Regional Scale Simulation of Uncertain Response of Transportation Infrastructure Soil-Structure Systems

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Outline

Introduction

Uncertain Inelastic Dynamics

Summary

Infrastructure Analysis

- Material behavior is nonlinear, inelastic
- Soil and Structure work together
- Modeling, epistemic uncertainty, analysis sophistication
- Parametric, aleatory uncertainty
  - Uncertain material behavior, parameters
  - Uncertain loads

Numerical Prediction under Uncertainty

- Modeling, Epistemic Uncertainty

  Modeling Simplifications

  Model sophistication for confidence in results

- Parametric, Aleatory Uncertainty

  $M\ddot{u}_i + C\dot{u}_i + K^{ep}u_i = F(t)$,

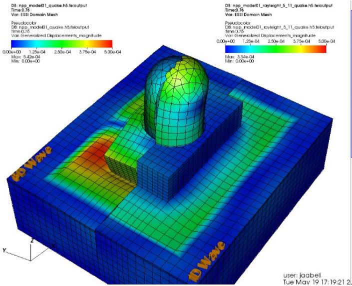
  Uncertain: mass M, viscous damping C and stiffness K^{ep}

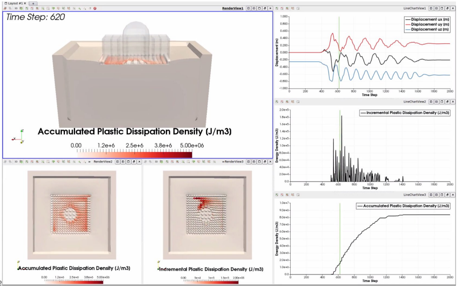
  Uncertain loads, $F(t)$

  Results are PDFs and CDFs for σ_{ij}, ϵ_{ij}, u_i, $\dot{u}_i$, $\ddot{u}_i$

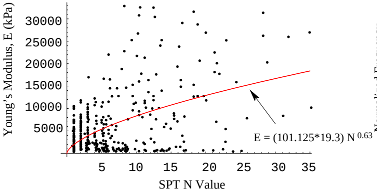
Modeling, Epistemic Uncertainty

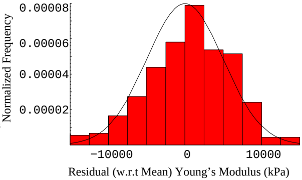
- Simplified modeling: 3D/2D/1D, 1C/2C/3C/6C; damping: viscous, elastic/el-pl, algorithmic
- Modeling simplifications are justifiable if one or two level higher sophistication model demonstrates that features being simplified out are not important (?!)



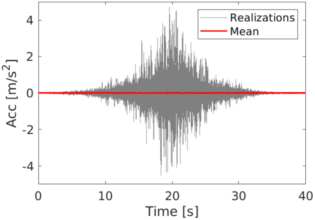


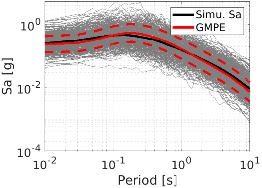
Parametric, Aleatory Uncertainty





(cf. Phoon and Kulhawy (1999B))





(cf. Wang et al. (2019))

Uncertainty Propagation

- Forward propagation of uncertainty, full probabilistic, nonlinear/inelastic Earthquake-Soil-Structure-Interaction, ESSI response in time domain (Jeremic et al 2011, Wang et al 2019)

- Backward propagation, sensitivity analysis, quantifies the relative importance of input uncertain parameters on the variance of the probabilistic system response (Sobol 2001, Sudret 2008, Jeremic et al 2021)

Forward Uncertain Inelasticity

- Incremental el–pl constitutive equation

$$\Delta\sigma_{ij} = E^{EP}_{ijkl}\ \Delta\epsilon_{kl} = \left[E^{el}_{ijkl} - \frac{E^{el}_{ijmn} m_{mn} n_{pq} E^{el}_{pqkl}}{n_{rs} E^{el}_{rstu} m_{tu} - \xi_* h_*} \right] \Delta\epsilon_{kl}$$

