Bayesian Inference for Mechanics–Based Digital Twinning of Bridges

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Digital Technology for Infrastructure Monitoring

Available Post-Earthquake Monitoring Solutions

Visual Inspection

- # of bridges × inspection time × chaos
- Inspection complexity
- Hidden damage
- No system-level insight

ShakeCast (USGS)

- Intensity-based metrics can be inaccurate!
- No/Limited damage localization & qunatification

Modal-Based SID

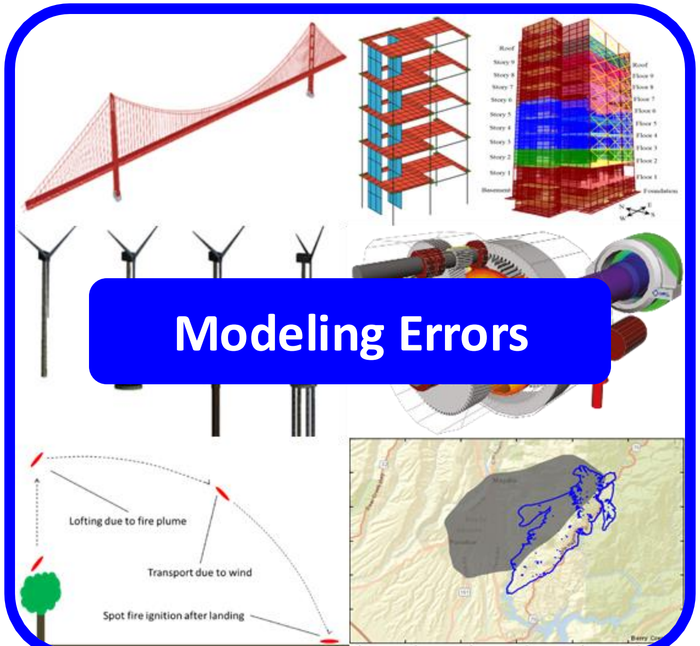
- Limited damage localization & quantification
- Based on ambient/ low-amplitude records

Point Monitoring

- Data not information!
- Require large number of sensors
- Maintenance and installation cost
- No system-level insight

Time-Domain Mechnics-based Model Updating

Digital World



Forward Modeling

- Uncertain Parameters
- Unknown Inputs

Real World

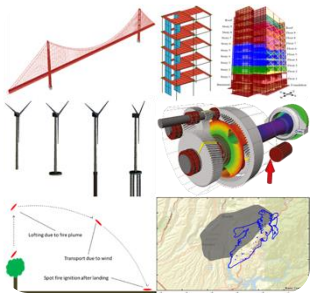
Model Updating

- Parameter Estimation
- Input Estimation
- Uncertainty Quantification

- **Digital Twinning**
- **Virtual Sensing**
- **Damage Diagnosis**

Bayesian Model Updating

Numerical Model



$$\hat{\mathbf{Y}} = \mathbf{f}(\mathbf{\Theta})$$

Measurement

$$\mathbf{Y} = \mathbf{y}$$

Simulation Error Model

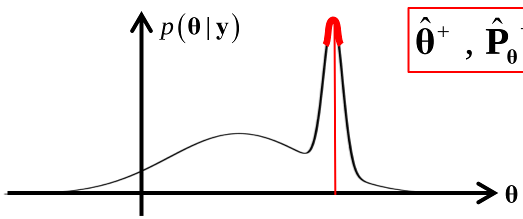
$$\mathbf{Y} - \hat{\mathbf{Y}}(\mathbf{\Theta}) \sim N(\mathbf{0}, \mathbf{R})$$

Likelihood Function

$$p(\mathbf{Y} = \mathbf{y} | \boldsymbol{\theta}) \sim N(\mathbf{0}, \mathbf{R})$$

Bayesian Updating

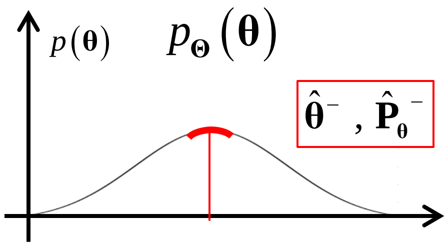
$$p(\boldsymbol{\theta}|\mathbf{y}) = \frac{p(\mathbf{y}|\boldsymbol{\theta})\ p(\boldsymbol{\theta})}{p(\mathbf{y})}$$



$\hat{\boldsymbol{\theta}}^{+}, \hat{\mathbf{P}}_{\boldsymbol{\theta}}^{+}$

Prior Information

$p_{\mathbf{\Theta}}(\boldsymbol{\theta})$



$\hat{\boldsymbol{\theta}}^{-}, \hat{\mathbf{P}}_{\boldsymbol{\theta}}^{-}$

First-Order Approximation: Kalman Filtering

Numerical Model

$$\hat{\mathbf{y}} = \mathbf{f}(\hat{\boldsymbol{\theta}}^-)$$

$$\hat{\mathbf{P}}_{\mathbf{y}} = \left(\frac{\partial \mathbf{f}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\bigg|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^-}\right) \hat{\mathbf{P}}_{\boldsymbol{\theta}}^- \left(\frac{\partial \mathbf{f}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\bigg|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^-}\right)^T$$

