

# PACIFIC EARTHQUAKE ENGINEERING RESEARCH CENTER

## Probabilistic Seismic Evaluation of Reinforced Concrete Structural Components and Systems

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### ABSTRACT

An accurate evaluation of the structural performance of reinforced concrete structural systems under seismic loading requires a probabilistic approach due to uncertainties in structural properties and the ground motion (referred to as basic uncertainties). The objective of this study is to identify and rank significant sources of basic uncertainties and structural components with respect to the seismic demand (referred to as the Engineering Demand Parameters, EDP) of reinforced concrete structural systems. The methodology for accomplishing this objective consists of three phases. In the first phase, the propagation of basic uncertainties to a structural system with respect to its EDPs is studied using the first-order second-moment (FOSM) method and the tornado diagram analysis to identify and rank significant sources of basic uncertainties. In the second phase, the propagation of basic uncertainties to structural components with respect to their capacities is studied. For this purpose, the stochastic fiber element model is developed to build probabilistic section models such as the moment-curvature relationships at critical sections of the structural component. In the third phase, the propagation of uncertainty in the capacities of structural components to the structural system with respect to its EDPs is studied. Using the FOSM method combined with probabilistic section models, EDP uncertainties induced by structural components are estimated to identify and rank significant components. Several case studies demonstrate the effectiveness and robustness of the developed procedure of propagating uncertainties.

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### 1 Introduction

### 1.1 GENERAL

The behavior of reinforced concrete (RC) structural members (or components), especially the inelastic behavior, depends on various geometric and material parameters. Most of these parameters are of a random nature, and consequently, uncertainty exists in the behavior of the RC members in terms of the strength and ductility. Therefore, a realistic estimate of the behavior of the RC structural system that is an assembly of a number of structural components requires a probabilistic approach for an appropriate treatment of uncertain structural properties, especially under seismic loading.

An accurate yet practical evaluation of the structural behavior due to seismic loading is one of the critical issues of the emerging performance-based earthquake engineering (PBEE) methodology. In particular, the estimation of the seismic loss and the corresponding repair cost of the structural system depend on an accurate and realistic estimate of the performance of the structural system. Uncertainty in the loss estimation of the structural system, mainly due to uncertainties in the ground motion and structural properties, can be costly because it is directly related to the repair cost. In that regard, it is important to identify and rank both sources of uncertainty and structural components that are relatively significant to the performance of the structural system.

The probabilistic analysis of RC structural components and systems has been the focus of a number of research efforts. One of the earliest works is that of Shinozuka (1972),

who investigated the effect of uncertain material properties on the strength of plain concrete structures. Several studies were concentrated on RC members such as beams or columns. Frangopol et al. (1996), Mirza and MacGregor (1989), and Grant et al. (1978) conducted strength analyses of RC beam-column members by considering uncertainties in material properties and cross-sectional dimensions. More recent research has focused on the probabilistic evaluation of RC structural systems. Chryssanthopoulos et al. (2000), Ghobarah and Aly (1998), and Singhal and Kiremidjian (1996) recently proposed systematic ways of evaluating RC framed structures by considering the uncertainties in ground motions and material properties. However, only a few studies were performed within the context of PBEE. Porter et al. (2002), among those few studies, investigated the sensitivity of loss estimate of an RC building to major uncertain parameters. Despite a large number of previous studies on probabilistic evaluation of RC structures, efforts on identifying relative significance of different sources of uncertainty and/or structural components with respect to the performance (or demand) of the structural system are scarce.

### 1.2 OBJECTIVES AND SCOPE

This study has three objectives that eventually aim at the main goal of the present study: to identify significant sources of basic uncertainties and structural components with respect to the seismic demand (referred to as the Engineering Demand Parameter, EDP) of an RC structural system. The following are specific objectives of this study:

- To understand the propagation of basic uncertainties to a structural system with respect to its seismic demands.
- To understand the propagation of basic uncertainties to structural components that form the structural system with respect to the strength and the deformation capacity of the component.
- To understand the propagation of uncertainty in the capacity of structural components

to the structural system with respect to its EDPs.

The basic uncertainty is an observable uncertainty, i.e., statistical information can be collected for it. For example, material properties, member dimensions, and soil properties are all basic uncertainties because one can physically test or measure them.

The methodology for accomplishing the objective of this study is developed for RC framed structure within the framework of the PBEE methodology being developed by the Pacific Earthquake Engineering Research (PEER) center. Two different modeling schemes are used to analyze structural components and systems. The first is a fiber element modeling, known for its accurate estimation of the inelastic structural response. The second utilizes plastic-hinge modeling that is widely used in practice due to its simplicity. Different probabilistic methods are used to understand propagation of uncertainties. Monte Carlo simulation is used for evaluation of the strength and the deformation capacity of structural components, while the first-order second-moment (FOSM) method and a method of deterministic sensitivity analysis using a tornado diagram<sup>1</sup> are used for evaluation of structural systems.

### 1.3 OVERVIEW

This report consists of six chapters beginning with the introduction in Chapter 1 and concluding with Chapter 6. Chapter 2 presents the background, literature review, and the methodology to support the core chapters. The core chapters of this report are Chapters 3 to 5. Each core chapter presents works related to each of the three objectives discussed in the previous section. There are three categories of uncertainty being discussed in this report, namely basic uncertainty, uncertainty in the capacity of structural components, and EDP uncertainty of structural systems. Each core chapter presents the relationship between two

<sup>&</sup>lt;sup>1</sup>Tornado diagram, commonly used in decision analysis, consists of a set of horizontal bars (swings) where the length of each bar represents the output sensitivity to a given input variable. These bars are displayed in the descending order of the bar length from the top to the bottom. This wide-to-narrow arrangement of the swings eventually resembles a tornado.

components of these uncertainties in terms of their propagation and identifies (or ranks) the relative significance of an individual random variable or a structural component. Fig. 1.1 shows an overview of the present study focusing on the three core chapters.



