

# EXPERIMENTAL AND ANALYTICAL STUDIES OF FIXED-BASE AND SEISMICALLY ISOLATED LIQUID STORAGE TANKS

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## ABSTRACT

Experimental and analytical studies were conducted on seismically isolated and fixed-base liquid storage tanks. Shaking table tests were performed on a vertical cylindrical open-top steel tank 6ft in height and diameter, protected by four Triple Pendulum™ bearings. Extensive parameter studies were performed, including comparisons of fixed-base and isolated model response to simulated ground motions of varying magnitude, time scale, and combination of components. Tests were repeated with three different water levels and for an empty tank. The responses of principal interest were base shear, water sloshing dynamics, tank deformation, tank uplift, and acceleration amplification at the top tank rim. The results of the experimental program were used to perform the corresponding parameter studies and to validate tank mechanical analog models used in design of liquid storage tanks. Experiment results show significant reductions in base shear, tank deformation, tank uplift, and acceleration amplification for seismically isolated tests as compared to corresponding fixed-base tests. Water sloshing dynamics show an increase in amplitude and complexity for multi-component excitation.

## INTRODUCTION

Seismic isolation, a technology that already demonstrated its effectiveness in improving seismic performance of buildings and bridges, is now being applied more and more to liquid storage tanks. Both elastomeric and Friction Pendulum™ systems have been successfully implemented in numerous projects worldwide. However, little experimental work has been done to verify satisfactory earthquake performance of seismically isolated liquid storage tanks. Also, little experimental work has been done to validate the simplistic analytical models used to design traditional and seismically isolated liquid storage tanks. The primary objective of the present research project is experimental validation of simple numerical models used in the industry for seismic structural design of traditional and seismically isolated liquid storage tanks. It is common practice to estimate earthquake forces and moments in liquid storage tanks using either a one-degree-of-freedom mechanical analog model, commonly referred to as the Housner model, or one of its derivatives. The validity of the mechanical analog model for a non-isolated tank has been tested experimentally, but little experimental work has been done to test the validity of the mechanical analog model as applied to a seismically isolated tank. No shaking table experiments have ever been performed on a tank isolated with Triple Pendulum™ bearings. None of the shaking table tests performed on liquid storage tanks in the past have looked at the response to multi-component excitation. The present research project aims to fill that knowledge gap by performing single- and multi-component earthquake simulations on the shaking table comparing fixed-base and seismically isolated tank response. Response parameters of primary interest are

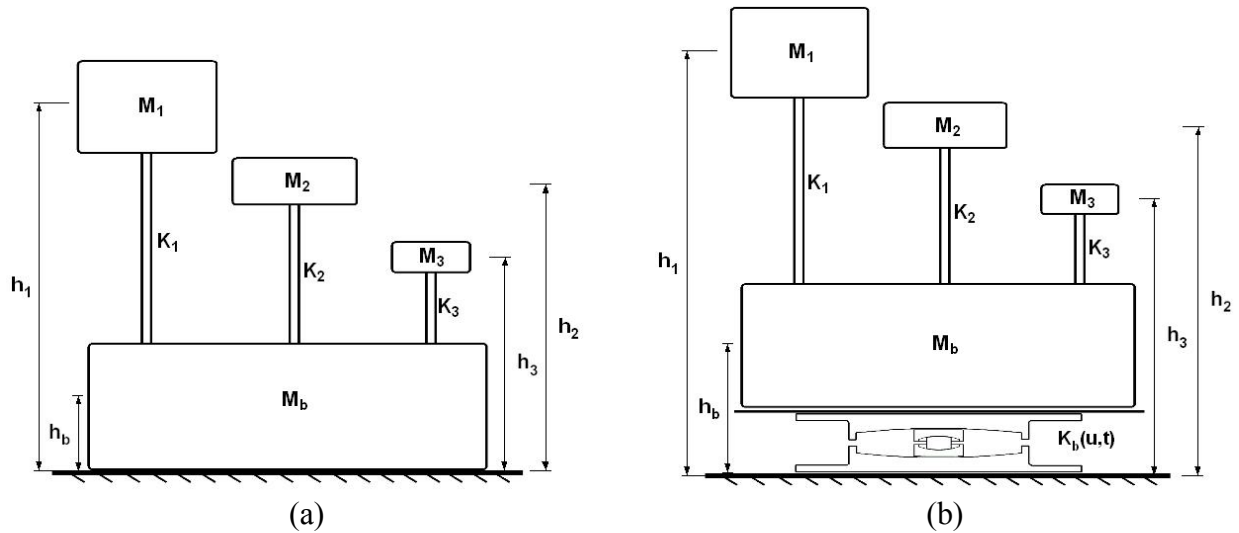
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base shear, tank uplift, tank top rim deformation, acceleration amplification at the top rim, as well as water surface dynamics.