FE Response Sensitivity Matrix

Measurement

$$\mathbf{Y} = \mathbf{y}$$

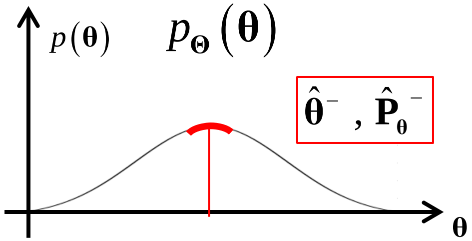
Simulation Error Model

$$\mathbf{Y} - \hat{\mathbf{Y}}(\boldsymbol{\Theta}) \sim N(\mathbf{0}, \mathbf{R})$$

Likelihood Function

$$p(\mathbf{Y} = \mathbf{y} | \boldsymbol{\theta}) \sim N(\mathbf{0}, \mathbf{R})$$

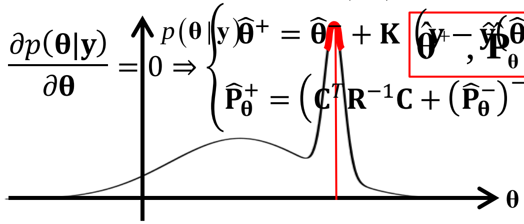
Prior Information



Bayesian Updating

$$p(\boldsymbol{\theta}|\mathbf{y}) = \frac{p(\mathbf{y}|\boldsymbol{\theta})\ p(\boldsymbol{\theta})}{p(\mathbf{y})}$$

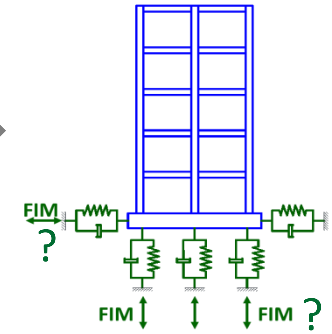
$$\frac{\partial p(\boldsymbol{\theta}|\mathbf{y})}{\partial \boldsymbol{\theta}} = 0 \Rightarrow \begin{cases} \hat{\boldsymbol{\theta}}^+ = \hat{\boldsymbol{\theta}}^- + \mathbf{K}\left(\mathbf{y} - \hat{\mathbf{y}}(\hat{\boldsymbol{\theta}}^-)\right) \\ \hat{\mathbf{P}}_{\boldsymbol{\theta}}^+ = \left(\mathbf{C}^T \mathbf{R}^{-1} \mathbf{C} + (\hat{\mathbf{P}}_{\boldsymbol{\theta}}^-)^{-1}\right)^{-1} \end{cases}$$



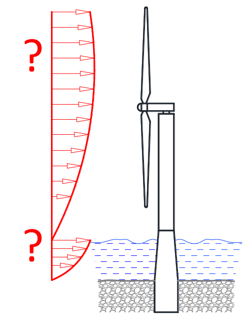
Unknown Input Excitation: Output-Only Model Updating

Unknown Input

Unknown Base Excitation



Unknown Force Excitation



Problem Statement:

$$\mathbf{M}(\boldsymbol{\theta})\ddot{\mathbf{u}}_k(\boldsymbol{\theta}) + \mathbf{C}(\boldsymbol{\theta})\dot{\mathbf{u}}_k(\boldsymbol{\theta}) + \mathbf{r}_k(\mathbf{u}_{1:k}(\boldsymbol{\theta}), \boldsymbol{\theta}) = \mathbf{L}\mathbf{f}_{1:k}^{unknown}$$

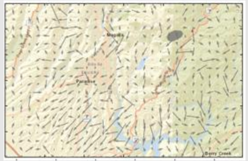
$$\text{Find} \quad \hat{\boldsymbol{\theta}}, \hat{\mathbf{f}}_{1:k}^{unknown} \mid \quad \left[\hat{\boldsymbol{\theta}}, \hat{\mathbf{f}}_{1:k}^{unknown}\right] = \underset{\boldsymbol{\theta}\in\Theta, \mathbf{f}\in\mathcal{F}}{\arg\max}\; p\left(\boldsymbol{\theta}, \mathbf{f}_{1:k}^{unknown} \mid \mathbf{y}_{1:k}\right)$$