Fig. 1.1 Overview.

In Chapter 2, the background of the present study is introduced including the review of previous works. In particular, the PBEE methodology being developed by the PEER center is summarized. General discussion of sources of uncertainty in the structural analysis is presented. Literature on characterizing sources of uncertainties that are considered in the present study is reviewed. Finally, a systematic procedure of probabilistic evaluation of structural systems, which consists of component evaluation and system evaluation, is described.

In Chapter 3, the propagation of basic uncertainties through a structural system with respect to its EDP is discussed. Descriptions of the FOSM and the tornado diagram methods, and the corresponding procedures of sensitivity analyses of a structural system are presented. Propagation of uncertainty is demonstrated for a case-study building (referred to as UCS) using both methods. Moreover, significant random variables to EDPs of UCS are identified. In Chapter 4, the propagation of basic uncertainties through structural components with respect to their capacity is discussed. First, a newly developed stochastic fiber element model is presented. This model combines the conventional fiber element model and one of the random field representation methods (the mid-point method) along with Monte Carlo simulation. Second, the model is used for a probabilistic evaluation of structural components considering spatial variability of random variables where a study of strength variability of an RC column due to uncertainties in structural properties is presented. Finally, structural components of a ductile RC frame (referred to as VE) are evaluated to develop probabilistic moment-curvature and shear force-distortion relationships at critical cross sections of the structural components.

In Chapter 5, the propagations of uncertainties in the strength and the deformation capacities of structural components through a structural system with respect to EDPs of the structural system are discussed. First, the procedure is demonstrated by a ductile portal frame to estimate the probability distribution of the lateral strength of the frame. Second, the EDP sensitivities of VE to uncertainty in the capacity of structural components using probabilistic component models developed in Chapter 4 are presented. Finally, significant structural components to EDPs of VE are identified according to the FOSM method.

In Chapter 6, the summary and conclusion of the study are presented. Recommendations for future extension of this study are also outlined.

## **2** Propagation of Uncertainty

### 2.1 INTRODUCTION

Almost all input parameters in the structural analysis such as mass, damping, material properties, boundary conditions, and applied load are uncertain. They are uncertain either because of the inherent physical randomness (or variability) or because of our state of knowledge. The former is called *aleatory* uncertainty and the latter is called *epistemic* uncertainty. By definition, aleatory uncertainty is irreducible and epistemic uncertainty is reducible by improving our state of knowledge. Regardless of the type of uncertainty, these uncertainties make the corresponding structural response also uncertain. This process is viewed as propagation of uncertainty in input parameters through the structural system. In this study, the propagation of uncertainty is studied within the framework of performance-based earthquake engineering (PBEE) methodology. However, the presented methodology in this study doesn't have to be limited to a particular PBEE framework such as the one formulated within the PEER center, as it can be equally applied to other structural performance evaluation process.

This chapter introduces the definitions and the background of PBEE, together with the PBEE methodology developed within the PEER center. Also discussed is how uncertainty is treated in the PBEE methodology of this study. Finally, the scope of this study in the context of the adopted ranges of uncertainty is discussed.

### 2.2 PERFORMANCE-BASED EARTHQUAKE ENGINEERING

After the events of big earthquakes in the mid-1990s, namely the 1994 Northridge and 1995 Kobe earthquakes, the structural engineering community realized that the amount of damage, the economic loss due to downtime (or loss of use), and repair cost of structures were unacceptably high, even though those structures complied with applicable seismic codes to satisfy only the life-safety performance objective. Accordingly, structural engineers and researchers started to think about a new design philosophy. FEMA 273 (1997) and Vision 2000 by SEAOC (1995) are known as the publications that reflect the pioneering work to formulate the PBEE methodologies.

The definition of PBEE is widespread in the literature (Bertero and Bertero 2002; Ghobarah 2001; SEAOC 1995). PBEE is defined such that it consists of development of conceptual, preliminary, evaluation, and final design; control of construction quality; and the maintenance of the structure such that the stated performance objectives are achieved when it is subjected to one of the stated levels of seismic hazard. The performance objectives may be a level of stress not to be exceeded, a force or deformation limit state at a member level, or a damage state at the system level. For example, Vision 2000 identifies performance levels as fully operational, operational, life safe, and near collapse. The levels of seismic hazard defined in Vision 2000 include frequent, occasional, rare, and very rare events. These events reflect Poisson-arrival events with probability of exceedance stated as 50% in 30 years, 50% in 50 years, 10% in 50 years, and 10% in 100 years, respectively. Figure 2.1 shows possible combinations of performance objective and seismic hazard level that can be used as design criteria.

### 2.3 PBEE METHODOLOGY DEVELOPED WITHIN PEER CENTER

The PEER Center, based at the University of California, Berkeley, is one of three federally funded earthquake engineering research centers in the United States. PEER has focused on



Fig. 2.1 Vision 2000 recommended seismic performance objectives for building, after SEAOC (1995).

developing a PBEE methodology for the past 7 years as a part of a 10-year research program. The key features of PEER's PBEE methodology are: (1) explicit calculation of system performance and (2) rigorous probabilistic calculation (http://www.peertestbeds.net). The performance of the whole system is explicitly calculated and expressed in terms of the direct interest of various stakeholder groups such as monetary values, downtimes, and injuries and deaths. Unlike earlier PBEE methodologies, forces and deformations of components are indicative of, but not the same as, the system performance. Rigorous probabilistic calculation implies that the performance is calculated and expressed in a probabilistic manner without relying on expert opinion. Uncertainties in earthquake intensity, ground motion detail, structural response, physical damage, and economic and human loss are explicitly considered in the methodology. While its overview is described by Porter (2003), PEER's PBEE methodology is summarized in this section as it provides a background of the present study.

