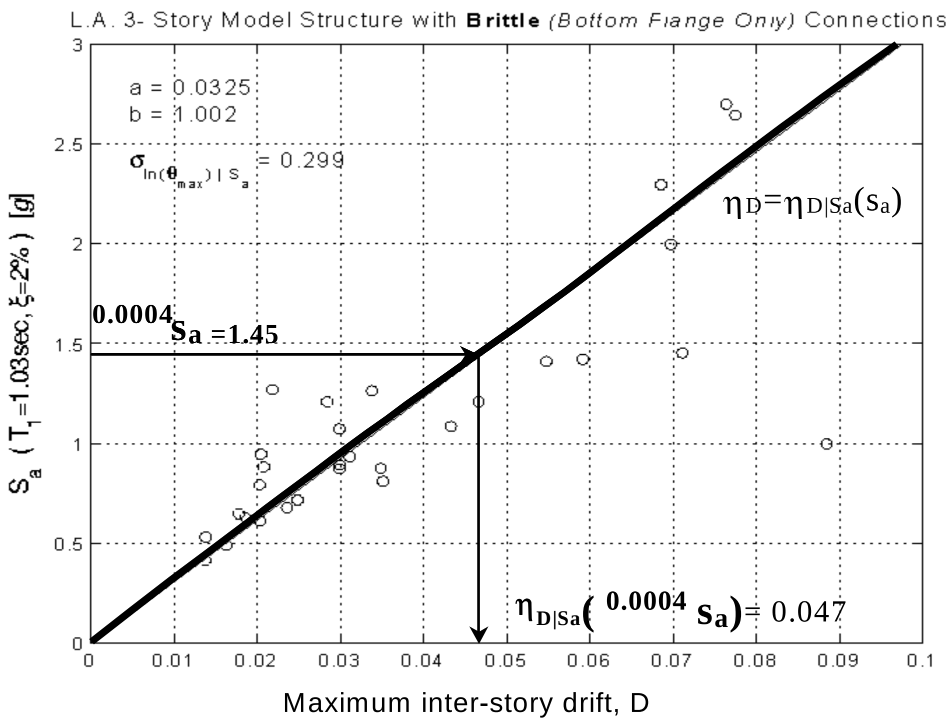

Fig. 2.6 Spectral acceleration plotted versus maximum interstory drift angle, and the power-law function fitted to the plotted data points (a line on the two-way logarithmic paper)

The next step is to calculate the demand factor, $\exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$. As mentioned in Section 2.2.3.3, $\beta_{D|S_a}$ is equal to the standard deviation of the (natural) logarithm of the demand given spectral acceleration, $\sigma_{\ln D|S_a}$. $\sigma_{\ln D|S_a}$ that is denoted by $\sigma_{\ln\theta_{\max}|S_a}=0.299$ on the graph in Figure 2.6 ($\theta_{\max}$ stands for maximum interstory drift angle, which is in fact the demand parameter D used in this example), is estimated by the standard error of the regression. The hazard slope parameter k is reported in Figure 2.5 as 3.0. Also the median demand-spectral acceleration slope parameter b is equal to 1.0 (Fig. 2.6). Now that the necessary parameter estimates for factored demand estimation are obtained, we are ready to calculate the demand factor or $\exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$:

$$\gamma = e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a}} = \mathrm{e}^{(\frac{1}{2})(\frac{3}{1})(0.3^2)} = 1.144$$

Finally, the factored demand is calculated by multiplying the median demand $\eta_{D|S_a}({}^{0.0004}s_a) = 0.047$ and the demand factor, $e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a}} = 1.144$:

$$D \cdot \gamma = \eta_{D|S_a}({}^{0.0004}s_a) \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a}} = (0.047)(1.144) = 0.0538 \qquad (2.21)$$

2.2.3.7.2 Factored capacity $C \cdot \phi$

As mentioned before, this numerical example demonstrates the assessment of structural performance for the limit state of global collapse, using maximum interstory drift angle as the displacement-based parameter. Therefore, the displacement-based capacity is represented by maximum interstory drift angle at the onset of global collapse. Similar to factored demand estimation, the estimation of factored capacity consists of two parts: (a) calculating the median capacity η_C for the collapse limit state and (b) calculating the capacity factor. Recalling from the previous parts of this numerical example in Chapter 1, the median (drift) capacity η_C for the collapse limit state and its dispersion parameter β_C (i.e., the standard deviation of the natural logarithm of maximum interstory drift angle capacity values) are equal to:

$$\eta_C \cong 0.07$$
$$\beta_C = 0.20$$

Hence, the capacity factor or $\exp(-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2)$ can be calculated as:

$$\phi = e^{-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_C} = \mathrm{e}^{-(\frac{1}{2})(\frac{3}{1})(0.2^2)} = 0.94$$

Finally, the factored capacity is calculated by multiplying the median capacity $\eta_C = 0.07$ by the capacity factor, $\exp(-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2) = 0.94$:

$$C \cdot \phi = \eta_C \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_C} = (0.07)(0.94) = 0.0658 \qquad (2.22)$$

Comparing the factored demand from Equation 2.21 and factored capacity from Equation 2.22, we can observe that:

$$D \cdot \gamma = 0.0538 \leq C \cdot \phi = 0.0658$$

We can conclude that the structure satisfies the design criteria in Equation 2.19 for an allowable (mean annual) frequency of 0.0004 (i.e., 10% in 50 years) corresponding to the collapse limit state. This conclusion implies that the limit state frequency (mean annual frequency of exceeding the collapse limit state) is less than 0.0004 per annum.

2.2.4 Fragility/Hazard Format: An *IM*-Based Probabilistic Format

In the previous sections, we outlined in detail the main components of DCFD format, which is a displacement-based probabilistic design/assessment format. Here, we are going to discuss the fragility/hazard format, a design/assessment format that stems from the *IM*-based framework equation derived in Chapter 1. One main advantage of an *IM*-based design format is that design and/or assessments are performed in the spectral acceleration ordinate and do not explicitly involve the displacement-based response.

The design criterion for the fragility/hazard format (see e.g., DOE 1020, 1994, and Kennedy and Short, 1994) is tested by comparing "fragility" curves to "hazard" curves for a given allowable probability level. The hazard curves represent the probabilistic ground motion intensity or, in general terms, the "loading" characteristics, whereas the "fragility" curves represent the probabilistic structural capacity or the structural "resistance."

As with the DCFD design format, the first step in developing a design format is to set the design criteria. The (*IM*-based) design criterion can be stated similar to the one in Equation 2.6 for DCFD format for a given allowable annual probability, P_0:

$$H_{LS,S_a-based} \leq P_0$$

where the limit state frequency, $H_{LS,S_a-based}$, is calculated from the *IM*-based expression for limit state frequency in Chapter 1 (Equation 1.48):

$$H_{LS,S_a\_based} = H_{S_a}(\eta_{S_{a,C}}) \cdot e^{\frac{1}{2} \cdot k^2 \cdot \beta^2_{S_{a,C}}} \tag{2.23}$$

The expression for the *IM*-based limit state frequency can be substituted in the design criterion (Equation 2.6):

$$H_{S_a}(\eta_{S_{a,C}}) \cdot e^{\frac{1}{2} \cdot k^2 \cdot \beta^2_{S_{a,C}}} \leq P_0 \tag{2.24}$$

where $S_{a,C}$ is an *IM*-based random variable representing the limit state capacity (or spectral acceleration capacity in short) and k is the parameter reflecting the steepness of the hazard curve for spectral acceleration. The authors have discussed in a separate paper (Jalayer and Cornell, 2003a) few alternative methods for estimating the statistical properties (i.e., median, $\eta_{S_{a,C}}$, and standard deviation of the natural logarithm, $\beta_{S_{a,C}}$) of this random variable using nonlinear dynamic analysis procedures such as incremental dynamic analysis (Vamvatsikos and Cornell, 2001). Recalling from Chapter 1, the spectral acceleration hazard can be approximated (at least locally) by a power-law relationship:

$$\lambda_{S_a}(x) = k_0 \cdot x^{-k} \tag{2.25}$$

Therefore, $\lambda_{S_a}(\eta_{S_{a,C}})$, the mean annual frequency of exceeding the median spectral acceleration capacity, can be calculated from the above expression and then substituted in Equation 2.24:

$$k_0 \cdot \eta_{S_{a,C}}{}^{-k} \cdot e^{\frac{1}{2} \cdot k^2 \cdot \beta^2_{S_{a,C}}} \leq P_0 \tag{2.26}$$