Bayesian Model Updating Applications

Cross-Disciplinary Problems

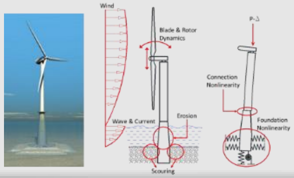
Wildfire Engineering

Wildfire Simulation and Data Assimilation

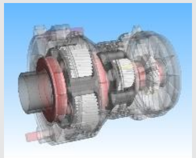


Digital Twinning of Offshore Wind Systems

Remote Monitoring of Offshore Structures

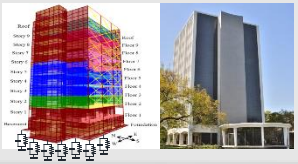


Diagnosis & Prognosis of Drivetrains

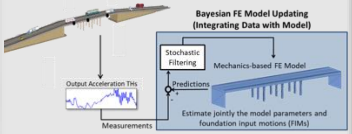


Digital Twinning of Civil Structures

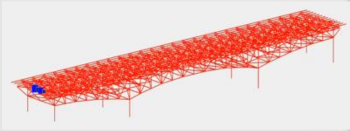
Soil-Structure Model Updating



Rapid Post-Earthquake Assessment



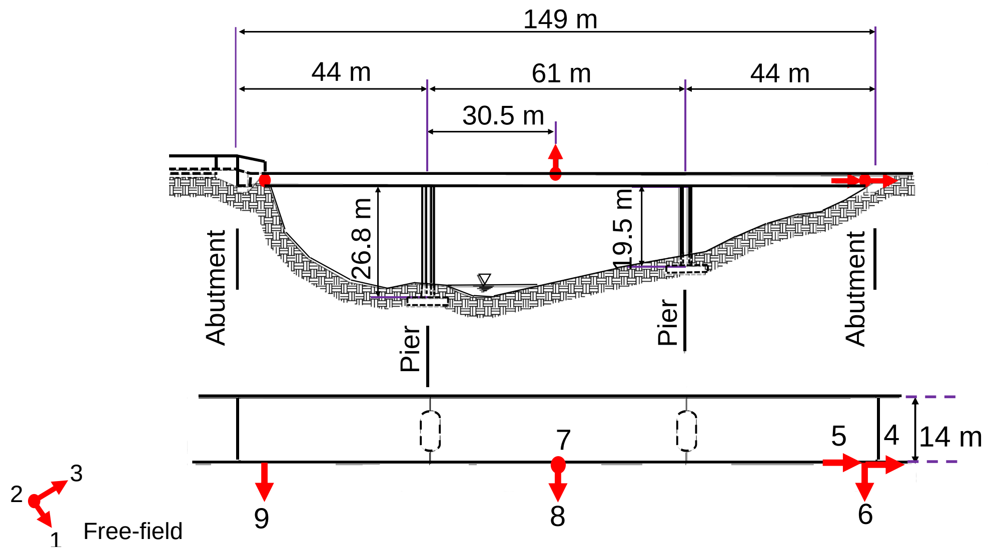
Operational Monitoring of Aging Bridges



Output-Only Model Updating Using Seismic Records

San Roque Canyon (SRC) Bridge (Santa Barbara, CA)