PEER's PBEE methodology consists of four phases: hazard analysis, structural anal-

ysis, damage analysis, and loss analysis, as illustrated in Figure 2.2 where p[X] refers to the probability density of X and p[X|Y] refers to the conditional probability density of X given the event of Y = y. (2.1) is the mathematical expression of the methodology.

$$p[DV|O,D] = \iiint p[DV|DM]p[DM|EDP]p[EDP|IM]p[IM|O,D]dIMdEDPdDM$$
(2.1)

where

O = Location of the structure

D = Design of the structure

- IM = Intensity measure of earthquake site effects
- EDP = Engineering demand parameter as a measure of structural response
- DM = Measure of physical damage of various members
- DV = Decision variable that is the performance parameter of interest such as repair cost

**Hazard Analysis** In the hazard analysis phase, parameters related to the location and design of the target structure are considered to develop the probabilistic seismic hazard, p[IM|O,D]. These parameters include magnitude, mechanism, and distance of nearby faults from the structure, and soil conditions of the site. Ground motion intensity is represented by IM such as the damped elastic spectral acceleration at the fundamental period of the structure. The probabilistic seismic hazard is usually expressed in the form of the annual exceedance frequency of various levels of IM, given the location and design of the structure.

**Structural Analysis** In the structural analysis phase, a computational model of the structure is developed to estimate the structural responses in terms of selected EDPs, subjected to a given IM (p[EDP|IM]). EDPs may include local parameters such as member forces or



Fig. 2.2 PEER's PBEE analysis methodology.

deformations, or global parameters such as floor acceleration and displacement, and interstory drift. Since IM is a random variable, EDP is also a random variable. In addition to IM uncertainty, uncertainties in parameters defining the structural model can be considered in the structural analysis phase. Uncertainties in the mass, damping, stiffness, and strength of the structure are among those.

**Damage Analysis** In the damage analysis phase, a set of fragility functions of structural and non-structural components of the target structure is developed to produce the probability of various damage levels in terms of DM, conditioned on the structural response given in terms of EDP (p[DM|EDP]). Physical damage of a specific component is defined relative to that of the undamaged state, considering the particular repair effort (or cost) required to restore the component to its undamaged state. Structural and non-structural components may include beams, columns, non-structural partitions, window glasses, or building contents such as laboratory equipment or computers. Fragility functions can be developed by laboratory experiments or by mathematical models describing physical phenomena.

Loss Analysis The loss analysis phase is the last phase of the PEER's PBEE methodology, where all uncertainties in previous analysis phases are integrated to develop the probabilistic estimation of structural performance in terms of DV, conditioned on DM (p[DV|DM). As mentioned earlier, the system performance of the structure is expressed in terms of the direct interest of stakeholder groups such as monetary values, downtimes, and injuries and deaths. The final product of this phase is in the form of the exceedance frequency of various levels of DV. Finally, decision-makers decide whether the current design and location are acceptable or not based on this final product of the PBEE methodology.

### 2.4 UNCERTAINTY IN PEER'S PBEE

Uncertainty is considered at each analysis phase and it propagates to the next analysis phase. In the hazard analysis phase, the exact location, magnitude, mechanism of nearby faults, and soil properties of the site of the target structure are uncertain. Consequently, the levels of seismic hazard intensity or IM that the designed structure will experience are uncertain. The details of ground motion profiles given those IMs are also uncertain.

Uncertainty in the hazard analysis phase, expressed in terms of IM, propagates to the structural analysis phase and causes EDP uncertainty even if a mathematical model of the structure is deterministic. In addition, sources of uncertainty in the structural model itself exist. They include mass, damping, material properties such as steel strength and modulus of elasticity, concrete strength and initial modulus of elasticity, and construction geometry such as beam and column dimensions, and location of reinforcing bars. Moreover, additional uncertainty exists in the selection of the element type and other modeling assumptions, which is often referred to as the modeling uncertainty. Similarly, in addition to the propagated uncertainty from the previous phases, the damage analysis phase also has its own sources of uncertainty. They are experimental uncertainty if a laboratory experiment is performed and modeling uncertainty if a computational model is used to develop fragility function of a specific building content, or structural or non-structural component. Uncertainty in the loss analysis phase can also be characterized in a similar way.

There are several published works of PEER researchers related to uncertainty in PEER's PBEE methodology (Baker and Cornell 2003; Miranda and Aslani 2003; Porter et al. 2002). Porter et al. (2002) studied the relative impact of uncertainties in so-called major variables to the system performance of a non-ductile RC building, namely the Van Nuys building in California, through the PEER's PBEE analysis methodology. They considered uncertainty in the spectral acceleration as IM and details of ground motion (hazard analysis phase), in building mass, viscous damping, and force-deformation behavior (structural analysis phase), in component fragility (damage analysis phase), and in unit repair costs, and overhead and profit (loss analysis phase). Figure 2.3 illustrates the results of their study showing the relative importance of random variables to system performance in terms of the damage factor in this case. One of the deterministic sensitivity analysis methods, using the so-called tornado diagram, is used in their study. The tornado diagram, as shown in Figure 2.3, consists of a set of horizontal bars, one for each random variable. The length of each bar (referred to as swing) represents the variation in the output due to the variation in the respective random variable. Thus, a variable with larger effect on the output has a larger swing than those with lesser effect.

Recently, Baker and Cornell (2003) developed an approach to calculate total uncertainty of future repair costs of a structure using the first-order second moment (FOSM) method. This proposed approach works within the framework of PEER's PBEE methodology. They estimated means and standard deviations of several conditional random variables, namely, DV|DM and DM|EDP in (2.1), using the FOSM method to develop a single con-



### Fig. 2.3 Sensitivity of future repair cost to uncertain input parameters for Van Nuys building, California, after Porter et al. (2002).

ditional random variable (total repair cost in this case) given IM. Subsequently, the ground motion hazard is treated accurately using Monte Carlo simulation based on the assumption that ground motion uncertainty is the dominant contributor to total uncertainty of future repair cost.