## MECHANICAL ANALOG MODELS

It is common practice for structural engineers to estimate the seismic design values for liquid storage tanks using simple mechanical equivalent models. The fundamental model first presented by Graham and Rodriguez in 1952, but widely accepted only after the 1963 publication by H.W. Housner represents the liquid stored in the tank as oscillating masses producing forces and moments in the tank walls equivalent to those predicted by a full hydrodynamic analysis [Ibrahim, 2005]. It is theorized that the liquid volume can be separated into two bodies, one that participates in the open surface sloshing, called *convective*, and another, called *impulsive*, that moves uniformly with the tank as it undergoes lateral dynamic excitation. The Housner model assumes a rigid tank and takes into account only the first nonsymmetrical sloshing mode. Over the years more involved models have been proposed, including those accounting for the flexibility of the tank and the participation of higher sloshing modes in the response [Haroun, 1983; Veletsos, 1984; Wang, 2001]. It has been shown that for the purposes of estimating design stress demands on typical tanks, the inclusion of tank flexibility effects is not critical [Jaiswal, 2003].



**Fig. 1. (a) 3-mode, 1-D fixed-base and (b) 3-mode, 1-D seismically isolated mechanical analog models.**

The 3-mode, 1-D models used in the present study are shown in Fig. 1. Following the formulation by Veletsos [1984] with  $M_L$  being the total stored liquid mass, the key multi-mode model parameters are  $M_j = M_L \cdot \left[ \frac{2}{\lambda_j^2 - 1} \cdot \frac{R}{\lambda_j \cdot H} \cdot \tanh\left(\lambda_j \cdot \frac{H}{R}\right) \right]$  and  $\omega_j = \sqrt{\lambda_j \cdot \frac{g}{R} \cdot \tanh\left(\lambda_j \cdot \frac{H}{R}\right)}$ , where  $\lambda_j$  are the roots of  $dJ_1(\lambda_j \cdot r/R)/dr_{r=R} = 0$ , and  $J_1$  is the Bessel function of the first kind of the first order. The values of  $\lambda_1$  through  $\lambda_5$  are 1.841, 5.335, 8.535, 11.205, and 14.850.  $R$  is the tank radius and  $H$  is the water depth. Modal periods can be obtained by the simple relationship  $T_j = 2\pi/\omega_j$ . The impulsive mass  $M_0$  is most easily found by  $M_0 = M_L - \sum_{j=1}^n M_j$ ,

where  $n$  is the number of modes considered. The base mass  $M_b$  includes  $M_0$  and the mass of the tank itself. If a numerical time stepping algorithm is to be employed, the modal stiffnesses should be found simply by  $K_j = M_j \cdot \omega_j^2$ . Modal heights for mechanical analog model used to evaluate the equivalent overturning moment of lateral pressures on the tank wall can be calculated using  $h_j = H \cdot \left[ 1 - \frac{1}{\lambda_j} \cdot \frac{R}{H} \cdot \tanh\left(\frac{\lambda_j}{2} \cdot \frac{H}{R}\right) \right]$ .

For correct evaluation of the overturning moment at the base of the tank, the effect of the water pressure acting on the base plate of the tank must be included by using  $h'_j = h_j + \Delta h_j$  as the modal moment arm, where  $\Delta h_j = R/(\lambda_j \cdot \sinh(\lambda_j \cdot H/R))$ . The evaluation of  $h_b$  and  $\Delta h_b$  is much more computationally involved and the use of tabulated results is recommended [Veletsos, 1984]. Once the above parameters are established, conventional structural analysis is performed to find modal forces as  $M_i \cdot A_i$  and modal moments as  $M_i \cdot A_i \cdot h'_i$ . The square root of the sum of the squares (SRSS) combination rule can then be implemented to find the total response.