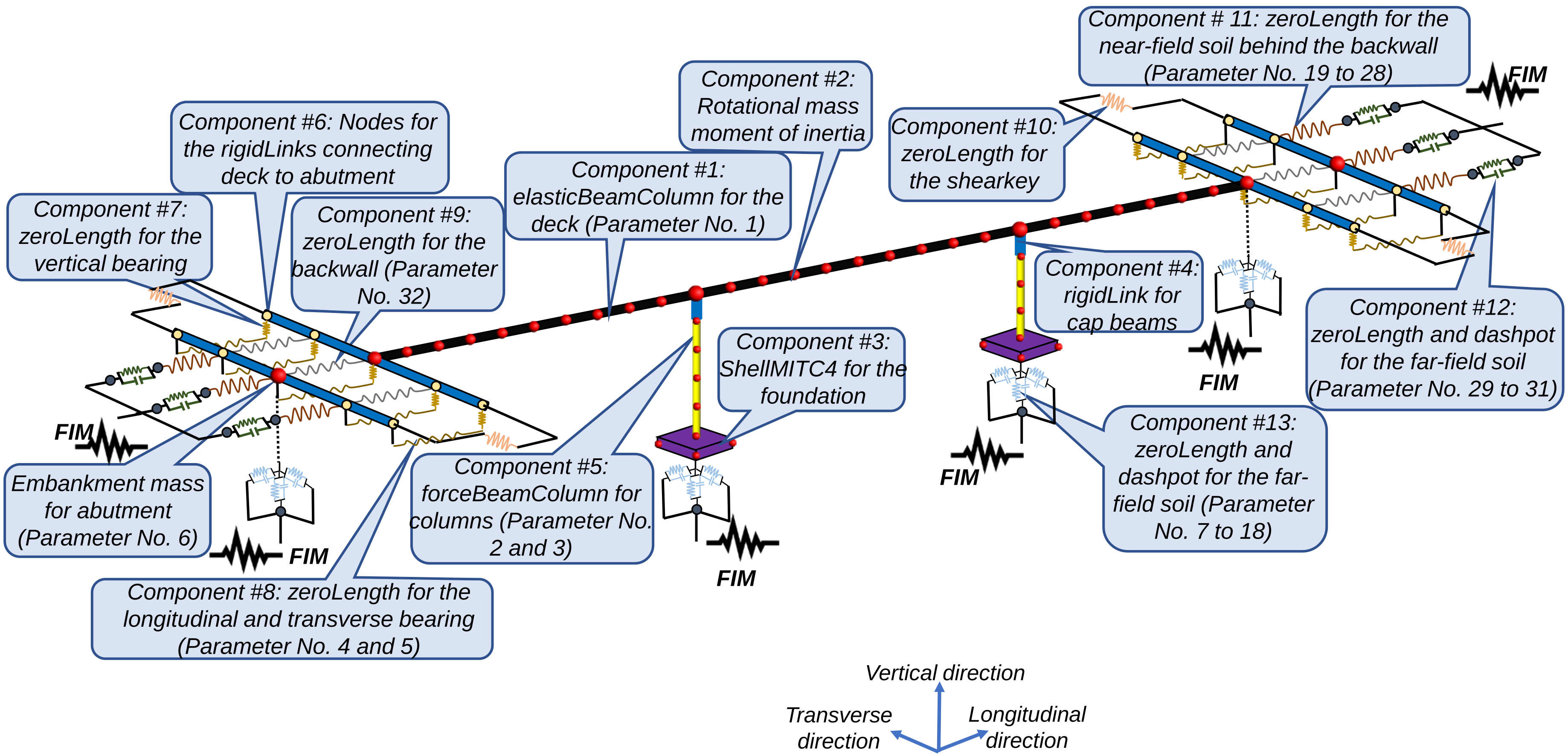
Bridge Layout



Recorded Earthquakes at Bridge Station

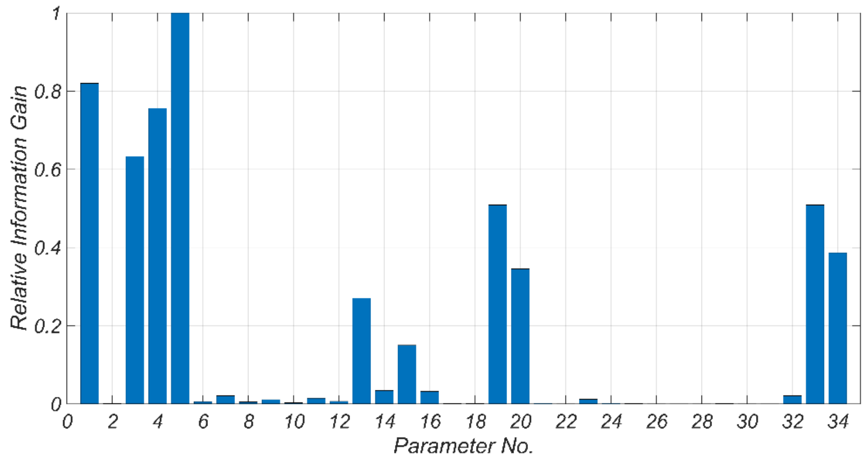
No.	Earthquake	Date	Distance (km)	PGA (g)	PSA in trans-verse direction (g)	PSA in vertical direction (g)	PSA in longi-tudinal direc-tion (g)
1	San Simeon	12/22/2003	187.0	0.015	0.045	0.042	0.022
2	IslaVista	05/09/2004	27.2	0.016	0.026	0.047	0.013
3	IslaVista	05/29/2013	18.0	0.041	0.060	0.150	0.040
4	Montecito	04/23/2017	9.5	0.022	0.024	0.045	0.014
5	Santa Cruz	04/05/2018	67.9	0.016	0.021	0.058	0.019