### 2.5 UNCERTAINTY IN THIS REPORT

The focus of this study is on the structural analysis phase of PEER's PBEE methodology, in particular for RC structures. Accordingly, uncertainty of interest is related to this phase. Initially, seismic hazard (or IM) uncertainty enters into the structural analysis phase as it represents uncertainty in the hazard analysis phase performed prior to the structural analysis phase. Then, the structural analysis phase has its own sources of uncertainty in modeling assumptions such as material properties, boundary conditions and structural geometry. This section summarizes findings of other researchers on uncertainties of various parameters in structural analysis. Only those related to applications in this study are presented in this section.

### 2.5.1 Uncertainty in Hazard Analysis Phase

There are various ways of characterizing IM of an earthquake. The typical IMs are measured peak ground motions and damped elastic responses in terms of acceleration, velocity, and displacement. There have been efforts to define a proper IM of an earthquake in relation to EDP (Taghavi and Miranda 2003; Cordova et al. 2001), where IM is strongly correlated to EDP such that IM becomes an indicator of EDP.

Among those typical IMs, the damped elastic spectral acceleration at the fundamental period of the structure is commonly used because it is strongly correlated to various EDPs and its probability function of occurrence is readily available. The Earthquake Hazard Program in the U.S. Geological Survey (USGS) provides US national maps showing earthquake ground motions that have a specified probability of being exceeded in 50 years (http://eqhazmaps.usgs.gov). The peak ground acceleration and the damped elastic spectral acceleration at the fundamental period of the structure are used as IMs in these maps.

Another type of uncertainty in seismic hazard is the ground motion profile (referred to as details of ground motion by Porter et al. (2002)). Unlike any other uncertainty addressed in this study, it is not straightforward to characterize the uncertainty in the ground motion profiles using methods other than Monte Carlo simulations. In that regard, a large number of ground motion profiles are used to obtain a set of outputs (e.g., EDP in this study) to be subsequently post-processed for their statistics. The incremental dynamic analysis (IDA) (Vamvatsikos and Cornell 2002) is one of the methods that can explicitly deal with this uncertainty using a set of ground motion profiles. For such set, one may use either recorded ground motions or simulated ones that are generated from a mathematical model. Since the comparison between using recorded (and possibly scaled to modify IM) versus simulated ground motion is still an on-going research in earthquake engineering, a set of recorded ground motions is employed in the study presented in this report.

### 2.5.2 Uncertainty in Structural Analysis Phase

Sources of uncertainty associated with the structural analysis phase include structural geometry, material properties, modeling assumptions, and construction errors. Structural geometry includes, for example, beam and column dimensions, and locations and sizes of reinforcing bars. Material properties include parameters defining individual material constitutive models such as concrete compressive strength and initial modulus of elasticity, or steel yield strength, ultimate strength, and initial modulus of elasticity. Several research efforts have been focused on studying the effect of uncertainty in material properties or structural geometry on the behavior of structural components or systems (Chryssanthopoulos et al. 2000; Singhal and Kiremidjian 1996; Frangopol et al. 1996; Grant et al. 1978; Knappe et al. 1975). Modeling assumptions include gravity load, mass, viscous damping, force and displacement boundary conditions, time step integration scheme, soil-foundation interface, and three-dimensional effects such as floor eccentricities between center of mass and center of rigidity. Selection of element type is also a source of uncertainty, since different elements use different assumptions and approximations in their element formulation.

### 2.5.2.1 Uncertainty in Concrete Properties

Mirza et al. (1979) proposed probability distributions of various static strength parameters of concrete by regression analyses. Of interest among those are the compressive strength and the initial modulus of elasticity. They suggested that the compressive strength of concrete has normal distribution with the mean computed by

$$\bar{f}'_c = 0.675 f'_c + 1,100 \le 1.15 f'_c$$
 in psi unit (2.2)

where  $f'_c$  is the design compressive strength of concrete and  $\bar{f}'_c$  is the mean of the compressive strength. Dispersions of the distribution are suggested as coefficients of variations (COV) of 10%, 15%, and 20% for excellent, average, and poor quality control, respectively, for strength levels below 4,000 psi. For concrete with an average strength above 4,000 psi,

standard deviations are suggested as 400 psi, 600 psi, and 800 psi also for excellent, average, and poor quality control. On the other hand, the initial tangent modulus of elasticity is suggested to have normal distribution with the mean computed by a regression equation

$$\bar{E}_c = 60,400\sqrt{f'_c} \text{ in psi unit}$$
(2.3)

and COV = 8%. They also realized that a strong correlation between compressive strength and the initial modulus of elasticity exists as indicated by a high correlation coefficient in the range of 0.88 to 0.91 as the results of various regression analyses. It should be noted that the suggestions Mirza et al. (1979) discussed above do not include long term effects on concrete such as creep and shrinkage.

Kappos et al. (1999) studied uncertainty in the ductility of confined RC members and suggested COV = 32-36% for ultimate strain of confined concrete where ultimate strain is defined by the strain corresponding to 85% of the compressive strength of the corresponding unconfined concrete along the descending branch of the stress-strain relationship.

### 2.5.2.2 Uncertainty in Reinforcing Steel Properties

Mirza and MacGregor (1979a) proposed probability distributions of various mechanical properties of reinforcing bars. In particular, yield strength, ultimate strength, and modulus of elasticity of reinforcing steel are of interest. They suggested the lognormal distributions for yield and ultimate strengths. Suggested values of means and COVs for yield strength are respectively 48.8 ksi and 10.7% for Grade 40 bars and 71.0 ksi and 9.3% for Grade 60 bars. They observed that an increase in mean ultimate strength of reinforcing steel over the yield strength is on the order of 55% with COV remaining approximately unchanged. For example, the suggested mean and standard deviation of ultimate strength is 110.1 ksi and 10.2 ksi (COV = 9.3%), respectively, for Grade 60 bars. It is also suggested that the probability distribution of the modulus of elasticity of Grade 40 or 60 reinforcing steel is normal with the mean 29,200 ksi and COV = 3.3%. The Joint Committee for Structural Safety (JCSS) (1996) proposed probability distributions of yield strength, ultimate strength, and ultimate strain of reinforcing steel. Normal distribution is suggested for all three properties. It is suggested that for yield strength, mean is computed by