Also by [Veletsos, 1984], the water sloshing height along the tank wall is obtained by  $d_s(t) = R \cdot \left[ \sum_{j=1}^n \frac{2}{\lambda_j^2 - 1} \cdot \frac{A_j(t)}{g} \right]$ , where  $A_j(t)$  is the  $j^{\text{th}}$  sloshing mode spectral acceleration. The value of the maximum sloshing is estimated by the codes AWWA D-100 & D-103, NZSEE, and Eurocode 8 as  $d_{s,max} = 0.84 \cdot R \cdot A_1$ . The codes AWWA D-110 & D-115, ACI 350.3, and API 650 use the more conservative estimate of  $d_{s,max} = R \cdot A_1$  [Jaiswal, et. al, 2003].

## EXPERIMENTAL PROGRAM

An extensive shaking table test program was carried out to investigate the effectiveness of Triple Pendulum bearings in improving liquid storage tank seismic performance and to provide data for the calibration of the analytical models described in Chapter 2. A total of 264 tests were performed on the same specimen. Such a large number of tests were made possible by the non-destructive nature of testing a seismically isolated structure of sufficiently small weight, which limits the damage to the sliding surface lining. The testing of fixed-base cases also proved non-destructive, as the supporting base and the tank behaved elastically for all but a handful of full magnitude tests. During several full magnitude tests some slip in the connections of the supporting base was observed, but simple re-tightening the loosened connections allowed for the continuation with further tests. The possibility of conducting such a large number of tests on the same specimen allowed us to vary many of the parameters of interest, including the water depth, the time and magnitude scaling of the ground motion signal, base condition (fixed or isolated), and the number of simultaneously imposed ground motion components X, Y, Z, and their various combinations, such as X-Y, X-Z, and X-Y-Z.

### Test Specimen

An upright open-top cylindrical steel tank, 6ft in diameter and height, weighing 1600 lb, was chosen for the experiment, see Fig. 2. A single standard A36 carbon steel 1/4" thick plate was used for the wall of the tank, with a double square-groove weld at the joint. The top rim of the tank was reinforced with a 1/4" steel flat bar ring. The bottom of the tank was made with a single 6 1/2" diameter plate also 1/4" in thickness. A double fillet weld was used around the

circumference of the cylindrical wall to join with the bottom plate. The tank was positioned directly on the supporting base, the centers coinciding, and was not restrained against small horizontal or vertical movement. The supporting base for the tank consisted of two concrete blocks, each 17kips, 10'x10'x14" in dimension, atop a structural steel frame, 3kips. The weight of the concrete blocks was required to enforce stable sliding friction coefficients on the bearing sliding surfaces. The steel frame was used as a transition between the concrete blocks and the four short supports, each consisting of an FP bearing atop a load cell, with appropriate transition steel plates in between the individual components. The concrete blocks and the steel frame were clamped together by four 1-¼" steel rods passing through pre-made holes. Maximum allowable torque was applied to the nut at the end of each rod to ensure as tight a connection as possible.

The Triple Pendulum™ bearings used in this study were manufactured by Earthquake Protection Systems in Vallejo, California. The radius of the inner sliding surface was 3", and that of both outer sliding surfaces was 39". The maximum displacement capacity of the bearings, which is based just on the geometry of the bearings, was 9.66". The sliding friction coefficients were determined to be approximately 5% for the inner sliding surface, and approximately 11% for both of the outer sliding surfaces. Near-perfect horizontal and vertical alignment of the four bearings was achieved by assembling the lower part of the supporting base (from load cell to steel frame) in the upside-down position. When this first stage of construction was completed, the frame was flipped and brought over to the earthquake simulator platform. The construction of the specimen was completed after the specimen was secured to the table.



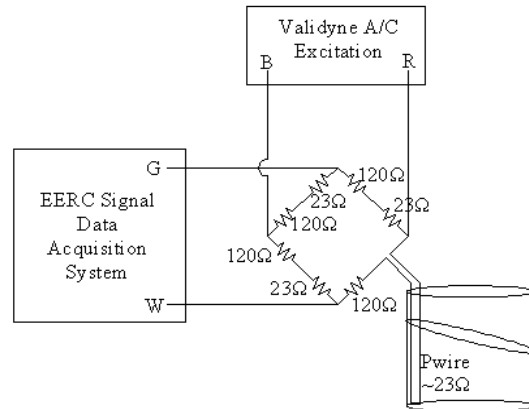
**Fig. 2. Fully instrumented test specimen on the shaking table.**