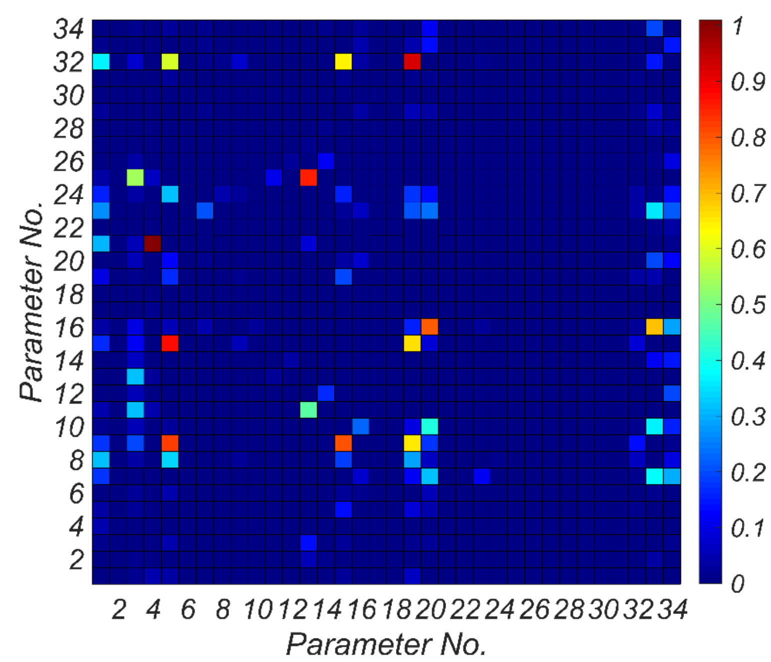
Finite Element Model



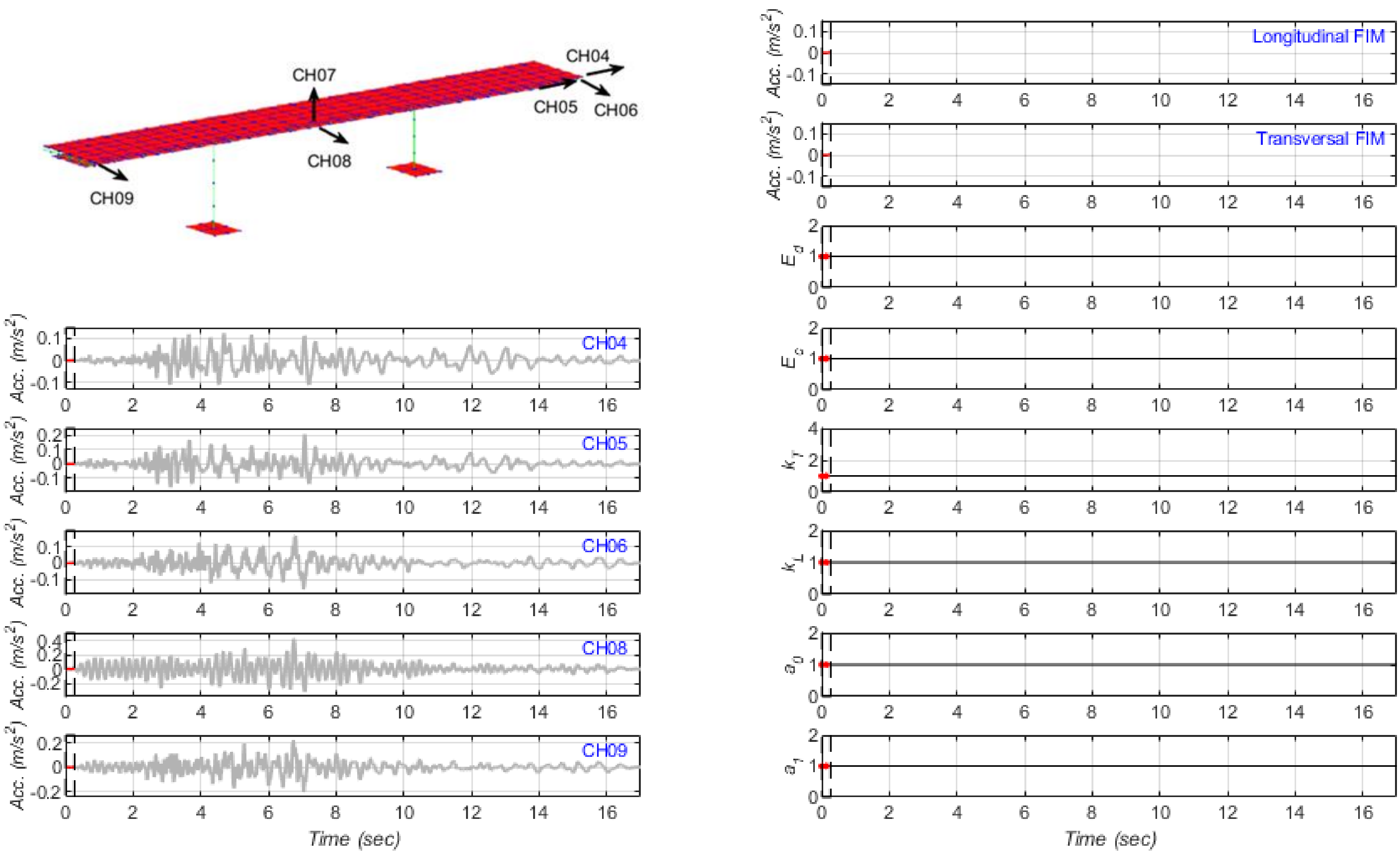
Uniqueness of Solution: Identifiability Analysis

No.	Description	No.	Description
1	Elastic modulus of deck	18	Rotational soil-foundation damping coefficient under pier about the vertical axis
2	Compressive strength of column	19	Longitudinal soil-foundation stiffness under abutment
3	Initial elastic modulus of column	20	Longitudinal soil-foundation damping coefficient under abutment
4	Transverse elastomeric shear stiffness of bearing pad	21	Transverse soil-foundation stiffness under abutment
5	Longitudinal elastomeric shear stiffness of bearing pad	22	Transverse soil-foundation damping coefficient under abutment
6	Embankment mass for abutment	23	Vertical soil-foundation stiffness under abutment
7	Vertical soil-foundation stiffness under pier	24	Vertical soil-foundation damping coefficient under abutment
8	Vertical soil-foundation damping coefficient under pier	25	Rotational soil-foundation stiffness under abutment about its longitudinal axis
9	Longitudinal soil-foundation stiffness under pier	26	Rotational soil-foundation damping coefficient under abutment about the longitudinal axis
10	Longitudinal soil-foundation damping coefficient under pier	27	Rotational soil-foundation stiffness under abutment about the vertical axis
11	Transverse soil-foundation stiffness under pier	28	Rotational soil-foundation damping coefficient under abutment about the vertical axis
12	Transverse soil-foundation damping coefficient under pier	29	Far-field soil-embankment stiffness in longitudinal direction
13	Rotational soil-foundation stiffness under pier about the longitudinal axis	30	Far-field soil-embankment radiation damping coefficient in the longitudinal direction
14	Rotational soil-foundation damping coefficient under pier about the longitudinal axis	31	Far-field soil-embankment material damping coefficient in the longitudinal direction
15	Rotational soil-foundation stiffness under pier about the transverse axis	32	Soil-backwall initial stiffness in the longitudinal direction
16	Rotational soil-foundation damping coefficient under pier about the transverse axis	33	Mass proportional Rayleigh damping coefficient
17	Rotational soil-foundation stiffness under pier about the vertical axis	34	Stiffness proportional Rayleigh damping coefficient