$$\bar{f}_y = f_y + 2\sigma \tag{2.4}$$

where  $\bar{f}_y$  and  $f_y$  are mean and nominal yield strengths, respectively, and  $\sigma$  is the standard deviation that is assumed as 30 MPa (4.4 ksi). Similar to the suggestions of Mirza and MacGregor, JCSS suggested that mean of ultimate strength is 50% higher than that of yield strength while the standard deviation of the ultimate strength is assumed as 40 MPa (5.8 ksi). The correlation coefficient between yield strength and ultimate strength is assumed to be 0.85. For ultimate strain of reinforcing steel, JCSS suggested COV = 9% while any suggestion on the mean of ultimate strain is not found in (Joint Committee for Structural Safety 1996). The correlation coefficient between ultimate strain and yield strength, and ultimate strain and ultimate strength are suggested as -0.50 and -0.55, respectively. These suggestions imply that an increase of either yield strength or ultimate strength of reinforcing steel may lead to a reduction in ultimate strain. It is noted that Mirza and Skrabek (1991) suggested using normal distribution with mean = 15% and COV = 20% for ultimate strain of reinforcing steel.

### 2.5.2.3 Uncertainty in Member Geometry

Mirza and MacGregor (1979b) studied variation in the geometry of RC members such as beam and column cross-section dimensions, bar location, and slab thickness. Of interest in this study is uncertainty in reinforcing steel placement of RC columns. In that respect, the location of longitudinal reinforcing bars is dictated by the cover thickness of concrete. Based on measurement of steel placement errors in 232 rectangular columns of 12 in-situ concrete buildings of various size in the Toronto-Hamilton area in Canada, they suggested that the cover thickness can be described by a normal distribution with a mean given by

$$\bar{t} = t_{sp} + 0.25 + 0.004h$$
 in inch (2.5)

where  $\bar{t}$ ,  $t_{sp}$ , and h are the mean and specified cover thickness, and the dimension of the long side of the column cross section, respectively, and standard deviation of 0.166 inch.

### 2.5.2.4 Uncertainty in Modeling Assumptions

Quantification of building mass in a dynamic structural analysis depends on several factors, namely materials used in construction, structural dimensions, locations of non-structural elements, and a numerical model of the structure such as choice of its nodal coordinates. Uncertainty in structural mass is an integration of uncertainties in those factors. Ellingwood et al. (1980) suggested that the probability distribution of dead load is normal with a mean value equal to the nominal dead load and COV = 10%. In this study, the probability distribution of building mass is assumed to be normal with a mean value equal to mass computed from the nominal self weight and the superimposed dead load.

A comprehensive discussion of uncertainty in the viscous damping ratio is presented by Porter et al. (2002) including a summary of earlier experimental studies by Taoko (2003), Camelo et al. (2001), and McVerry (1979). Based on these experimental studies, Porter et al. (2002) suggested a reasonable estimate of COV of the damping ratio in the range of 30% to 40%, while the mean of damping ratio is reported in the range of 1.1% to 11%. In this study, the mean and COV of damping ratio are assumed to be 5% and 40%, respectively.

#### 2.5.2.5 Excluded Uncertainty

Not all possible uncertainties are considered in this study. Among excluded ones are those due to soil-foundation interface modeling, three-dimensional effect, non-structural components, and gravity load including its role in P- $\Delta$  effect, all of which may significantly affect EDP uncertainty. The propagation of these uncertainties should be studied in the future extension of the presented study. Some other sources of uncertainties whose effects on EDPs are assumed to be small such as the tensile strength of concrete are considered as deterministic at their best estimate such as the mean or the median.

### 2.5.2.6 Spatial Variability

Specified uncertainties in the structural analysis phase should, in general, spatially vary. This is commonly referred to as the *spatial variability* of uncertainty. Lee and Mosalam (2004) pointed out the importance of considering the spatial variability of material and geometrical properties in estimating the strength of RC columns. In this report, the spatial variability of structural properties is only considered in Chapter 4 for the evaluation of structural components, as it is not practical to consider it in the evaluation of structural systems due to the expected high computational effort.

### 2.6 METHODOLOGY OF PROPAGATING UNCERTAINTY IN THIS STUDY

This section describes a systematic method for propagating uncertainty, in particular, from the basic uncertainty to uncertainties in the capacity of structural components and from that to EDP uncertainty of a structural system. These two types of propagation of uncertainty are covered in Chapters 4 and 5, respectively.

#### 2.6.1 Overview

By definition, a structural system consists of a number of structural components. In fact, almost all structural systems consist of a few repeatedly used structural components (referred to as typical components) such as typical columns and beams. If each of these typical structural components is investigated separately, the result can be portable to many types of structural systems, as if the generic structural components were tested in the laboratory and the results are subsequently used in estimating the behavior of structural systems. Once the understanding of the expected level of damage for different sets of boundary conditions is established for typical structural components, the damage of an entire structural system can be estimated in a systematic manner. Accordingly, the methodology of evaluating the propagation of basic uncertainties to the structural system with respect to EDP consists of two phases: (1) component evaluation phase where the propagation of basic uncertainties to structural components with respect to their capacity is evaluated and (2) system evaluation phase where the propagation of uncertainty in the capacity of structural components to the structural system with respect to EDP is evaluated.

### 2.6.2 Component Evaluation

#### 2.6.2.1 Identifying Typical Structural Components

The first step in the methodology is to identify typical structural components of a given structural system using the conventional definitions of beam and column. For example, the length of a typical beam is defined by the distance between two column centerlines, while the height of a typical column is defined by the distance between the centerlines of slabs or beams, whichever the column meets, at both ends of the column. The strength and the deformation capacities of a typical structural component are defined and evaluated at given force boundary conditions that can be identified by a linear elastic analysis of the entire structural system. The gravity load and a properly distributed lateral load such as the one that resembles the shape of the first vibrational mode of the structural system along its height are applied in this analysis. Figure 2.4 depicts the process of identifying typical structural components. It should be noted that structural components with identical dimensions could be subjected to different force boundary conditions depending on the location of the structural components in the structural system.