### **Instrumentation**

The test specimen was carefully instrumented to record the performance of the shaking table and the response of the water tank – supporting structure system. A total of 124 data channels were utilized. The instruments used, include: accelerometers, three kinds of displacement transducers, and parallel wire water depth gauges. 46 accelerometers with range  $\pm 5$  g's were installed

throughout the test specimen on the shaking table surface, the isolator plates, the steel frame, the top concrete block, and the top and bottom tank rims. In the majority of these locations the accelerometers were arranged to record all three components of acceleration. 18 wire pots with 40” range were installed on the laboratory reaction steel frame and targeted at the specimen steel frame, the top concrete block, and the top tank rim. For displacement measurements requiring higher precision, DCDT’s and Novotechnik Displacement Transducers were used for recording tank uplift, tank “walking” on the top concrete block, fixed-base support deformation, and tank top rim deformation. 12 channels were dedicated to the load cells, which recorded moments as well as axial and shear forces below each bearing.

The dynamic variations in the water surface during the tests were measured by 7 parallel wire (Pwire) gauges, which were developed especially for this experimental program. The fundamental principle of operation of the Pwire gauge is very similar to that of a full-bridge strain gauge. The change in the water level changes the resistance in the circuit, causing the bridge to become unbalanced. The signal is recorded, and the variations in the resistance of the circuit are calibrated to a change in water elevation to provide a water depth reading. A near-linear performance was achieved over a range of up to  $\pm 36$ ” with noise on the order of  $\pm 1$ ”. The calibration and performance of the Pwire gauges was verified by processing video footage from two HD cameras mounted on the top tank rim. A schematic representation of the implemented system is shown in Fig. 3. The design, calibration, and implementation of the Pwire gauges are described in much more detail a separate publication [Calugaru and Mahin, 2010].



**Fig. 3. Schematic representation of the parallel wire depth gauge used in the experiment.**

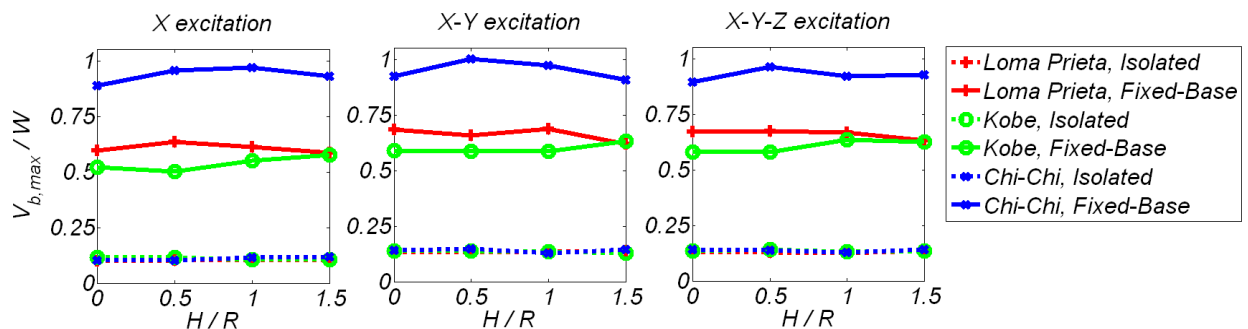
### **Facilities and Procedures for Shaking Table Tests**

The facility used in the present research project is the Earthquake Simulator Laboratory at the Pacific Earthquake Engineering Research Center (PEER) in Richmond, CA. The earthquake simulator (shaking table) platform measures 20ft x 20ft and is a 12” reinforced concrete slab stiffened by heavy transverse ribs below its bottom surface. Three translational (two horizontal, one vertical) and three rotational (roll, pitch, and yaw) degrees of freedom are provided by eight horizontal and four vertical actuators. Horizontal displacement of the shaking table has a  $\pm 5.0$ ” performance limit, while that for vertical displacement is  $\pm 2.0$ ”. Velocity limit of the actuators is about 30 in/sec and the acceleration limit is about 3g.

Three near-fault ground motions from the PEER NGA database were selected for the experiment. Kobe (Takatori record), Loma Prieta (Los Gatos record), and Chi-Chi (028) were the ground motion records selected. The primary guideline for the selection of particular ground motions was to obtain strong response for all three components, but to have a difference in relative magnitude of the ground motions at different time scales of simulation. The choice of ground motions was also guided by the limitations of the shaking table. Full magnitude tests were possible for length scale of 2 and higher for Chi-Chi, and length scale 3 and higher for Kobe and Loma Prieta. In the following discussion and figures, for all ground motions the stronger component is the X component. For Chi-Chi the North-South component is X and the East-West component is Y. For Loma Prieta and Kobe, the 000 component is X and the 090 component is Y.