After some simple re-arrangements, with the objective of separating the “load” and “resistance” sides, Equation 2.26 takes the following form:

$$\eta_{S_{a,C}} \cdot e^{-\frac{1}{2} \cdot k \cdot \beta^2_{S_{a,C}}} \leq \left(\frac{P_0}{k_0} \right)^{-\frac{1}{k}} \tag{2.27}$$

Recalling from Equation 2.5, the right-hand side of the equation is in fact the spectral acceleration ${}^{P_0}s_a$ for a hazard level equal to the allowable probability P_0 (or spectral acceleration for allowable probability in short), which was defined in Section 2.2.2 in the beginning of this chapter:

$$^{P_0}s_a \leq \eta_{S_{a,C}} \cdot e^{-\frac{1}{2} \cdot k \cdot \beta^2_{S_{a,C}}} \qquad (2.28)$$

which is the closed-form expression for the fragility/hazard format. A similar closed-form expression, but with the exponential term (with a positive sign) applied to $^{P_0}s_a$, has been used in the current draft of ISO seismic criteria for offshore structures (Banon et al. 2001). It will be shown below how this expression (Equation 2.28) relates to the fragility and hazard curves. Similar to the DCFD format, the left-hand side of this expression represents the "factored demand for the allowable annual probability, P_0," and the right-hand side represents the "factored capacity." However, if compared to the expression for the DCFD format in Equation 2.12, one can observe that the demand factor representing the dispersion in displacement-based demand is missing in the demand side of the expression. Nonetheless, the factored capacity looks similar to that of the DCFD format except for the fact that the *b* value is missing from the capacity factor. This is to be expected since the *b* value represents the (log) slope of the displacement-based demand parameter versus spectral acceleration; and the fragility/hazard format does not explicitly involve the displacement-based demand. Therefore, the design criterion based on the fragility/hazard format can be stated in terms of the *IM*-based factored capacity being less than or equal to the *IM*-based factored demand for a given allowable annual probability, P_0. The following sections will discuss fragility and hazard curves and how they can be employed in order to make parameter estimates for the fragility/hazard format in Equation 2.28.

2.2.4.1 Hazard curves

The hazard function, $H_{S_a}(s_a)$, for a given spectral acceleration value, s_a, can be defined as the mean annual frequency of exceeding the spectral acceleration value, s_a. The hazard function $H_{S_a}(s_a)$ is discussed in more detail in Chapter 1. Figure 2.7 illustrates a schematic hazard curve. As mentioned before, the hazard curve is approximated by a power-law relation, $H_{S_a}(s_a) = k_0 \cdot s_a^{-k}$ where parameter *k* represents the steepness of the hazard curve. Strictly speaking, *k* is the slope of the power-law hazard curve plotted on a two-way logarithmic paper. The slope parameter *k* may be estimated as the local slope of the hazard curve (Fig. 2.7) in the region of hazard/spectral acceleration values that are of interest.

In the context of the fragility/hazard format, the hazard curve represents the probabilistic characteristics of “load” or demand. It is demonstrated in Figure 2.7 how the “factored demand for the allowable probability P_0” in the fragility/hazard format, which is denoted by ${}^{P_0}s_a$, can be derived from the hazard curve. As shown in the figure, ${}^{P_0}s_a$ is the spectral acceleration with a mean annual frequency of exceedance (hazard) equal to, P_0.

2.2.4.2 Fragility curves

The structural fragility for a specified limit state is defined as the conditional probability of exceeding the limit state capacity for a given level of ground motion intensity (conditional probability of failure in short). If the ground motion intensity is represented in terms of the spectral acceleration, the fragility can be expressed as:

$$F_{LS}(s_a) = P[S_a \geq S_{a,C} \mid S_a = s_a] = P[S_{a,C} \leq s_a] \quad (2.29)$$

where $F_{LS}(s_a)$ is the structural fragility at spectral acceleration s_a for limit state *LS*. It can be observed from the above equation that structural fragility is expressed as the probability that the random variable $S_{a,C}$ is less than or equal the given value, s_a. In other words, fragility is the cumulative distribution function of the random capacity, $S_{a,C}$. If it is assumed that the probability distribution of the spectral acceleration capacity, $S_{a,C}$, is lognormal with median, $\eta_{S_{a,C}}$, and standard deviation of the natural logarithm, $\beta_{S_{a,C}}$, fragility can be expressed in terms of the standardized Gaussian distribution function:

$$F_{LS}(s_a) = P[S_{a,C} \leq s_a] = \Phi(\ln(\frac{s_a}{\eta_{S_{a,C}}}) \Big/ \beta_{S_{a,C}}) \quad (2.30)$$

It can be observed from the above equation that structural fragility for limit state, LS, can be plotted as a function of spectral acceleration. For a certain limit state, a monotonically increasing “fragility curve” can be plotted. A schematic fragility curve is shown in Figure 2.7. According to Equation 2.30:

$$F_{LS}(\eta_{S_{a,C}}) = \Phi(\ln(\frac{\eta_{S_{a,C}}}{\eta_{S_{a,C}}}) \Big/ \beta_{S_{a,C}}) = \Phi(0) = 0.50$$

Therefore, median spectral acceleration capacity $S_{a,C}$ is marked on the figure as the spectral acceleration corresponding to a fragility of 50%. Also according to Equation 2.30:

$$F_{LS}(\eta_{S_{a,C}} \cdot e^{-\beta_{S_{a,C}}}) = \Phi(\ln(\frac{\eta_{S_{a,C}} \cdot e^{-\beta_{S_{a,C}}}}{\eta_{S_{a,C}}}) \Big/ \beta_{S_{a,C}}) = \Phi(\text{-}1) = 0.16$$

$$\beta_{S_{a,C}} = -\ln \frac{\eta_{S_{a,C}} \cdot e^{-\beta_{S_{a,C}}}}{\eta_{S_{a,C}}} = \ln F_{LS}{}^{-1}(0.50) - \ln F_{LS}{}^{-1}(0.16)$$

Therefore, the standard deviation of the (natural) logarithm of $S_{a,C}$ is marked on the figure as the difference between the spectral accelerations (on the logarithmic paper) corresponding to fragility values 16% and 50%.

The fragility curve for a specific limit state represents the probabilistic characteristics of structural resistance or capacity for that limit state. Once the fragility curve is available for a limit state, the “factored capacity” according to the fragility/hazard format, $\eta_{S_{a,C}} \cdot \exp(-\frac{1}{2} \cdot k \cdot \beta^2{}_{S_{a,C}})$, can be calculated based on the parameter estimates for k, $\eta_{S_{a,C}}$, and $\beta_{S_{a,C}}$, obtained from the hazard and fragility curves (Fig. 2.7).

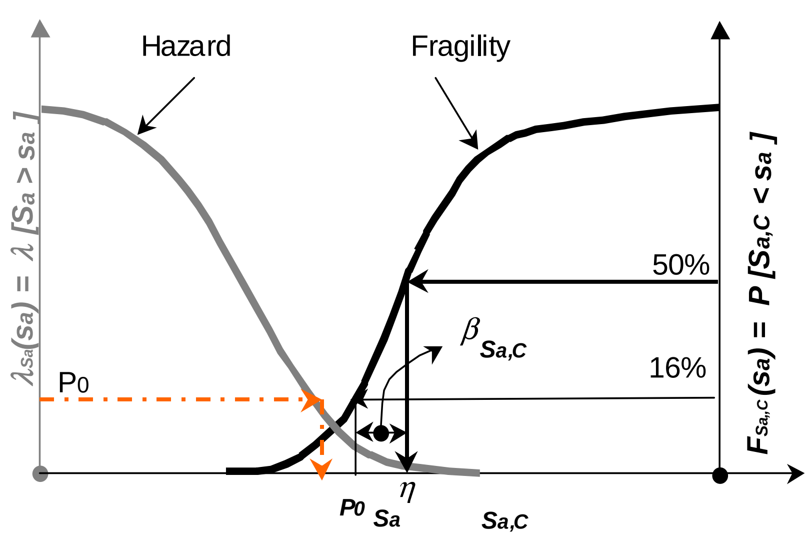

Fig. 2.7 A schematic plot of hazard and fragility curves. The basic parameters of hazard/fragility format are also shown.