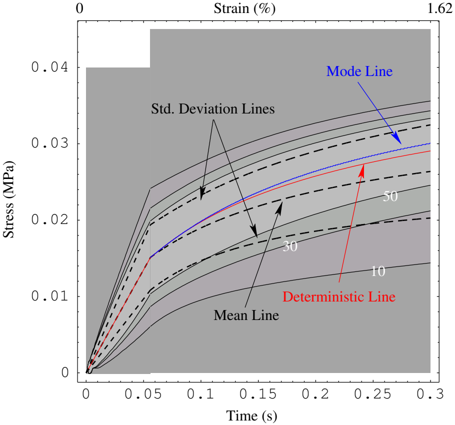
- Dynamic Finite Elements

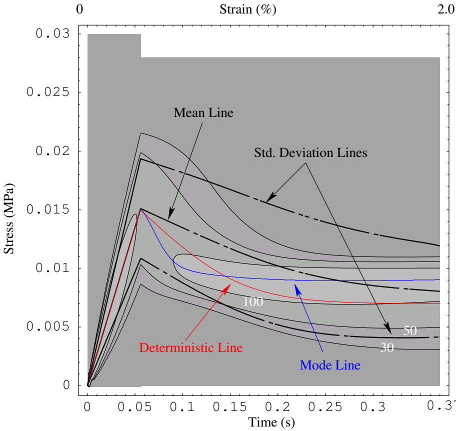
$$M\ddot{u}_i + C\dot{u}_i + K^{ep} u_i = F(t)$$

- Material behavior (LHS) is uncertain

- Loads (RHS) are uncertain

Cam Clay with Random G, M and p_0





Stochastic Elastic-Plastic FEM (SEPFEM)

Dynamic Finite Elements $M\ddot{u}_i + C\dot{u}_i + K^{ep}u_i = F(t)$

- Input random field/process(non-Gaussian, heterogeneous/ non-stationary): Multi-dimensional Hermite Polynomial Chaos (PC) with known coefficients

- Output response process: Multi-dimensional Hermite PC with unknown coefficients

- Galerkin projection: minimize the error to compute unknown coefficients of response process

- SEPFEM eliminates Monte-Carlo inefficiency and inaccuracy

Stochastic Elastic-Plastic Finite Element Method

- Material uncertainty expanded into stochastic shape funcs.
- Loading uncertainty expanded into stochastic shape funcs.
- Displacement expanded into stochastic shape funcs.
- Jeremić et al. 2011

$$\begin{bmatrix} \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_0 > K^{(k)} & \dots & \sum_{k=0}^{P_d} < \Phi_k \Psi_P \Psi_0 > K^{(k)} \\ \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_1 > K^{(k)} & \dots & \sum_{k=0}^{P_d} < \Phi_k \Psi_P \Psi_1 > K^{(k)} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_P > K^{(k)} & \dots & \sum_{k=0}^{M} < \Phi_k \Psi_P \Psi_P > K^{(k)} \end{bmatrix} \begin{bmatrix} \Delta u_{10} \\ \vdots \\ \Delta u_{N0} \\ \vdots \\ \Delta u_{1P_u} \\ \vdots \\ \Delta u_{NP_u} \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{P_f} f_i < \Psi_0 \zeta_i > \\ \sum_{i=0}^{P_f} f_i < \Psi_1 \zeta_i > \\ \sum_{i=0}^{P_f} f_i < \Psi_2 \zeta_i > \\ \vdots \\ \sum_{i=0}^{P_f} f_i < \Psi_{P_u} \zeta_i > \end{bmatrix}$$