## RESULTS AND ANALYSIS OF SHAKING TABLE TESTS

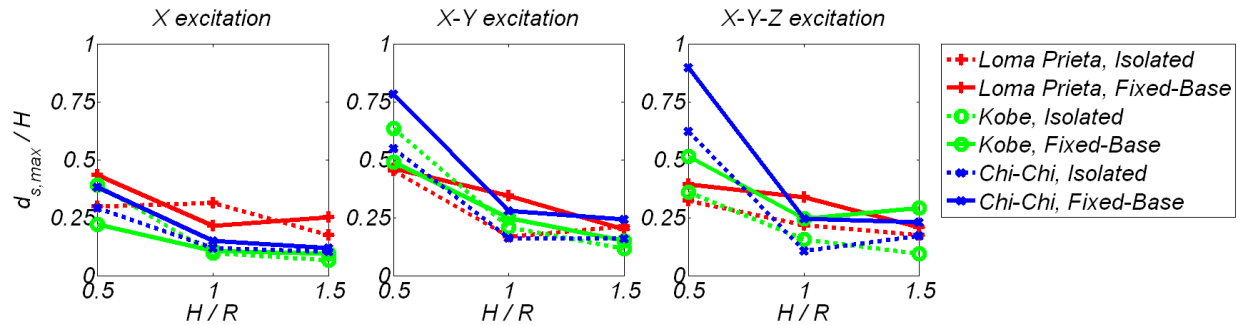
There is significant reduction in base shear for seismically isolated cases compared to fixed-base, see Fig. 4. This reduction is approximately by a factor of 5 for Kobe, a factor of 6 for Loma Prieta, and up to a factor of 9 for the Chi-Chi ground motion. The normalized base shear for seismically isolated cases is limited by 0.11, which is approximately the friction coefficient of the larger sliding surfaces for the given weight. Another interesting result is that the weight-normalized base shear is essentially independent of the H/R ratio, which is evident from the near-zero slope of the curves on the H/R range from 0 to 1.5. This indicates that there is a linear relationship between base shear and specimen weight. Comparing base shear results for single and two component excitation, there is an increase in normalized base shear response for fixed-base and isolated cases between the single X component excitation and two component X and Y excitation. This increase is by up to 0.1 for fixed-base cases for all three ground motions. Comparing base shear results for two and three component excitation, essentially no difference in maximum base shear is observed.



**Fig. 4. Maximum normalized base shear results for Loma Prieta, Kobe, and Chi-Chi ground motions; all length scale 4, half magnitude scale.**

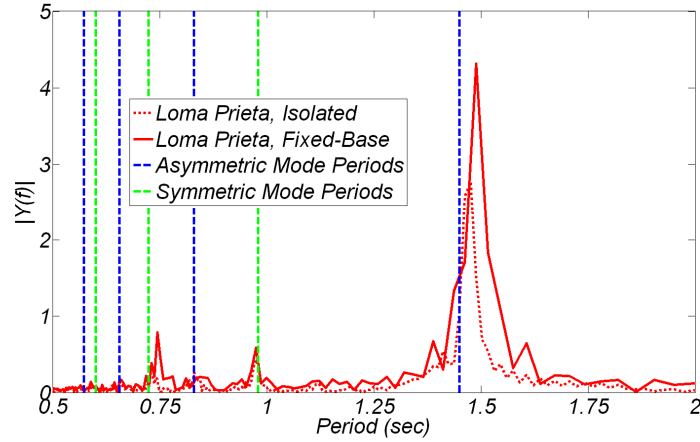
Fig. 5 depicts maximum water sloshing results for 72 tests with length scale 4, magnitude scale  $\frac{1}{2}$ , presented as a function of the H/R ratio. The plotted maximum water sloshing height is normalized by the corresponding water depth for each test. Water sloshing results do not appear to present trends as uniform as do the results for base shear. Results for the Chi-Chi earthquake show noticeable reduction in the maximum water sloshing height for the seismically isolated tests compared to the fixed-base tests, especially for tests with the multi-component excitation.