2.2.4.3 The IM-based limit state frequency in terms of the fragility and hazard functions

It was demonstrated in the previous sections that fragility and hazard curves are helpful graphic tools for estimating the *IM*-based factored demand and capacity. Moreover, it will be shown in this section that fragility and hazard curves can also be used in order to calculate the limit state frequency. The *IM*-based limit state frequency can be derived from the following equation (Equation 1.39):

$$H_{LS,S_a-based} = \int P[x \geq S_{a,C}] \cdot \left| dH_{S_a}(x) \right| \tag{2.31}$$

where the first term in the integrand is nothing but the fragility $F_{LS}(s_a)$ at a spectral acceleration equal to s_a from Equation 2.29. Therefore, the limit state frequency in Equation 2.31 can also be written as:

$$H_{LS,S_a-based} = \int F_{LS}(x) \cdot \left| dH_{S_a}(x) \right| \tag{2.32}$$

where the *IM*-based limit state frequency is derived in terms of fragility and hazard. This equation states that the mean annual frequency of exceeding a limit state can be calculated as the area under the product of the structural fragility curve for that limit state multiplied by the (absolute value of) the increment in the spectral acceleration hazard.

2.2.4.4 Numerical example: Performance evaluation for an existing building

In the numerical example presented earlier for the DCFD format, we presented an assessment of the performance of an existing three-story frame for the collapse limit state for an allowable probability of $P_0 = 4\times10^{-4}$ (2% in 50 years). Here, we are going to use the same example in order to make probabilistic assessments based on the fragility/hazard format. Based on the fragility/hazard design format, we are going to calculate the *IM*-based factored demand for the allowable probability $P_0 = 4\times10^{-4}$, and then compare it to the factored capacity for the collapse limit state.

Factored demand: The *IM*-based factored demand for a given probability $P_0 = 4\times10^{-4}$ is equal to ${}^{P_0=0.0004}s_a$ (Equation 2.28). Back in Section 2.2.3.7, the spectral acceleration with $P_0 = 4\times10^{-4}$

frequency of exceedance was found to be equal to ${}^{P_0=0.0004}s_a = 1.45[g]$. Therefore the factored demand for an allowable probability $P_0 = 4\times10^{-4}$ is equal to:

$$F.D.(P_0) = {}^{P_0=0.0004}s_a = 1.45\,[g]$$

Factored capacity: In order to calculate the factored capacity for fragility/hazard format, we assume that the (uncertain) spectral acceleration capacity has median and (fractional) standard deviation equal to:

$$\eta_{S_{a,C}} \cong 2.15[g]$$

$$\beta_{S_{a,C}} \cong 0.20$$

Recalling from the previous numerical example for DCFD format, the slope parameter k for spectral acceleration hazard curve is equal to 3 (Fig. 2.5). Now that we have obtained the parameter estimates for estimates for k, $\eta_{S_{a,C}}$, and $\beta_{S_{a,C}}$, we can calculate the factored capacity:

$$F.C. = \eta_{S_{a,C}} \cdot \exp(-\frac{1}{2}\cdot k \cdot \beta^2{}_{S_{a,C}}) = 2.15.\exp(-\frac{1}{2}\times3\times0.2^2) = 2.15x0.94 = 2.0[g]$$

Comparing the factored demand for allowable probability, $P_0 = 4\times10^{-4}$, with the factored capacity, we can observe that:

$$F.D.(P_0) = 1.45[g] \le F.C. = 2\ [g]$$

Hence, we can conclude that the fragility/hazard design criterion is satisfied for an allowable annual probability of $P_0 = 4\times10^{-4}$ (i.e., 10% in 50 years) for the global collapse limit state. However, it should be noted that the parameter estimates used in this section for the spectral acceleration capacity are only for the sake of demonstration. In practical applications, nonlinear dynamic analysis procedures (Vamvatsikos and Cornell 2001) can be implemented in order to build the structural fragility curve(s). Then, the factored capacity can be calculated using the structural fragility curve as it is shown in Section 2.2.4.2. A numerical application of this format is demonstrated in Cordova et al. (2000), in which a design format similar to the one in Equation 2.28 is employed for seismic assessment of a composite moment frame for global collapse limit state.

2.3 RANDOMNESS AND UNCERTAINTY, THE SOURCES OF UNCERTAINTY

The design/assessment formats introduced in Section 2.2 only considered the randomness (or aleatory) type of uncertainty in demand and capacity parameter estimations. This type of uncertainty results in record-to-record variability in demand and capacity estimations. However, it is of interest to include the uncertainty due to incomplete knowledge (epistemic uncertainty) in the estimation of spectral acceleration hazard, demand, and capacity. As seen in Chapter 1, consideration of the uncertainty due to incomplete knowledge affects the mean estimate of the limit state frequency and/or the confidence statements that can be made about the bounds on estimates of the limit state frequency. Therefore, it is desirable to measure the epistemic uncertainty involved in the estimation of the parameters, and also to represent such uncertainty in the design or the assessments. One way to do this is to simply replace H_{LS} in the previous section (Section 2.2) everywhere by its mean estimate, $\overline{H}_{LS}$. As per Equation 1.86 in Chapter 1:

$$\overline{H}_{LS} = \nu \cdot \overline{P}[D > C] = \overline{H}_{S_a}(S_a^{\eta_C}) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot(\beta^2{}_{RD}+\beta^2{}_{UD})} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\cdot(\beta_{RC}^2+\beta_{UC}^2)} \tag{2.33}$$

In which $\overline{H}_{S_a}$ is the mean estimate of the hazard curve, $\beta_{RC}{}^2 + \beta_{UC}{}^2$ and $\beta_{RD}{}^2 + \beta_{UD}{}^2$ are the total aleatory and epistemic uncertainty variances in demand and capacity, respectively. It is then clear when comparing this to Equation 2.1 that both of the DCFD formats introduced in Section 2.2 can be "upgraded" to include epistemic uncertainty by simply replacing H_{LS} by its mean estimate, $\overline{H}_{LS}$, and the aleatory uncertainty variances, β^2, for demand given spectral acceleration ($D \mid S_a$) and capacity by their total β^2's, i.e., the total aleatory and epistemic variances. For example, Equation 2.2 transforms into the following:

$$\eta_{D|^{P_0}s_a} \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot(\beta_{RD|S_a}^2+\beta_{UD|S_a}^2)} = \eta_C \cdot e^{-\frac{1}{2}\frac{k}{b}\cdot(\beta_{RC}^2+\beta_{UC}^2)} \tag{2.34}$$

where it is understood that the spectral acceleration ${}^{P_0}s_a$ is obtained from the mean estimate of the hazard curve at P_0. Therefore, it is implied that the allowable limit state frequency corresponds to the mean estimate of the limit state frequency. While not precisely in this format, DOE 1020 is based on using such a mean estimate approach with combined or total (aleatory plus epistemic) variances used for the demand and capacity.

In this section we chose to outline a hybrid scheme. The DCFD format is extended to account for the epistemic uncertainty in demand and capacity parameter estimations. This is achieved by associating a level of confidence with the frequency of exceeding the limit state. This format has recently been implemented for performance evaluation of existing steel moment-resisting structures in FEMA 351.