Sensitivity Analysis

- The ANalysis Of VAriance representation (Sobol 2001)
- Total variance of the probabilistic model response $y = f(\boldsymbol{X})$

$$D = \mathrm{Var}[f(\boldsymbol{X})] = \int_{I^n} f^2(\boldsymbol{x})\, d\boldsymbol{x} - f_0^2$$

- Sobol' indices $S_{i_1 \ldots i_s}$, fractional contributions from random inputs $\{X_{i_1}, \ldots, X_{i_s}\}$ to the total variance D: $S_{i_1 \ldots i_s} = D_{i_1 \ldots i_s}/D$
- Total sensitivity indices, influence of input parameter X_i

$$S_i^{\mathrm{total}} = \sum_{\mathscr{S}_i} D_{i_1 \ldots i_s}$$

Sobol-Sudret Sensitivity Analysis

- PC expansion of response in ANOVA form (Sudret 2008)
- Multi-dimensional PC bases $\{\Psi_j(\boldsymbol{\xi})\}$ decomposition

$$\Psi_j(\boldsymbol{\xi}) = \prod_{i=1}^{n} \phi_{\alpha_i}(\xi_i)$$

- ANOVA representation $\rightarrow$ PC-based Sobol' indices $S^{PC}_{i_1 \ldots i_s}$

$$S^{PC}_{i_1 \ldots i_s} = \sum_{\boldsymbol{\alpha} \in \mathscr{S}_{i_1, \ldots, i_s}} y^2_{\boldsymbol{\alpha}} \boldsymbol{E}\left[\Psi^2_{\boldsymbol{\alpha}}\right] / D^{PC}$$

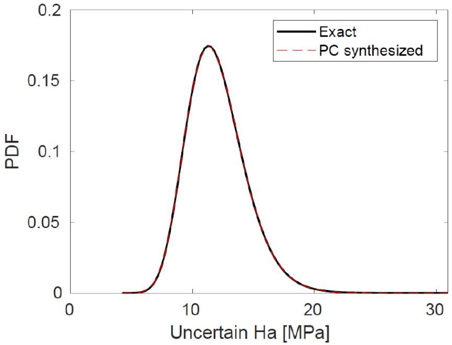
- Total Sobol' indices $S^{PC,\mathrm{total}}_{j_1 \ldots j_t}$

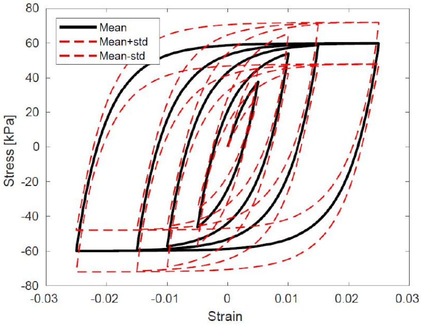
$$S^{PC,\mathrm{total}}_{j_1 \ldots j_t} = \sum_{(i_1, \ldots, i_s) \in \mathscr{S}_{j_1, \ldots, j_t}} S^{PC}_{i_1 \ldots i_s}$$

- Sobol-Sudret sensitivity indices within SEPFEM are analytic and inexpensive

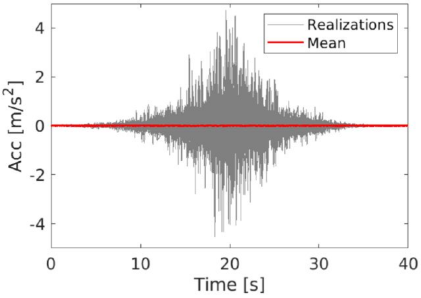
Stochastic Material Parameters

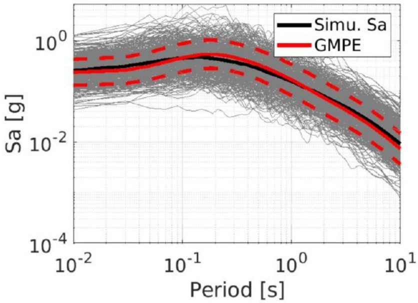
Log-normal distributed random field with PC Dim. 3 Order 2