Displacement boundary conditions of a typical structural component (modeled as a planar beam element) are defined such that all three degrees of freedom, namely translations



Fig. 2.4 Process of identifying typical structural components by elastic analysis.

in lateral and vertical directions and the rotation about the out-of-plane axis, are fixed at one end, while only the rotational degree of freedom is fixed at the other end of the component. Force boundary conditions are defined at free degrees of freedom only, as depicted in Figure 2.4, where force boundary conditions for some of the typical structural components are identified as axial loads and lateral loads whose directions are perpendicular to the length of the component. The axial load at each typical structural component consists of two parts: one is due to the gravity load and the other is due to lateral loads applied to the structural system. The former is referred to as  $P_{ai}$ , i = 1, 2, and 3, and the latter is referred to as  $\alpha_j P_{lj}$ , j = 1, 2, and 3, in Figure 2.4 where  $\alpha_j$  and  $P_{lj}$  are the constant of proportionality and the lateral load, respectively. In other words,  $\alpha_j$  is the ratio of the axial force in the component induced by applied lateral loads to the structural system to the shear force in the component. It should be noted that lateral loads are applied to the structural system such that approximately  $\alpha_j = 0$  in all beams, as depicted in Figure 2.4. In this way, the force boundary conditions in structural components under the ground shaking can be better described by eliminating the effect of *being pushed*.

The forces applied to a typical structural component are determined such that they can closely reproduce the axial force, shear force, and bending moment diagrams of the corresponding component based on the linear elastic analysis of the structural system. It is noted that the assumed displacement and force boundary conditions of the typical structural component are based on anti-symmetric bending moments at both ends of the component with the inflection point in the middle of the component length. Even though the bending moments at both ends of the component based on the linear elastic analysis of the structural system are not necessarily anti-symmetric, they are usually close to anti-symmetric for regular framed structures under lateral loads when no distributed load along the length of the structural component is applied. This specific setup of displacement and force boundary conditions makes it possible to estimate correlations between two end cross sections in terms of parameters describing force-deformation relationships. Using an elastic analysis as a basis of identifying typical structural components of the structural system is one of the many possible ways of defining the force boundary conditions of structural components. It is selected in this study because it is both simple and efficient to address variation in the axial load of the typical structural component due to applied lateral load to the structural system. However, a possible influence of nonlinear behavior, such as load redistribution due to damage of structural components, on changing the force boundary conditions of typical structural components is not considered. Nevertheless, an elastic analysis is the choice of this study because its objective is to demonstrate a systematic approach of understanding propagation of basic uncertainties to the structural system. Moreover, this specific way of identifying typical structural components can be replaced by other working approaches without affecting the developed general methodology. A possible solution for this issue is suggested in Section 2.6.4.

#### 2.6.2.2 Stochastic Fiber Element Model

Each identified typical structural component is evaluated to develop *probabilistic section models* that describe the probability distribution of a force-deformation relationship of a typical structural component such as the moment-curvature relationship or the shear forcedistortion relationship at critical cross sections of the component. In general, a critical cross section is defined as the one with the largest force (e.g., bending moment) or deformation (e.g., curvature) demand. In that regard, both end cross sections of a structural component are defined as critical cross sections in this study because their moment or curvature demands are the largest when no distributed load along the length of the structural component is applied, which is assumed to be the case in this study. It is noted that one of the two critical cross sections represents a cross section under a positive bending while the other one is under a negative bending. For the process of developing probabilistic section models, the *stochastic fiber element model* is developed in this study, such that spatial variability of the material and geometrical properties in the structural model is accounted for in the conventional
(deterministic) fiber element model. This model is developed in the framework of Monte Carlo simulation using a random field representation method as discussed in Chapter 4.

#### 2.6.2.3 Probabilistic Moment-Curvature Relationship

Among force boundary conditions of each typical structural component, the lateral load is monotonically increased (the part of the axial load proportional to the lateral load is increased accordingly), while the constant part of the axial load due to the gravity load is applied simultaneously until the ultimate deformation capacity is reached at the critical cross section of the typical structural component. The moment-curvature relationship at this cross section is idealized as a multilinear relationship defined by several moment-curvature pairs. Figure 2.5 shows a trilinear moment-curvature relationship defined by three critical points, namely the yielding point, the peak point, and the ultimate point. The yielding point  $(\varphi_y, M_y)$  is defined by the moment and curvature corresponding to the first yielding of any longitudinal reinforcing bar. The peak point  $(\varphi_p, M_p)$  is defined by the maximum moment and its corresponding curvature. The ultimate point  $(\varphi_u, M_u)$  is defined by the moment and curvature corresponding to the ultimate compressive strain of a confined concrete fiber or the fracture strain of a steel fiber whichever occurs first (cf. Section 4.2.3).



Fig. 2.5 Idealization of a moment-curvature curve.