The negative slope of the curves indicates a reduction in maximum water sloshing height normalized by the corresponding water depth with increasing H/R ratio. This trend is also present for the two other ground motions, but with a few exceptions. This negative slope in the figures above suggests that the relationship between the water sloshing height and water depth is not linear, and the fact that the curves change slope between H/R=0.5 and H/R=1.5 suggests that the relationship between water sloshing height and water depth is not quadratic. Results for the Kobe and Loma Prieta ground motions don't show as clear a reduction in maximum sloshing height for the seismically isolated cases.



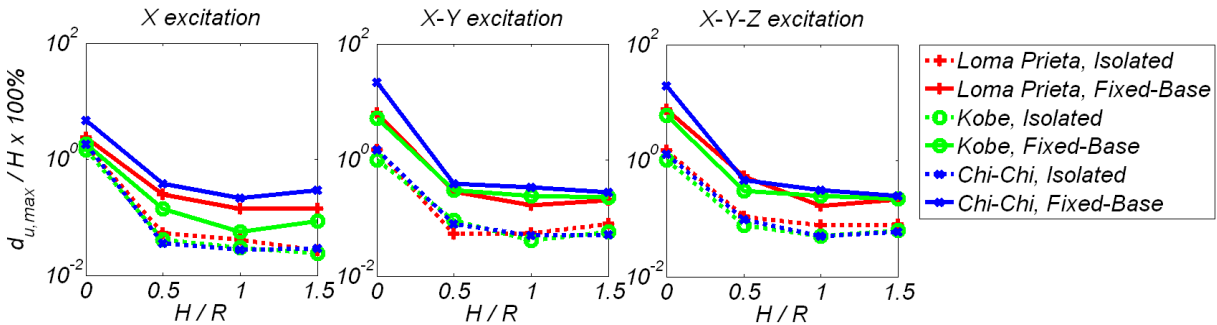
**Fig. 5. Maximum normalized water sloshing results for Loma Prieta, Kobe, and Chi-Chi ground motions; all length scale 4, half magnitude scale.**

A look at the Fast Fourier Transform (FFT) study of water sloshing height, presented in Fig. 6, leads to several interesting discoveries. Firstly, the most striking observation, is that the first mode sloshing, at around 1.45 seconds, is clearly carrying the most energy and is the most distinguishable for both fixed-base and isolated tank configuration tests. Secondly, notice that the symmetric sloshing modes make a strong appearance on the FFT plots, particularly the first and the second symmetric modes. The contribution of these modes to sloshing dynamics is often overlooked in engineering analysis. A third interesting discovery to be made, is that sloshing height results for one and multi-component excitation tests lead to very similar sloshing energy content distributions between the sloshing modes. Notice, also, that the tank fixity condition does not seem to affect the sloshing mode natural periods. Finally, notice that isolated tank tests FFT plot has an overall lower spectrum amplitude than the fixed-base tank tests. This seems to indicate that seismic isolation effectively limits the amount of ground motion energy passing to the body of liquid.



**Fig. 6. FFT study of water sloshing height for Loma Prieta X-Y-Z excitation, length scale 4, half magnitude scale; 36 inch water depth.**

Fig. 7 depicts maximum tank uplift results for 72 tests with length scale 4, half magnitude scale tests, presented as a function of the H/R ratio. For H/R = 0, tank uplift is normalized by 1. Significant reduction in tank uplift for the seismically isolated cases as compared to the fixed-base cases is evident. This reduction is especially obvious for the multi-component excitation test results. The magnitude of this reduction ranges between a factor of approximately 1.5 and 4. Normalized tank uplift decreases with increasing H/R ratio, especially between 0 and 18 inches of water depth. In the range H/R=0.5 to H/R=1.5 there is essentially no change in normalized tank uplift. This indicates that the relationship between tank uplift and water depth is nearly linear.