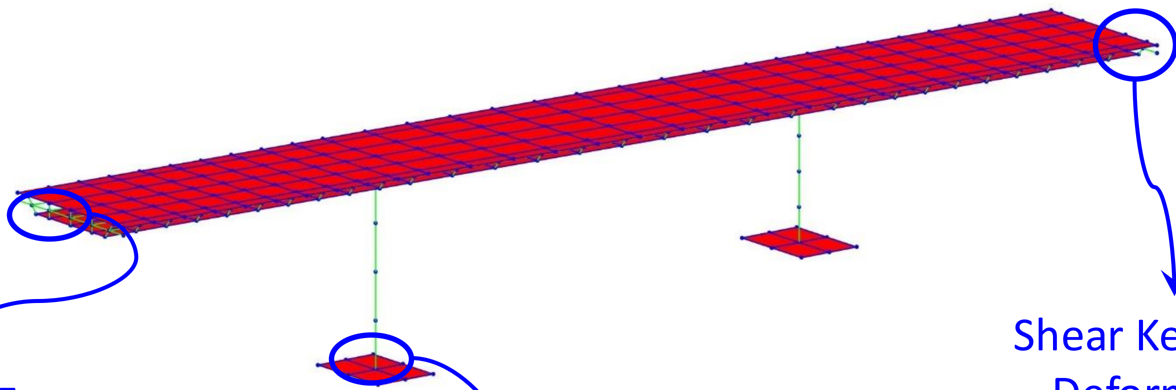




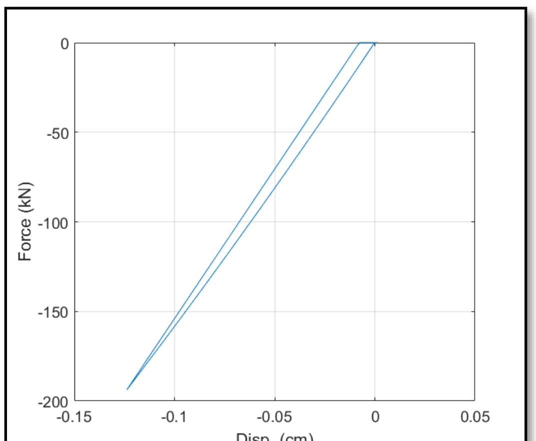
San Simeon Earthquake Record



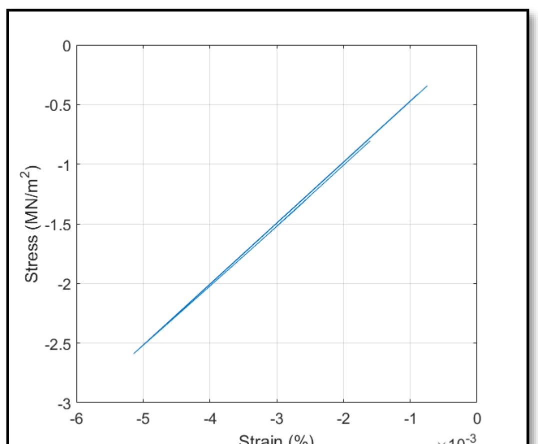
Virtual Sensing



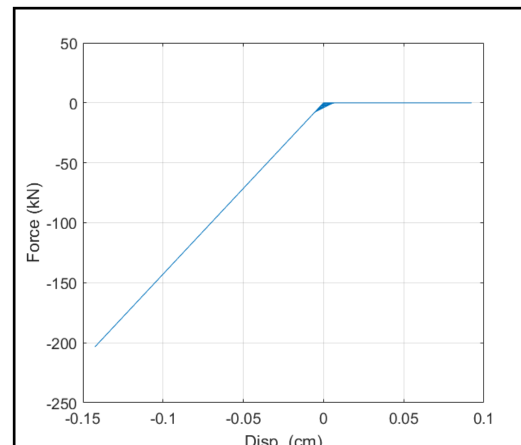
Passive Soil Force-Deformation



Concrete Fiber Material Response



Shear Key Force-Deformation



Limitation: Modeling Error or Model-Form Uncertainty

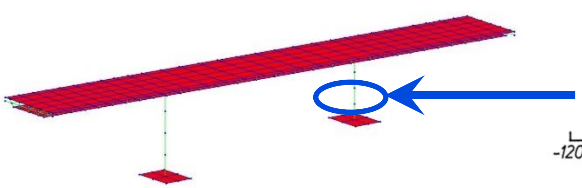
$$\mathbf{M}(\boldsymbol{\theta})\ddot{\mathbf{u}}_k + \mathbf{C}(\boldsymbol{\theta})\dot{\mathbf{u}}_k + \mathbf{r}_k(\mathbf{u}_{1:k}, \boldsymbol{\theta}) = -\mathbf{M}(\boldsymbol{\theta})\mathbf{L}\ddot{\mathbf{u}}_k^g$$