Monte Carlo simulation produces random samples for each of the six parameters defining the trilinear moment-curvature relationship, and the probabilistic distributions of these parameters can be estimated using simple statistics. In this study, the means, variances, and covariances of the six parameters are estimated by sample means, sample variances, and sample covariances, respectively. Let  $\mathbf{X} = [X_1, X_2, \dots, X_6]^T = [M_y, M_p, M_u, \varphi_y, \varphi_p, \varphi_u]^T$  be a vector containing the six random variables, i.e., the six parameters, having mean  $\mu_i$  and variance  $\sigma_i^2$ ,  $i = 1, \dots, 6$ . The sample mean  $\bar{X}_i$  and sample variance  $S_i^2$  of  $X_i$  are given by

$$\bar{X}_{i} = \frac{1}{N} \sum_{k=1}^{N} X_{ik}$$
(2.6)

$$S_i^2 = \frac{1}{N-1} \sum_{k=1}^{N} \left( X_{ik} - \bar{X}_i \right)^2$$
(2.7)

where  $X_{ik}$  is the  $k^{th}$  sample of random variable  $X_i$  and N is the sample size. Sample covariance  $S_{ij}$  of  $X_i$  and  $X_j$  is given by

$$S_{ij} = \frac{1}{N-1} \sum_{k=1}^{N} \left[ (X_{ik} - \bar{X}_i)(X_{jk} - \bar{X}_j) \right]$$
(2.8)

where  $X_{ik}$  and  $X_{jk}$  are the  $k^{th}$  samples of random variables  $X_i$  and  $X_j$ , respectively. It should be noted that  $\bar{X}_i$ ,  $S_i^2$ , and  $S_{ij}$  are unbiased estimates of the true means, variances, and covariances with  $X_j$  of  $X_i$ , respectively. The final step of developing the probabilistic moment-curvature relationship is estimating the distribution type (e.g., normal distribution) to each of the six parameters. Rational judgment based on, e.g., a histogram or a q-q plot<sup>1</sup> is required for this process. The process of developing a probabilistic moment-curvature relationship is illustrated in Figure 2.6.

 $<sup>^1{\</sup>rm The}$  quantile-quantile (q-q) plot is a graphical technique for determining if two sets come from populations with a common distribution.



Fig. 2.6 Process of developing a probabilistic moment-curvature curve.

# 2.6.2.4 Probabilistic Shear Force-Distortion Relationship

The conventional fiber element model (including the stochastic fiber element model developed in this study) does not consider either the axial force-shear force or the bending moment-shear force interactions at the critical cross sections of the structural components. Therefore, probabilistic shear force-distortion relationships at the critical cross section of a typical structural component is developed using Response 2000 (2000) software based on the modified compression field theory (Vecchio and Collins 1986) instead of the stochastic fiber element model. Due to the limitation of Response 2000, Monte Carlo simulation is not appropriate for developing a probabilistic shear force-distortion relationship. Instead, the FOSM method (cf. Section 3.2.1) is adopted.

Similar to the idealized moment-curvature relationship (cf. Figure 2.5), a computed shear force-distortion curve is idealized as a trilinear relationship. Three points defining the idealized shear force-distortion curve are the cracking point, the peak point, and the ultimate point. The cracking point ( $\gamma_c$ ,  $V_c$ ) corresponds to the initiation of the shear crack. The peak point ( $\gamma_p$ ,  $V_p$ ) is defined by the maximum shear force and its corresponding distortion. The ultimate point ( $\gamma_u$ ,  $V_u$ ) corresponds to the fracture of the transverse steel. The means, standard deviations, and correlation coefficient matrices of parameters defining the cracking, peak, and ultimate points are estimated using the FOSM method to develop a probabilistic shear force-distortion relationship.

## 2.6.3 System Evaluation

Although the stochastic fiber element model can be used for probabilistic evaluation of any framed structure, it is not practical to use it for a complete structural system due to the large computational demand of Monte Carlo simulation combined with the fiber element model. Instead, a plastic hinge model (or a lumped plasticity model) (D'Ambrisi and Filippou 1999) is used to develop a computational model of the structural system. The behavior of the plastic hinge model is defined by probabilistic moment-curvature and shear force-distortion relationships obtained from the component evaluation phase. Moreover, instead of Monte Carlo simulations, the FOSM method can be used to compute EDP uncertainty of the structural system to avoid the need for a large computational effort. Figure 2.7 illustrates the procedure of the system evaluation using probabilistic section models of typical structural components in the context of the FOSM method to estimate EDP uncertainty.

In the process of propagating uncertainty from the typical structural component capacity in terms of critical section behavior to the system EDP, a practical approach can be followed. In that regard, one should note that EDP uncertainty induced by uncertainty in one of the structural components is a measure of sensitivity of EDP to the corresponding component. For example, in Figure 2.7, uncertainties in the peak displacement induced by uncertainties in Column 1 and in Beam 4 can be estimated using the FOSM method and the corresponding measures of sensitivity such as COV can be computed. From these measures, the relative significance of each component to the system EDP can be identified and ranked accordingly.

## 2.6.4 Suggested Iterative Evaluations of Structural Components and System

As discussed in Section 2.6.2.1, the procedure of identifying typical structural components based on the linear elastic analysis of the structural system cannot consider an influence of



Fig. 2.7 System evaluation procedure using probabilistic section models and the FOSM method.

nonlinear behavior due to damage of structural components. Possible errors caused by this simplification will propagate to the system evaluation phase and affect the uncertainty estimation of the structural system. One way of reducing the possible errors and overcoming this drawback is to use an iterative process of the component and system evaluations such that, for example, the methodology described in Sections 2.6.2 and 2.6.3 is considered as the first iterative process. In this hypothetical case, information regarding forces and deformations of the structural system at the end of the system evaluation phase will be used to modify the boundary conditions of typical structural components in the beginning of the component evaluation phase for the second iteration. Moreover, structural components identified as insignificant to the selected EDPs in the previous iteration can be treated as deterministic structural component. This iterative process is schematically described in Figure 2.8. It should be noted that the suggested iterative approach is not pursued in this report because it is not within the scope of the present study.



Fig. 2.8 The suggested iterative process of propagating basic uncertainties to the structural system.

# 2.7 CONCLUDING REMARKS

The propagation of basic uncertainties to the structural system with respect to its EDP is evaluated within a general framework of the PBEE method. In this chapter, the PEER's PBEE methodology and the treatment of uncertainty in this methodology are introduced as the background of the present study. The focus of this study is on the hazard analysis and the structural analysis phases, as two important phases of several ones forming PEER's PBEE methodology. Accordingly, the corresponding sources of uncertainties are identified and characterized based on the previous literatures.