The DCFD format presented in this section focuses mainly on the consideration of epistemic uncertainty in the structural demand and capacity parameter estimations. However, it (implicitly) takes into account the epistemic uncertainty in the seismic hazard estimations by incorporating the "mean" estimate for the hazard instead of the "median" estimate.

2.3.1 A Confidence-Based DCFD Format

In Chapter 1, the mean annual frequency of exceeding a limit state was derived by taking into account the uncertainty due to both aleatory and epistemic uncertainty. In such derivations, the limit state frequency was an uncertain quantity and could assume a range of possible values represented by a central value (median) and a dispersion measure (standard deviation of the natural logarithm):

$$\hat{H}_{LS} = \hat{H}_{S_a}(S_a^{\eta_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RD}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RC}}$$

$$\beta_{H_{LS}} = \sqrt{\beta_{UH}{}^2 + \frac{k^2}{b^2}\beta_{UD}{}^2 + \frac{k^2}{b^2}\beta_{UC}{}^2} \qquad (2.35)$$

where $\hat{H}_{LS}$ is the median estimate of the limit state frequency and $\hat{H}_{S_a}(.)$ is the median estimate of the spectral acceleration hazard. $\beta_{H_{LS}}$ is the dispersion measure (standard deviation of the natural logarithm) for the limit state frequency; it contains the epistemic uncertainty-related dispersion terms for hazard, demand, and capacity. We note that these could be used to develop one or more DCFD formats that treat the epistemic uncertainty in hazard, demand, and capacity in a more uniform manner. Here, however, we chose to develop the hybrid scheme introduced above. Suppose we assume that there is no epistemic uncertainty in the estimation of the median spectral acceleration hazard (i.e., $\beta_{UH} = 0$), the dispersion term $\beta_{\lambda_{LS}}$ in Equation 2.35 would be simplified to:

$$\beta_{H_{LS}} = \frac{k}{b}\sqrt{\beta_{UD}{}^{2} + \beta_{UC}{}^{2}} = \frac{k}{b}\beta_{UT} \tag{2.36}$$

where β_{UT} is the dispersion parameter representing the total epistemic uncertainty in displacement-based demand and capacity. In order to account for the epistemic uncertainty in the estimation of hazard, we substitute the "median" estimate of the spectral acceleration hazard, $\hat{H}_{LS}$, in Equation 2.35 by the "mean" estimate of the spectral acceleration hazard, $\hat{H}_{S_a} \cdot \exp(\frac{1}{2}\beta_{UH}^{2})$. The resulting median estimate for the limit state frequency is denoted by "$\hat{H}_{LS}$" (in order to distinguish it from the median hazard $\hat{H}_{LS}$ in Equation 2.35):

$$"\hat{H}_{LS}" = \overline{H}_{S_a}(S_a^{\eta_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RD}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RC}} = \overline{H}_{S_a}(S_a^{\eta_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RT}} \tag{2.37}$$

where the bar "–" represents the mean estimate; and (parallel to β_{UT}) β_{RT} is the dispersion parameter representing total aleatory uncertainty in displacement-based demand and capacity parameters. It should be noted that the hazard curves provided by the seismologists are usually in terms of the mean estimates of the annual frequency of exceedance, or "mean hazard" in short.

Now, we can build a confidence interval around the "median" estimate for the limit state frequency reflecting the epistemic uncertainty in the estimation of the demand (given S_a) and capacity parameters. The limit state frequency corresponding to the confidence level, *x*, denoted by, H_{LS}^{x}, can be expressed as:

$$H_{LS}^{x} = "\hat{H}_{LS}" \cdot e^{K_x \cdot \beta_{H_{LS}}} \tag{2.38}$$

where K_x is the standard Gaussian variate associated with the probability x of <u>not</u> being exceeded. Values for K_x are tabulated in standard probability tables under the normal distribution as a function of the number of standard deviations above or below the mean. Substituting the "median" estimate for the frequency of exceeding the limit state from Equation 2.37 into Equation 2.38, one obtains the upper %*x* confidence limit H_{LS}^{x} of the limit state frequency:

$$H_{LS}^{x} = \overline{H}_{S_a}(S_a^{\eta_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RT}} \cdot e^{K_x \cdot \beta_{H_{LS}}} \tag{2.39}$$

Recalling from Chapter 1 and earlier sections in this chapter the mean annual frequency of exceeding (hazard), the spectral acceleration corresponding to median displacement-based capacity can be calculated from the following power-law expression:

$$\overline{H}_{S_a}(S_a^{\eta_C}) = k_0 \cdot (S_a^{\eta_C})^k = k_0 \cdot (\frac{\eta_C}{a})^{-\frac{k}{b}} \qquad (2.40)$$

Clearly this result represents a theoretically inconsistent treatment of the total epistemic uncertainty, since the uncertainty in hazard, β_H, is incorporated in $\overline{H}$, while the uncertainty in capacity and demand (given S_a) is represented via the confidence factor $e^{K_x \cdot \beta_{H_{LS}}}$. The main objective of this hybrid formulation of DCFD format is to focus on structural epistemic uncertainties. More precisely, one should say that this represents an *x%* confidence limit on H_{LS} given the mean hazard curve. After substituting the estimate for $\overline{H}_{S_a}(S_a^{\eta_C})$ from Equation 2.40 into Equation 2.39, H_{LS}^x or the limit state frequency corresponding to the confidence level *x* is derived as:

$$H_{LS}^x = k_0 \cdot \left(\frac{\eta_C}{a}\right)^{-\frac{k}{b}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RT}} \cdot e^{K_x \cdot \beta_{\lambda_{LS}}} \qquad (2.41)$$

Parallel to the derivation of the DCFD format in Section 2.2.3 (Equation 2.6), the design criterion can be tested by comparing the limit state frequency H_{LS}^x corresponding to a confidence level *x* to an allowable probability, P_0:

$$H_{LS}^x \leq P_0 \qquad (2.42)$$

Substituting the expression for H_{LS}^x from Equation 2.41 into the design criterion in Equation 2.42 above:

$$k_0 \cdot \left(\frac{\eta_C}{a}\right)^{-\frac{k}{b}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RT}} \cdot e^{K_x \cdot \beta_{\lambda_{LS}}} \leq \mathrm{P}_0 \qquad (2.43)$$

Similar to the derivation of the DCFD format in Section 2.2.3, we make some re-arrangements in the above inequality mainly in order to separate the demand and capacity sides:

$$\eta_C \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2{}_{RT}} \cdot e^{-K_x \cdot \frac{b}{k}\beta_{\lambda_{LS}}} \geq a \cdot \left(\frac{P_0}{k_0}\right)^{-\frac{b}{k}} \tag{2.44}$$

Recalling the relationship between the dispersion parameter $\beta_{H_{LS}}$ for the limit state frequency and β_{UT} that measures total epistemic uncertainty in Equation 2.36:

$$\beta_{UT} = \frac{b}{k}\beta_{H_{LS}} \tag{2.45}$$

Replacing the above in Equation 2.44, it is simplified to:

$$\eta_C \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2{}_{RT}} \cdot e^{-K_x \cdot \beta_{UT}} \geq a \cdot \left(\frac{P_0}{k_0}\right)^{-\frac{b}{k}} \tag{2.46}$$

Now, we multiply both sides of the inequality in $\exp(-\frac{1}{2}\frac{k}{b}\beta^2{}_{UT})$ (in order to make it look similar to DCFD):

$$\eta_C \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2{}_{RT}} \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2{}_{UT}} \cdot e^{-K_x \cdot \beta_{UT}} \geq a \cdot \left(\frac{P_0}{k_0}\right)^{-\frac{b}{k}} \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2{}_{UT}} \tag{2.47}$$

After further simplifications noting that: $\eta_{D|S_a}({}^{P_0}s_a) = a \cdot \left(\frac{P_0}{k_0}\right)^{-\frac{b}{k}}$, and also breaking up the total variance terms into the corresponding demand and capacity variance terms:

$$\eta_C \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2{}_{RC}} \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2{}_{UC}} \geq \eta_{D|S_a}({}^{P_0}s_a) \cdot e^{\frac{1}{2}\frac{k}{b}\beta^2{}_{RD}} \cdot e^{\frac{1}{2}\frac{k}{b}\beta^2{}_{UD}} \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2{}_{UT}} e^{K_x \cdot \beta_{UT}} \tag{2.48}$$