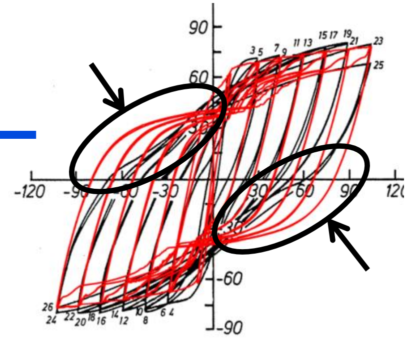
$$(\mathbf{M}(\boldsymbol{\theta}) + \delta\mathbf{M})\ddot{\mathbf{u}}_k + (\mathbf{C}(\boldsymbol{\theta}) + \delta\mathbf{C}_k)\dot{\mathbf{u}}_k + \mathbf{r}_k(\mathbf{u}_{1:k}, \boldsymbol{\theta}) + \delta\mathbf{r}_k = -(\mathbf{M}(\boldsymbol{\theta}) + \delta\mathbf{M})\mathbf{L}\ddot{\mathbf{u}}_k^g + \mathbf{f}_k^{unknown}$$

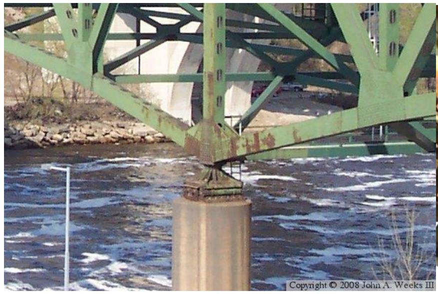
$$\mathbf{M}(\boldsymbol{\theta})\ddot{\mathbf{u}}_k + \mathbf{C}(\boldsymbol{\theta})\dot{\mathbf{u}}_k + \mathbf{r}_k(\mathbf{u}_{1:k}, \boldsymbol{\theta}) + \boldsymbol{\omega}_k = -\mathbf{M}(\boldsymbol{\theta})\mathbf{L}\ddot{\mathbf{u}}_k^g$$

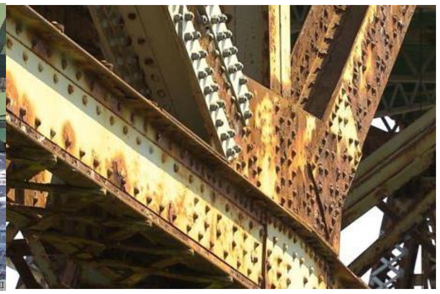
$\boldsymbol{\omega}_k$: **Modeling errors lumped at the structural level**

Jointly estimate $\boldsymbol{\theta}$ and higher statistics of $\boldsymbol{\omega}_k$ @ measurement locations



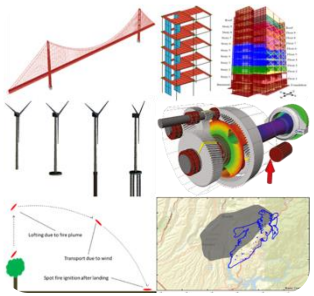






Adaptive Bayesian Inference for Model Updating

Numerical Model



$$\hat{\mathbf{Y}} = \mathbf{f}(\mathbf{\Theta})$$

Measurement

$$\mathbf{Y} = \mathbf{y}$$

Simulation Error Model

$$\mathbf{Y} - \hat{\mathbf{Y}}(\mathbf{\Theta}) \sim N(\boldsymbol{\mu}, \mathbf{R})$$

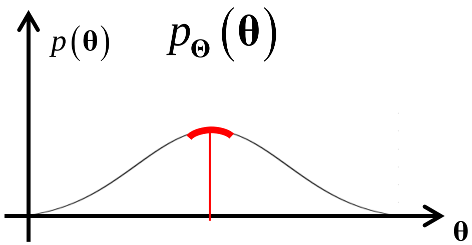
Likelihood Function

$$p(\mathbf{Y} = \mathbf{y}|\boldsymbol{\theta}) \sim N(\boldsymbol{\mu}, \mathbf{R})$$

Modeling Errors

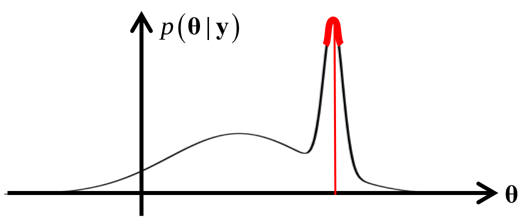
Prior Information

$$p_{\mathbf{\Theta}}(\boldsymbol{\theta})$$

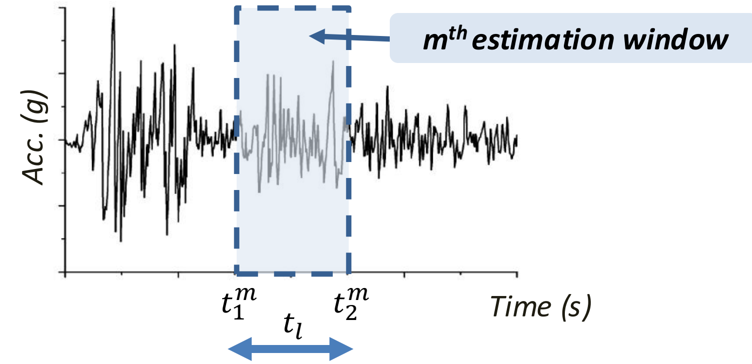


Bayesian Updating

$$p(\boldsymbol{\theta}, \boldsymbol{\mu}, \mathbf{R}|\mathbf{y}) = \frac{p(\mathbf{y}|\boldsymbol{\theta}, \boldsymbol{\mu}, \mathbf{R})\, p(\boldsymbol{\theta}, \boldsymbol{\mu}, \mathbf{R})}{p(\mathbf{y})}$$