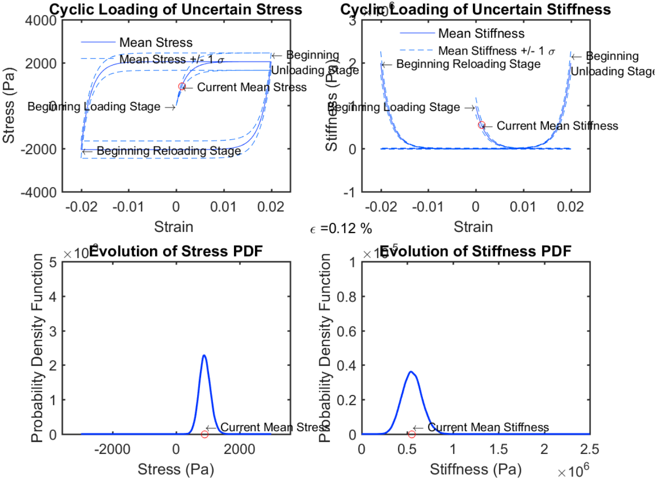


Stochastic Seismic Motions



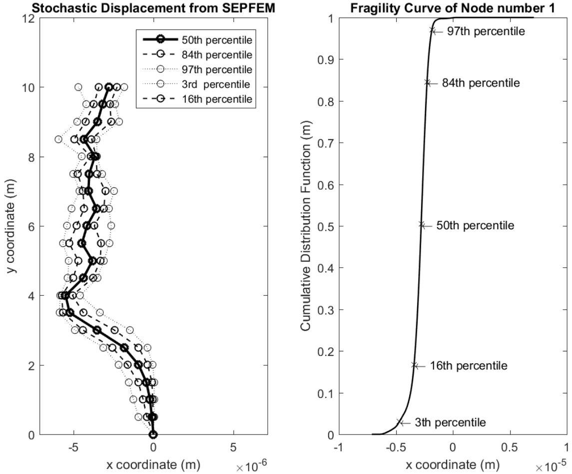


Probabilistic Elastic-Plastic Response



(MP4)

SEPFEM: Example in 1D



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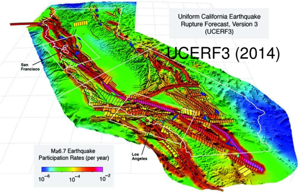
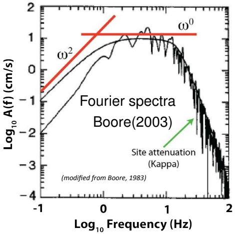
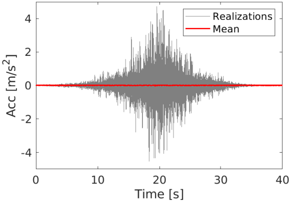
Application: Seismic Hazard

Seismic source characterization



Stochastic ground motion



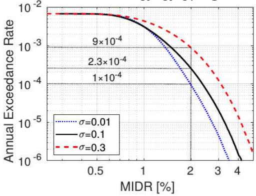


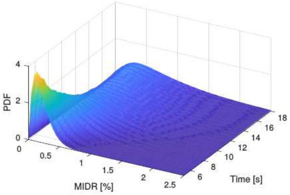
$$\lambda(EDP > z) = \sum N_i(M_i, R_i) P(EDP > z | M_i, R_i)$$

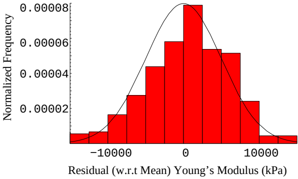
Uncertainty propagation SEPFEM

Uncertainty characterization Hermite polynomial chaos

EDP hazard/risk







Summary

- Probabilistically characterized seismic sources
- Uncertain soil-structure shear beam system
- Forward: uncertain motions through uncertain materials
- Backward: sensitivity analysis