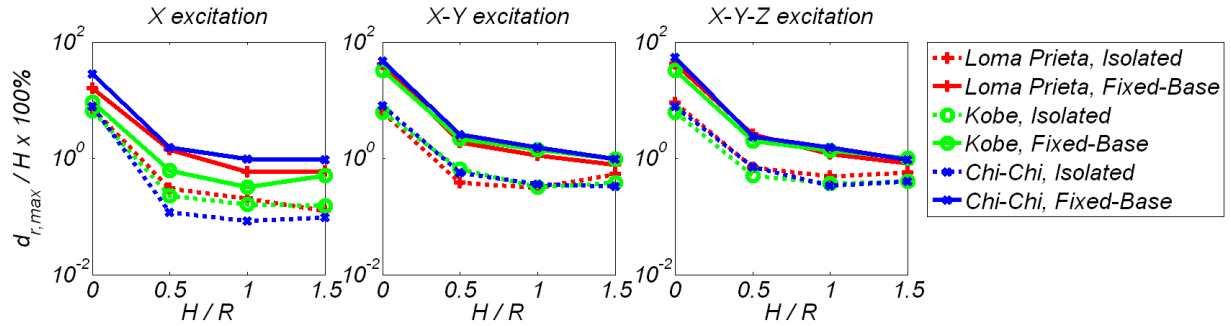


**Fig. 7. Maximum normalized tank uplift results for Loma Prieta, Kobe, and Chi-Chi ground motions; all length scale 4, half magnitude scale.**

Fig. 8 depicts maximum tank deformation results for 72 tests, as measured by 4 equally spaced DCDT's stretched diametrically across the top tank rim, with length scale 4, half magnitude scale tests, presented as a function of the H/R ratio. For H/R = 0, tank uplift is normalized by 1. The results for tank deformation follow some trends similar to the results for base shear, water sloshing, and tank uplift. Seismically isolated test cases show significant reduction in tank top rim deformation compared to fixed-base test cases for all three ground motions. The magnitude of this reduction varies with H/R ratio, and ranges between a factor of approximately 1.5 and 6. There is also significant increase in tank deformation for multi-component excitation as compared to single component excitation. The increase in top rim deformation with increasing

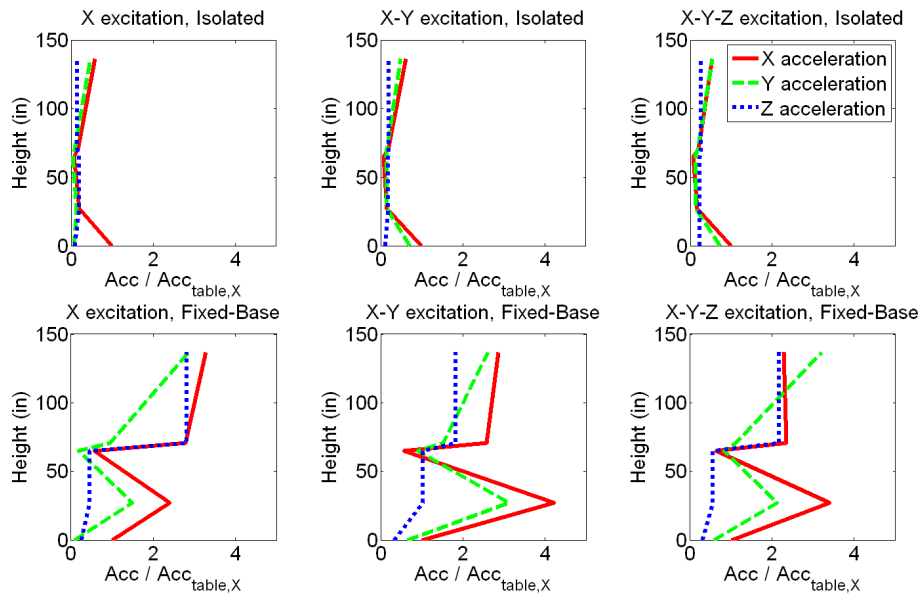


H/R ratio is not as pronounced as that of tank uplift; however the relationship between tank deformation and water depth is also nearly linear.



**Fig. 8. Maximum normalized tank uplift results for Loma Prieta, Kobe, and Chi-Chi ground motions; all length scale 4, half magnitude scale.**