Now, we can define the demand and capacity factors as:

$$\begin{aligned} \gamma = \gamma_R \cdot \gamma_U &= e^{\frac{1}{2}\frac{k}{b}\beta^2_{RD}} \cdot e^{\frac{1}{2}\frac{k}{b}\beta^2_{UD}} \\ \phi = \phi_R \cdot \phi_U &= e^{-\frac{1}{2}\frac{k}{b}\beta^2_{RC}} \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2_{UC}} \end{aligned} \tag{2.49}$$

Also the confidence factor corresponding to the confidence level, x, denoted by λ_x is defined as:

$$\lambda_x = e^{-\beta_{UT}(K_x - \frac{1}{2}\frac{k}{b}\beta_{UT})} \tag{2.50}$$

After demand, capacity, and confidence factors are substituted from Equations 2-49 and 2-50 into Equation 2.48:

$$\eta_{D|S_a}({}^{P_0}s_a) \cdot \gamma \leq \eta_C \cdot \phi \cdot \lambda_x \tag{2.51}$$

or:

$$\frac{\eta_{D|S_a}({}^{P_0}s_a) \cdot \gamma}{\eta_C \cdot \phi} \leq \lambda_x \tag{2.52}$$

This is the final form for the DCFD format that takes into account both aleatory and epistemic uncertainty. This looks very similar to Equation 2.12 for the DCFD format considering only the aleatory uncertainty, except for the confidence factor λ_x and also that the demand and capacity factors in Equation 2.51 also include the effect of epistemic uncertainty. As mentioned before, this format is implemented in FEMA 351 for the performance evaluation of existing steel moment-resisting structures. If Equation 2.51 is satisfied, one can say that the probability of failure is less than P_0 with confidence *x%.*

It is also interesting to re-arrange Equation 2.51 in terms of the ratio of the factored demand to factored capacity related to aleatory uncertainty (according to Equation 2.49):

$$\frac{F.D.(P_0)}{F.C.} = \frac{\eta_{D|S_a}({}^{P_0}s_a) \cdot \gamma_R}{\eta_C \cdot \phi_R} \leq e^{-K_X \cdot \beta_{UT}} \tag{2.53}$$

or:

$$F.D.(P_0) \cdot e^{K_X \cdot \beta_{UT}} \leq F.C. \tag{2.54}$$

which is the equivalent design criterion for the DCFD format taking into account the epistemic uncertainty. It should be noted that the factored demand and capacity in Equation 2.54 take into account only the aleatory uncertainty and are identical to those of Section 2.2.3.

The design criterion in Equation 2.53 can also be implemented in order to assess the level of confidence in an existing design for an allowable probability, P_0, by following the steps outlined below:

1. Calculate the factored demand for an allowable probability P_0 and also the factored capacity from Equation 2.12, taking into account only the aleatory uncertainty.

2. Find the ratio of the calculated factored demand to factored capacity.
3. Estimate the dispersion measure $\beta_{UT} = \sqrt{\beta_{UD}^2 + \beta_{UC}^2}$ accounting for the total uncertainty in the estimation of demand and capacity factors; examples appear in DOE 1020, FEMA 351, etc.
4. Solve the equation

$$\frac{F.D.(P_0)}{F.C.} = e^{-K_X \cdot \beta_{UT}} \tag{2.55}$$

   in order to find the corresponding Gaussian variate K_x. Note that Equation 2.55 is a special case of the design criteria in Equation 2.54 that holds at the onset of the limit state.
5. Find the corresponding confidence level x for the existing design.

2.3.1.1 Numerical example

The procedure outlined above for finding the confidence level corresponding to an existing design can be applied to the numerical example in Section 2.2.3.7 where the factored demand for an allowable probability of $P_0 = 0.0004$ and factored capacity for the collapse limit state were calculated. The ratio of the factored demand to factored capacity is equal to:

$$\frac{F.D.(0.0004)}{F.C.} = \frac{0.0538}{0.0658} = 0.817$$

We have used the tables in the FEMA 351 guidelines in order to estimate β_{UT}. For a three-story (low-rise) structure, the tables recommend the value $\beta_{UD} = 0.15$ accounting for the uncertainty in the estimation of the displacement-based response using nonlinear dynamic procedures for the collapse limit state. Also the guidelines recommend the value $\beta_{UC} = 0.15$ associated with the uncertainty in the estimation of the global dynamic collapse capacity for a low-rise structure. Therefore, β_{UT} can be estimated as:

$$\beta_{UT} = (0.15^2 + 0.15^2)^{\frac{1}{2}} = 0.212$$

The next step is to calculate the Gaussian variate K_x from Equation 2.55:

$$\frac{F.D.(0.0004)}{F.C.} = 0.817 = e^{-K_X \cdot 0.212}$$

which implies that: $K_x = 0.953$. Hence, the corresponding confidence level for $K_x = 0.953$ can be found from a normal distribution table:

$$x = \Phi(0.953) = 0.83$$

Hence, we can conclude that the confidence associated with the existing design of this structure is 83%. Precisely the same conclusion will be reached following the SAC-like format based on the definitions in Equations 2.49 and 2.50 for demand, capacity, and, confidence factors. The factored demand and capacity would differ in value as would their ratio, but the numerical confidence calculated via 2.50 would be the same. Yun et al. (2002) have followed a similar procedure to the one outlined above in order to estimate the confidence of a nine-story building (with both pre-Northridge and post-Northridge designs) in satisfying collapse prevention and immediate occupancy performance levels according to FEMA 273 guidelines.

2.4 SUMMARY AND CONCLUSIONS

A probabilistic framework for the assessment of the performance of structures under seismic excitations was developed in the Chapter 1. Chapter 2 discusses several of the many possible alternative design and assessment formats that stem from this probabilistic framework. The design formats discussed can all be traced back to a general probabilistic design criterion, which is satisfied when the frequency of exceeding a certain limit state is less than or equal to an allowable probability, P_0. A design format usually offers equivalent displacement-based or spectral acceleration-based criteria parallel to the general design criterion. The advantage of these equivalent criteria is that they are expressed in terms of structural response parameters and hence the resulting format can be incorporated more easily into the existing design codes.

These formats can be categorized based on the types of uncertainty involved in parameter estimations. The first category takes into account the randomness, also known as the aleatory uncertainty, in the assessment of demand and capacity. The second category takes into account both the randomness (aleatory uncertainty) in the estimation of the demand and capacity and also the uncertainty due to limited knowledge (epistemic uncertainty) in the estimation of the hazard, demand, and, capacity parameters.

Within the first category, the demand and capacity factor design (DCFD) format was discussed. This format is a (displacement-based) design format, analogous to the LRFD procedures, that stems directly from the expression for the mean annual frequency that the displacement-based demand exceeding capacity for a certain limit state. The DCFD format is based on a displacement-based design criterion in which the factored (displacement-based) demand (representing "load") for the allowable probability P_0 should be less than or equal to the factored (displacement-based) capacity (representing "resistance") for a certain limit state. Another format discussed under the first category is an *IM*-based format known as the fragility/hazard format, in which the fragility curves represent the structural "resistance" and the hazard curves represent the seismic "load." This format is based on a design criterion in which the spectral acceleration for a hazard value (i.e., frequency of exceedance) equal to the allowable probability P_0, is less than or equal to the factored capacity expressed in spectral acceleration terms. Each fragility curve is specific to a its corresponding limit state and can be used in order to obtain parameter estimates for the calculation of the factored capacity. The fragility/hazard format has been implemented for the design and evaluation of energy facilities (e.g., nuclear power-plants) in the DOE 1020 guidelines and for offshore structures in ISO guidelines.