Joint Parameter, Input, and Noise Estimation



$$p\left(\boldsymbol{\psi}_m, \boldsymbol{\mu}_m, \mathbf{R}_m \middle| \mathbf{y}_{t_1:t_2}\right) = \frac{p\left(\mathbf{y}_{t_1:t_2} \middle| \boldsymbol{\psi}_m, \boldsymbol{\mu}_m, \mathbf{R}_m\right) p\left(\boldsymbol{\psi}_m, \boldsymbol{\mu}_m, \mathbf{R}_m\right)}{p\left(\mathbf{y}_{t_1:t_2}\right)}$$

$$\left\{\widehat{\boldsymbol{\psi}}_m^+, \widehat{\boldsymbol{\mu}}_m^+, \widehat{\mathbf{R}}_m^+\right\} = \underset{\boldsymbol{\psi}_m, \boldsymbol{\mu}_m, \mathbf{R}_m}{\operatorname{argmax}}\; p\left(\boldsymbol{\psi}_m, \boldsymbol{\mu}_m, \mathbf{R}_m \middle| \mathbf{y}_{t_1^m:t_2^m}\right)$$

$$\left\{\widehat{\boldsymbol{\psi}}_m^+\right\} = \underset{\boldsymbol{\psi}_m}{\operatorname{argmax}}\; p\left(\boldsymbol{\psi}_m \middle| \boldsymbol{\mu}_m, \mathbf{R}_m, \mathbf{y}_{t_1^m:t_2^m}\right)$$

$$\widehat{\boldsymbol{\psi}}_m^+ = \widehat{\boldsymbol{\psi}}_m^- + \mathbf{K}_m\left(\mathbf{y}_{t_1^m:t_2^m} - \mathbf{h}_{t_1^m:t_2^m}\left(\widehat{\boldsymbol{\psi}}_m^-\right) - \widehat{\boldsymbol{\mu}}_{t_1^m:t_2^m}^+\right)$$

$$\mathbf{P}_{\boldsymbol{\psi},m}^+ = \mathbf{P}_{\boldsymbol{\psi},m}^- - \mathbf{K}_m \mathbf{P}_{\mathbf{yy},m} \mathbf{K}_m^T$$

$$\left\{\widehat{\boldsymbol{\mu}}_m^+, \widehat{\mathbf{R}}_m^+\right\} = \underset{\boldsymbol{\mu}_m, \mathbf{R}_m}{\operatorname{argmax}}\; p\left(\boldsymbol{\mu}_m, \mathbf{R}_m \middle| \mathbf{y}_{t_1^m:t_2^m}\right)$$

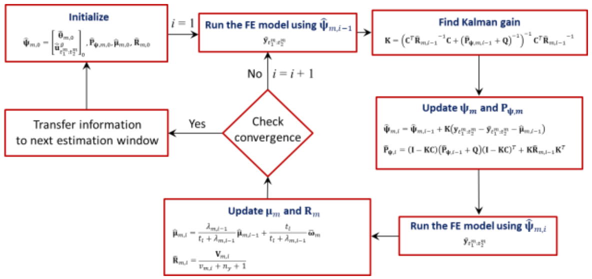
$$\widehat{\boldsymbol{\mu}}_m^+ = \frac{\lambda_m^-}{t_l + \lambda_m^-}\widehat{\boldsymbol{\mu}}_m^- + \frac{t_l}{t_l + \lambda_m^-}\overline{\boldsymbol{\omega}}_m$$

$$\widehat{\mathbf{R}}_m^+ = \frac{\mathbf{V}_m^+}{v_m^+ + n_y + 1}$$

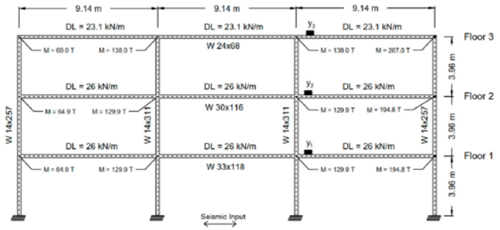
Fixed-Point Iterations

PEER Project Summary

- Advancing existing algorithms → Adaptive Bayesian inference for Joint parameter, input, and noise estimation



- Verification & Validation



- Real-world application (BRACE^2 project)
- Open-source codes for HPC environment
  - OpenSees integration
  - Nested parallelization
- Educational & support materials

CGS CSMIP-89324
Rio Dell - Hwy 101/Painter St. Overpass




Thank you!