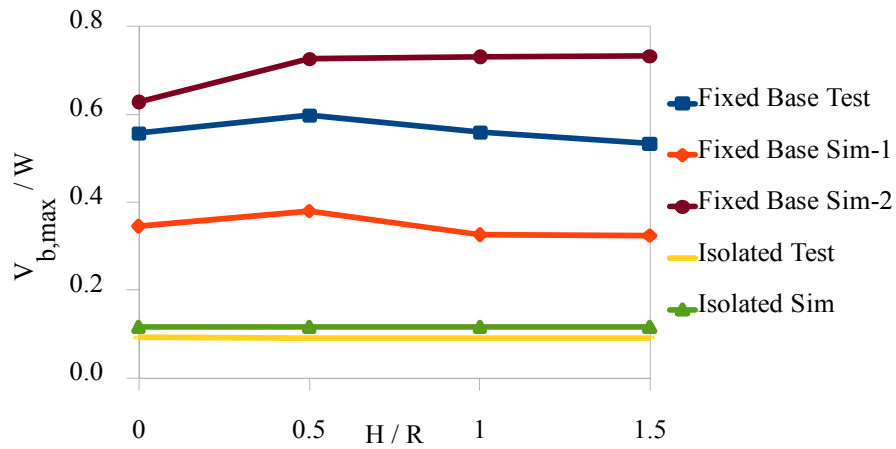
Fig. 9 depicts a sample acceleration amplification study performed with full magnitude scale, length scale 3, Chi-Chi ground motion simulations with fixed-base and isolated tank configurations. Each curve is normalized by the maximum table acceleration in the X direction, which is the maximum table acceleration for each test. Isolated tank configuration shows significant decreases in acceleration amplification through the tank shell. The peak acceleration at the tank top rim for the isolated tank configuration never exceeds the peak table acceleration, while for the fixed-base tank configuration the acceleration amplification factor can exceed 3. Notice that reduction of acceleration is achieved in the isolated tank configuration in all three accelerometer components and for all single- and multi-component excitation test cases. Interestingly, reduction in vertical (Z) acceleration is also achieved with isolation.



**Fig. 9. Maximum normalized acceleration envelopes using accelerometer data. Chi-Chi length scale 3, full magnitude scale.**

## VALIDATION OF MECHANICAL ANALOG MODELS

Numerical studies indicated that higher modes of sloshing do not contribute significantly to overall base shear, because even second mode mass is only approximately 3% of the first mode mass, while the accelerations of the first two modes are quite comparable for the ground motions considered, resulting in the second mode contributing only approximately 3% to total base shear. Fig. 10 compares the 1-mode model results for base shear with experimental data. The discrepancy between the model results for the two different acceleration inputs is due to the imperfect fixity of the fixed-base tank configuration, which introduced acceleration amplification between the surface of the table of the bottom of the tank. The two numerical simulation curves which use different acceleration input, in effect provide an envelope for the values obtained in the experiment. The isolated model base shear results compare well with test results, being only slightly conservative.



**Fig. 10. 1-mode, 1-D model validation for base shear; Loma Prieta, length scale 4, half magnitude scale; 36in water depth.**

Higher modes contribute more to the total sloshing behavior than to base shear. Interestingly, the maximum sloshing response in the numerical simulations and in the shaking table tests occurred only after the strongest earthquake pulses have acted. All other response parameters considered reached their maximum amplitude during the strong earthquake pulses. More discussion on 3-mode, 1-D and 2-D model validation can be found in [Calugaru and Mahin, 2010].

## SUMMARY AND CONCLUSIONS

Experimental results show significant reductions in base shear, tank uplift, tank deformation, and acceleration amplification in tank shell for the isolated as compared to fixed-base tank configuration. Similar reduction factors were achieved for these response parameters and ranged between 1.5 and 8. Base shear normalized by test specimen weight was found to be essentially constant for the full range of tested H/R ratios, indicating a nearly linear relationship between base shear and specimen weight. Tank uplift and deformation data suggest that these response parameters are linearly related to water depth. These response parameters taken directly without normalization showed near-linear increase with increasing H/R ratio. Significant increase in all

response parameters, including water sloshing height, was observed for multi-component excitation as compared to single-component excitation.

Water surface dynamics increased in complexity and non-linearity with increasing number of excitation components included in the simulation. Isolated test cases showed sloshing behavior most closely resembling simple first mode oscillation. Fixed-base test cases showed much more violent sloshing, with breaking waves and generally more chaotic dynamics. Unlike the other response parameters, water sloshing did not show a consistent decrease for isolated compared to fixed-base tank configuration tests. FFT studies of water sloshing height showed symmetric sloshing modes to be important contributors to water sloshing dynamics.

Mechanical analog model validation studies showed good comparison between experimental results for base shear and maximum water sloshing height and those predicted by simple numerical models. Inclusion of higher modes in the numerical models resulted in slightly better accuracy of prediction for water sloshing height, but practically no improvement for base shear. Studies of the basic 1-mode Housner's model numerically mounted on a TP bearing show that this approach provides reasonable approximations of base shear and water sloshing heights in the isolated system.

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