Within the second category of the design formats that also address the epistemic uncertainties, a more general form of the displacement-based DCFD format is discussed. This format associates a level of confidence with the estimated frequency of exceeding a limit state. This confidence level represents explicitly the epistemic uncertainties involved in the estimation of the demand and capacity parameters and implicitly (and approximately) the epistemic uncertainty in the hazard estimation. The displacement-based design criterion for this format is very similar to that of the DCFD considering only the aleatory type of uncertainty except for an additional factor that reflects the level of confidence in the estimation of the limit state frequency. The DCFD format can be used for both designing a building with a certain level of confidence and also determining the level of confidence associated with an existing design for an allowable limit state frequency. This format is implemented in the guidelines for the performance evaluation of existing and earthquake damaged buildings in FEMA 351.

As a final note, it should be mentioned that there are numerous ways to transform the probabilistic design criterion stated in Equation 2.6 into design criteria that are suitable for code implementation. This chapter discusses only the most commonly used of these formats.

Nevertheless, the fundamentals used for deriving these formats can be applied toward developing new design/assessment formats.

3 Summary and Conclusions

A closed-form analytic foundation for the design and assessment of structures under seismic loads was developed using basic probabilistic concepts (Chapter 1). This foundation forms the theoretical basis to alternative formats suitable for implementation in design and assessment guidelines (Chapter 2).

3.1 CHAPTER 1: A TECHNICAL FRAMEWORK FOR PROBABILITY-BASED DESIGN AND ASSESSMENT

A probabilistic foundation is developed based on simplifying assumptions. This results in an analytic closed-form expression for the mean annual frequency of exceeding specified structural performance levels or, more briefly, limit state frequency. The limit state frequency is derived by assuming that the parameters involved in the assessments have a stochastic nature, which is modeled by considering two different types of uncertainty. The first type identifies the more familiar "natural variability" in the parameters, and is referred to as "randomness" or, more precisely, the "aleatory uncertainty." The second type addresses limited knowledge and data, and is referred to as "uncertainty," or "epistemic uncertainty." This second kind of uncertainty can be reduced by acquiring more data (larger sample sizes) and/or by increasing the knowledge upon further research.

The derivation of the limit state frequency employs a probabilistic tool known as the "total probability theorem" (TPT) in order to decompose the derivations into smaller and less complex parts. Therefore, the process of evaluating the limit state frequency involves additional "interface" variables. Two alternative solution strategies for deriving the expression for limit state frequency are presented, namely, the displacement-based strategy and the ground motion intensity-based solution strategy. The displacement-based approach evaluates the limit state frequency as the frequency that a displacement-based demand variable exceeds the corresponding limit state capacity. The derivations in this case are performed in two steps: (1)

The first step evaluates the frequency that the displacement-based demand exceeds a given value by decomposing it with respect to the ground motion intensity level and then composes the results by integration over all possible intensity levels. This first step is done by employing the total probability theorem and an interface variable representing the ground motion intensity. This variable is referred to as the intensity measure (*IM)*. The assumptions made in this step of the derivation include approximating the frequency that the *IM* exceeds a certain level, also known as the "hazard" for the IM, by a power-law function, modeling the probability distribution of the displacement-based demand for a given level of ground motion intensity by a lognormal distribution, and assuming that this lognormal distribution is defined by a median (central value) that is itself a power-law function of the ground motion *IM* and a (log) standard deviation (dispersion measure) that is invariant with respect to the ground motion intensity. (2) The second and final step is to evaluate the frequency that the displacement-based demand exceeds capacity by decomposing it into (conditional) frequencies of exceeding given values for the limit state capacity and then composing these frequencies by integration over all possible values of capacity. In this step it is assumed that probabilistic distribution of the (displacement-based) capacity can be modeled with a lognormal distribution with constant median and standard deviation and also that capacity and demand are uncorrelated. The second or ground motion intensity-based approach evaluates the mean annual frequency that the *IM* variable exceeds the corresponding limit state capacity *IM* or more briefly the *IM* capacity for a specific limit state (also called "limit state frequency"). The derivation involves decomposing the limit state frequency into conditional limit state frequencies that the *IM* exceeds *IM* capacity for a given intensity measure and integrating the conditional limit state frequencies over all levels of ground motion intensity.

3.2 CHAPTER 2: PROBABILITY-BASED DEMAND AND CAPACITY FACTOR DESIGN (DCFD) FORMATS

The closed-form analytic expression(s) derived for the limit state frequency can be formed into alternative formats. These formats are alternative representations of the closed-form expression for the frequency of exceeding a certain limit state based on displacement-based or *IM*-based design/assessment criteria. These criteria, which are expressed in common structural engineering terms rather than the more abstract probabilistic ones, can be implemented in existing design and assessment procedures and guidelines.

Demand and capacity factor design (DCFD) represents a family of displacement-based design formats that are distinguished with regard to the types of uncertainties considered in the formulation of the limit state frequency. This format has already been implemented in FEMA 350 for the design of new steel moment-resisting frames, in FEMA 351 for the assessment of the existing steel moment-resisting frames, and in ISO guidelines for the design of offshore structures. The fragility/hazard format represents a ground motion intensity-based family of design formats, also capable of considering both types of uncertainty. The fragility/hazard format has a graphic representation based on fragility and hazard curves and has been implemented in various forms in the nuclear power plant PRAs and DOE 1020 seismic criteria (Kennedy and Short 1994). Consideration of the epistemic uncertainty in the development of these formats results in designing the structure with a certain degree of confidence or in assessing the level of confidence in the design of an existing structure for a given allowable probability level.

Appendix A: The Expected Value of Y^{α} Where Y is a Lognormal Random Variable:

Assume *lnY* is a normal random variable (i.e., *Y* is lognormal) with mean *m* and standard deviation σ. One can always write the following relationship for *Y* raised to a power, α:

$$Y^{\alpha} = e^{\alpha \ln Y}$$

lnY can be transformed into a standard normal variable *U*,

$$U = \frac{\ln Y - m}{\sigma} \tag{A.1}$$

for which, the standard normal probability density function (PDF) at *U=u* is equal to:

$$\phi(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} \tag{A.2}$$

Based on the linear relation between *lnY* and *U* (Equation A.1) and the standard normal PDF for *U* (Equation A.2), the PDF for normal random variable *lnY* at *lnY=x* can be obtained as:

$$f_{\ln Y}(x) = \frac{1}{\sigma}\phi(u) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}u^2} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{\ln Y - m}{\sigma})^2} \tag{A.3}$$

where *f(.)* denotes the PDF function. The expected value for a function *g(.)* of a continuous random variable *Z* can calculated as:

$$E[g(Z)] = \int_{-\infty}^{\infty} g(z) \cdot f_Z(z) \cdot dz \tag{A.4}$$

Therefore, the expected value of $g(Y) = Y^{\alpha}$ can be written as (using Equations A.2, A.3 and A.4):

$$E[Y^{\alpha}] = E[e^{\alpha \ln Y}] = \int_{-\infty}^{+\infty} e^{\alpha x} \cdot f_{\ln Y}(x) \cdot dx = \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\alpha x} \cdot e^{-\frac{1}{2}(\frac{x-m}{\sigma})^2} \cdot dx$$

After some algebraic operations, which involves adding and subtracting a few (necessary) square terms, the following equation is obtained:

$$E(e^{\alpha \ln Y}) = \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\alpha x} \cdot e^{\frac{(x-m)^2}{2\sigma^2}} dx = e^{m\alpha} \cdot \mathrm{e}^{\frac{1}{2}\alpha^2\sigma^2} \cdot \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-m-\alpha\sigma^2)^2}{2\sigma^2}} \cdot dx$$

We can recognize that the term inside the integral is nothing but the PDF for a normal variable with a mean equal to $m+\alpha\cdot\sigma^2$, and a standard deviation equal to, σ. Therefore, the resulting integral (from $-\infty$ to ∞) is equal to unity. Hence, the expected value of Y^α is simplified to the product of the following two terms:

$$E[Y^\alpha] = E[e^{\alpha \ln Y}] = e^{m\alpha}.\mathrm{e}^{\frac{1}{2}\alpha^2\sigma^2}$$

For a lognormal random variable, the mean of the logarithm of the variable is equal to the logarithm of the median of the variable (Benjamin and Cornell 1970):

$$\ln \eta_{\mathrm{Y}} = E[\ln Y]$$

where $\eta(.)$ denotes the median. Hence, for normal random variable *ln*Y with mean *m* and standard deviation σ, the expected values of Y^α can be written as:

$$E[Y^\alpha] = E[e^{\alpha \ln Y}] = e^{m\alpha} \cdot \mathrm{e}^{\frac{1}{2}\alpha^2\sigma^2} = e^{\alpha \ln \eta_{\mathrm{Y}}} \cdot \mathrm{e}^{\frac{1}{2}\alpha^2\sigma^2} = (\eta_{\mathrm{Y}})^\alpha \cdot \mathrm{e}^{\frac{1}{2}\alpha^2\sigma^2} \qquad \text{(A.5)}$$

Thus, the expected value of a lognormal random variable raised to a power α can be expressed as the product of the median value raised to the power times a *magnification factor*, which is an exponential function of the variance of *lnY* times $\frac{1}{2}\alpha^2$.

Appendix B: Statement of the Total Probability Theorem

Given a set of mutually exclusive and collectively exhaustive *events*, B_1, B_2, …, B_n, the probability $P[A]$ of another event A can always be expanded in terms of the following joint probabilities (Benjamin and Cornell 1970):

$$P[A] = P[B_1 \cap A] + P[B_2 \cap A] + \ldots + P[B_n \cap A] = \sum_{i}^{n} P[B_i \cap A] \quad \text{(B.1)}$$

Appendix C: Annual Frequency of Exceeding a Limit State — Demand and Capacity are Correlated

In Chapter 1 of this report, we derived a closed-form expression for the mean annual frequency of exceeding limit state capacity. However, the derivations were based on the assumption that the limit state capacity is not correlated with displacement-based demand. In this appendix we will derive the limit state frequency for a more general case in which demand and capacity are correlated. As it turns out, incorporating the correlation between demand and capacity in the formulations is simple and may be carried out by modifying the total fractional standard deviation in the closed-form expression for the limit state frequency. As in the report, here we will study the effect of correlation between demand and capacity in two categories, namely (a) when the aleatory part of the uncertainties (e.g., randomness due to record-to-record variability in demand and capacity) in demand and capacity are correlated and (b) the epistemic part of the uncertainties (e.g., due to imperfect knowledge in estimating structural model parameters) in demand and capacity are correlated. In fact, as mentioned in the appendix of a paper by the authors (Cornell et al. 2002), some deliberations on the correlation between demand and capacity in the FEMA/SAC project indicated significant correlation between the epistemic uncertainties in the estimation of the displacement-based demand at larger ground motion levels and (global) collapse capacity. We will discuss each of the above *a* and *b* cases separately without loss of generality, as they can be simply combined if needed.

C.1. CORRELATION BETWEEN ALEATORY UNCERTAINTIES IN DEMAND AND CAPACITY

The annual frequency of exceeding a limit state, H_{LS}, can be expressed as the limit state probability $P[D \geq C]$ times the occurrence rate parameter ν (Equation 1.28):

$$H_{LS} = \nu \cdot P_{LS} = \nu \cdot P[D \geq C] \tag{C.1}$$

In the report, we derived the limit state frequency in two steps: (1) by deriving the mean annual frequency of exceedance (MAF) for the displacement-based demand or the drift hazard and (2) by deriving the conditional probability that demand exceeds capacity for a given value of capacity and combining it with the MAF for the displacement-based demand in the first step. However, that approach is based on the assumption that demand and capacity are uncorrelated, and hence can be treated in two separate steps. In this appendix, we will derive the limit state frequency by conditioning the probability that demand exceeds capacity on spectral acceleration and then integrating it with respect to spectral acceleration:

$$H_{LS} = \nu \cdot P[D \geq C] = \nu \cdot \int_0^\infty P[D \geq C \mid S_a = x] \cdot f_{S_a}(x) \cdot dx = \int_0^\infty P[D \geq C \mid S_a = x] \cdot \mid dH_{S_a}(x) \mid \quad \text{(C.2)}$$

where $| dH_{S_a}(x) | = \nu \cdot dG_{S_a}(x) = \nu \cdot f_{S_a}(x) \cdot dx$, as it is explained in Section 1.4.7.3. The term $P[D \geq C \mid S_a = x]$ can be re-arranged as follows:

$$P[D \geq C \mid S_a = x] = P[\frac{D}{C} \geq 1 \mid S_a = x] = P[\frac{D(x)}{C} \geq 1] \quad \text{(C.3)}$$

It is assumed that displacement capacity is not correlated with spectral acceleration[1]. The term, $D(x)$, denotes the displacement-based demand for a given spectral acceleration value, *x*. Assuming that both conditional demand for a given spectral acceleration, $D(x)$, and capacity, C, are lognormal random variables, the ratio of the two variables, $D(x)/C$, is also a lognormal random variable with the following mean and standard deviation of the log:

$$mean(\ln \frac{D(x)}{C}) = \ln \eta_{\ln \frac{D|S_a}{C}} = mean(\ln D(x)) - mean(\ln C) = \ln \eta_{D|S_a}(x) - \ln \eta_C = \ln \frac{\eta_{D|S_a}(x)}{\eta_C} \quad \text{(a)}$$

$$\text{var}(\ln \frac{D(x)}{C}) = \text{var}(\ln D(x)) - \ln C) = \text{var}(\ln D(x)) - 2 \cdot \rho_{nD|S_a, \ln C} \cdot \sigma_{\ln D|S_a} \cdot \sigma_{\ln C} + \text{var}(\ln C)$$

$$= \beta_{\frac{D|S_a}{C}}{}^2 = \beta_{D|S_a}{}^2 - 2 \cdot \rho_{\ln D|S_a, \ln C} \cdot \beta_{D|S_a} \cdot \beta_C + \beta_C{}^2 \quad \text{(b)}$$

(C.4)

where we have made use of the fact that the mean (expected value) is a linear operator in order to expand the mean of the (log of) demand to capacity ratio (Equation C.4.a). We can observe that the mean of the logarithm of the demand to capacity ratio is equal to the logarithm of the ratio of

[1] This assumption makes the derivations more consistent with that described in the part 1 of the report.

the median demand (given spectral acceleration), $\eta_{D|S_a}(x)$, to median capacity, η_C. We arrived at this conclusion based on a property of a lognormal random variable, in which the mean of the logarithm is equal to the logarithm of the median. The variance term is also expanded into the sum of the variance of the log of demand for a given spectral acceleration, $\beta_{D|S_a}{}^2$, the variance of the log of capacity, $\beta_C{}^2$, and a correlation term that has the correlation factor between the (log) demand (given spectral acceleration) and (log) capacity, $\rho_{\ln D|S_a, \ln C}$. It should be noted that the fractional standard deviation, $\beta_{D|S_a}$, is assumed to be a constant and not a function of the spectral acceleration, which is one of the assumptions made in order to arrive at a closed-form solution for limit state frequency in Chapter 1. The standard deviation of the (log of) demand to capacity ratio is also a constant, if we assume that the fractional standard deviation in capacity and the correlation factor between (log) demand and capacity are constants and do not depend on the spectral acceleration level.

Having derived the statistical properties of the lognormal variable, $D(x)/C$, we can further expand the term $P[D \geq C \mid S_a = x]$ in Equation C.3:

$$P[D \geq C \mid S_a = x] = P[\frac{D(x)}{C} \geq 1] = P[\ln \frac{D(x)}{C} \geq 0] = 1 - \Phi(-\frac{\eta_{\ln \frac{D|S_a}{C}}}{\beta_{\frac{D|S_a}{C}}}) =$$

$$= 1 - \Phi(-\frac{\ln \frac{\eta_{D|S_a}(x)}{\eta_C}}{\beta_{\frac{D|S_a}{C}}}) = 1 - \Phi(-\frac{\ln a \cdot x^b - \ln \eta_C}{\beta_{\frac{D|S_a}{C}}}) = 1 - \Phi(\frac{\ln \frac{\eta_C}{a \cdot x^b}}{\beta_{\frac{D|S_a}{C}}}) \qquad \text{(C.5)}$$

where $\Phi(.)$ is the standard normal cumulative distribution function. We have also replaced $\eta_{D|S_a}(x)$ by $a \cdot x^b$ (Section 1.4.7). We can substitute the above expression for $P[D \geq C \mid S_a = x]$ in Equation C.2 in order to derive the limit state frequency:

$$H_{LS} = \nu \cdot P[D \geq C] = \int_0^\infty \{1 - \Phi(\frac{\ln \frac{\eta_C}{a \cdot x^b}}{\beta_{\frac{D|S_a}{C}}})\} \cdot |dH_{S_a}(x)| \qquad \text{(C.6)}$$

Comparing the above expression for the limit state frequency to that of the MAF of exceeding displacement-based demand value, d (i.e., drift hazard), in Equation 1.16, we can observe that the two expressions will be identical if η_C is replaced by d and $\beta_{D|S_a/C}$ is replaced by $\beta_{D|S_a}{}^2$. Therefore, we can use the resulting closed-form solution for the MAF of exceeding the demand value *d* in Equation 1.25 by replacing d with η_C and $\beta_{D|S_a}$ with $\beta_{D|S_a/C}$:

$$H_{LS} = H_{S_a}(S_a^{\eta_C}) \cdot \exp(\frac{1}{2} \cdot \frac{k^2}{b^2} \cdot \beta^2{}_{D|S_a/C}) \quad \text{(C.7)}$$

Replacing the expression for $\beta_{D|S_a/C}$ from C.4.b in the above equation:

$$\begin{aligned} H_{LS} &= H_{S_a}(S_a^{\eta_C}) \cdot \exp\{\frac{1}{2} \cdot \frac{k^2}{b^2} \cdot (\beta_{D|S_a}{}^2 - 2 \cdot \rho_{\ln D|S_a, \ln C} \cdot \beta_{D|S_a} \cdot \beta_C + \beta_C{}^2)\} \\ &= H_{S_a}(S_a^{\eta_C}) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta_{D|S_a}{}^2} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta_C{}^2} \cdot e^{-\frac{k^2}{b^2}\cdot\rho_{\ln D|S_a,\ln C}\cdot\beta_{D|S_a}\cdot\beta_C} \end{aligned} \quad \text{(C.8)}$$

which is the closed-form analytic solution for the limit state frequency taking into account the correlation between the aleatory uncertainties in demand (given spectral acceleration) and capacity. Comparing Equation C.8 to the closed expression for limit state frequency in Equation 1.38, we can observe that the two expressions are identical except for the exponential correlation term $\exp(-\frac{k^2}{b^2} \cdot \rho_{\ln D|S_a, \ln C} \cdot \beta_{D|S_a} \cdot \beta_C)$ appearing in Equation C.8. It can be argued that if the correlation factor is positive, the limit state frequency in Equation 1.38 overestimates the limit state frequency, whereas if the correlation factor is negative. the limit state frequency will be underestimated.

C.2. CORRELATION BETWEEN EPISTEMIC UNCERTAINTIES IN DEMAND AND CAPACITY

We will now base our derivation of the limit state frequency directly on the derivations outlined in Section 1.4.8.

The limit state frequency conditioned on the deviations due to epistemic uncertainty in spectral acceleration hazard, ε_{UH}, demand, ε_{UD}, and capacity, ε_{UC} can be calculated from Equation 1.82:

$$H_{LS|\varepsilon_{UH},\varepsilon_{UD},\varepsilon_{UC}} = k_0 \cdot \left(\frac{\hat{\eta}_C}{a}\right)^{\frac{-k}{b}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta_{RD}^2} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta_{RC}^2} \cdot \varepsilon_{UH} \cdot \varepsilon_{UD}^{\frac{k}{b}} \cdot \varepsilon_{UC}^{\frac{-k}{b}} \quad \text{(C.9)}$$

[2] It should be noted that we are using the assumption that η_C and $\beta_{D|S_a/C}$ are both constants (with respect to spectral acceleration) in order to make the above statement.

where $\hat{\eta}_C$ is the estimated median capacity. The above expression can be rewritten by treating the limit state frequency, spectral acceleration hazard, and the deviations in demand and capacity as random variables (Equation 1.83):

$$\tilde{H}_{LS} = \tilde{H}_{S_A}(S_a^{\hat{\eta}_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RD}} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{RC}} \cdot \tilde{\varepsilon}^{\frac{k}{b}}{}_{UD} \cdot \tilde{\varepsilon}^{\frac{-k}{b}}{}_{UC} \tag{C.10}$$

where

$$\tilde{H}_{S_A}(S_a^{\hat{\eta}_C}) = k_0 \cdot \left(\frac{\hat{\eta}_C}{a}\right)^{\frac{-k}{b}} \cdot \tilde{\varepsilon}_{UH}$$

where we have used the “tilde” symbol to distinguish the uncertain quantities from deterministic ones. We will now calculate the mean and standard deviation of the logarithm of limit state frequency from Equation C.10, taking into account the correlation between random variables $\tilde{\varepsilon}_{UD}$ and $\tilde{\varepsilon}_{UC}$ that represent epistemic uncertainty in demand and capacity. In doing so, we will use the statistical properties of $\tilde{\varepsilon}_{UD}$ and $\tilde{\varepsilon}_{UC}$ listed in Equations 1.60 and 1.77; we will also use the expressions for mean and standard deviation of two or more correlated random variables as in the previous section:

$$median(\tilde{H}_{LS}) = \tilde{H}_{S_A}(S_a^{\hat{\eta}_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RD}} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{RC}} \tag{C.11.a}$$

and,

$$\beta_{\tilde{H}_{LS}}{}^2 = \beta_{UH}{}^2 + \frac{k^2}{b^2}\beta_{UD}{}^2 - 2\cdot\frac{k}{b}\cdot\rho_{\ln\tilde{\varepsilon}_{UD}\cdot\ln\tilde{\varepsilon}_{UC}}\cdot\beta_{UD}\cdot\beta_{UC} + \frac{k^2}{b^2}\beta_{UC}{}^2 \tag{C.11.b}$$

where $\rho_{\ln\tilde{\varepsilon}_{UD}\cdot\ln\tilde{\varepsilon}_{UC}}$ denotes the correlation factor between the (log of) epistemic deviations in demand and capacity. The above expression is based on the assumption that neither of the epistemic deviations in demand and capacity is correlated with that of the spectral acceleration hazard. Hence the mean estimate for the limit state frequency can be derived as:

$$\overline{H}_{LS} = \overline{H}_{S_a}(s_a^{\hat{\eta}_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}(\beta^2{}_{RD}+\beta^2{}_{UD})} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot(\beta^2{}_{RC}+\beta^2{}_{UC})} \cdot e^{-\frac{k}{b}\cdot\rho_{\ln\tilde{\varepsilon}_{UD}\cdot\ln\tilde{\varepsilon}_{UC}}\cdot\beta_{UD}\cdot\beta_{UC}} \tag{C.12}$$

where $\overline{H}_{S_a}(s_a^{\hat{\eta}_C}) = \hat{H}_{S_A}(S_a^{\hat{\eta}_C}) \cdot e^{\frac{1}{2}\cdot\beta_{UH}{}^2}$ denotes the mean estimate of the spectral acceleration hazard. Comparing the above expression for the mean estimate of the limit state frequency in Equation 1.86, we will observe that they differ by the exponential correlation term, $\exp(-\frac{k^2}{b^2}\cdot\rho_{\ln\tilde{\varepsilon}_{UD}\cdot\ln\tilde{\varepsilon}_{UC}}\cdot\beta_{D|S_a}\cdot\beta_C)$. Similar to the previous section on aleatory uncertainties, the mean estimate for limit state frequency from Equation 1.85 will be underestimated if the epistemic uncertainties in demand and capacity are negatively correlated.

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