An overview of the methodology of evaluating the propagation of basic uncertainties to the structural system with respect to EDP is presented. This methodology consists of two phases, namely the component evaluation and the system evaluation. In the component evaluation phase, typical structural components that form the structural system are evaluated using the stochastic fiber element model to obtain probabilistic section models such as the probabilistic moment-curvature relationship at critical cross sections of the structural component. In the system evaluation phase, the structural system is evaluated using the probabilistic section models developed in the component evaluation phase to estimate EDP uncertainty of the system. EDP sensitivity to each structural component can be used to identify and rank relatively significant structural component to a specific EDP.

# 3 EDP Sensitivity Induced by Basic Uncertainty

## 3.1 INTRODUCTION

Probabilistic evaluation of structural performance subjected to seismic loading is an important procedure for damage and loss estimation in PBEE. Recently, several researchers studied propagation of uncertainty within the framework of PEER's PBEE methodology (Baker and Cornell 2003; Miranda and Aslani 2003; Porter et al. 2002). Porter et al. (2002) investigated sensitivity of loss estimate of an RC building to major uncertain parameters in PBEE, such as ground motion intensity, structural properties, member fragility, and unit repair cost. Baker and Cornell (2003) proposed a procedure for estimating total uncertainty in the repair cost of a structure using the FOSM method. Miranda and Aslani (2003) developed a methodology to evaluate the expected annual loss in buildings under seismic loadings using a component-based approach. However, research efforts focusing on the EDP sensitivity is rare. Such investigation can identify sources of uncertainty whose further investigation might reduce total uncertainty in EDP. On the other hand, one may ignore uncertainties of relatively insignificant variables by treating them as deterministic ones fixing their values at the best estimate, such as the mean or the median to reduce computational effort.

Various methods can be used for computing EDP uncertainty, e.g., Monte Carlo simulation and the FOSM method. Monte Carlo simulation can estimate the probabilistic distribution of EDP with good accuracy. However, this method might be computationally demanding, especially for a structural model consisting of a large number of degrees of freedom, a nonlinear time history analysis, or a large number of simulations. One of the challenges in the probabilistic evaluation of a structural system performance is the relatively high computational cost compared to a deterministic analysis of a structural system or a probabilistic analysis of a structural component. On the other hand, the FOSM method can be used as an approximate approach to estimate the mean and the standard deviation of an EDP. Due to its simplicity and efficiency, the FOSM method is useful for sensitivity study in situations where Monte Carlo simulation is not practical from a computational point of view.

The objective of this chapter is to study how basic uncertainties propagate through a structural system to affect EDP uncertainties, and to identify relatively important sources of uncertainty to a given EDP. The procedure of such identification is illustrated using a case study of an RC shear-wall building. In this application, relatively important random variables to both global and local EDPs of the building are identified. Although the FOSM method is used throughout this procedure, EDP uncertainty due to uncertainty in ground motion profile (or record-to-record variability) is computed by Monte Carlo simulation because the FOSM method is not applicable to this type of uncertainty.

# 3.2 METHODS OF SENSITIVITY ANALYSIS

Two different methods are used for sensitivity analysis of EDP in this chapter. One uses the FOSM method and the other uses a tornado diagram. These two methods are discussed in the following two sections.

## 3.2.1 First-Order Second-Moment Method

Let's consider the function Y = g(X) of a single random variable X having the mean  $\mu_X$  and variance  $\sigma_X^2$ . Provided that the derivatives of g(X) with respect to x exist, the first-order approximation of g(X) using Taylor series expansion evaluated at  $x_0$  is given as

$$Y \approx g_0 + \left(\frac{dg}{dx}\right)_0 (X - x_0) \tag{3.1}$$

where ()<sub>0</sub> denotes a function evaluated at  $x_0$ . The first moment of Y, i.e., the mean  $\mu_Y$ , can be derived from (3.1) as

$$\mu_Y = \mathcal{E}[g(X)] \tag{3.2}$$

$$\approx \mathrm{E}\left[g_0 + \left(\frac{dg}{dx}\right)_0 (X - x_0)\right]$$
(3.3)

$$\approx \operatorname{E}[g_0] + \left(\frac{dg}{dx}\right)_0 \operatorname{E}\left[(X - x_0)\right]$$
 (3.4)

$$\approx g_0 + \left(\frac{dg}{dx}\right)_0 (\mu_X - x_0) \tag{3.5}$$

Especially when  $x_0 = \mu_X$  (this is a typical choice of FOSM method),  $\mu_Y$  can be given as

$$\mu_Y \approx g(\mu_X) \tag{3.6}$$

The second moment of Y, i.e., variance  $\sigma_Y^2$ , can be derived from (3.1) as

$$\sigma_Y^2 = \mathbf{E}\left[g^2(X)\right] - \mu_Y^2 \tag{3.7}$$

$$\approx \mathbf{E}\left[g_0^2 + \left\{\left(\frac{dg}{dx}\right)_0 (X - x_0)\right\}^2 + 2g_0\left(\frac{dg}{dx}\right)_0 (X - x_0)\right] - \mu_Y^2 \tag{3.8}$$

$$\approx g_0^2 + \left(\frac{dg}{dx}\right)_0^2 \mathbf{E}\left[(X - x_0)^2\right] + 2g_0 \left(\frac{dg}{dx}\right)_0 (\mu_X - x_0) - \mu_Y^2 \tag{3.9}$$

$$\approx g_0^2 + \left(\frac{dg}{dx}\right)_0^2 \sigma_X^2 + 2g_0 \left(\frac{dg}{dx}\right)_0 (\mu_X - x_0) - \mu_Y^2$$
(3.10)

For  $x_0 = \mu_X$ , (3.10) is given as

$$\sigma_Y^2 \approx \left(\frac{dg}{dx}\right)_0^2 \sigma_X^2 \tag{3.11}$$

The approximations in (3.6) and (3.11) are called the first-order second-moment (FOSM) method (Melchers 1999). It should be noted that in (3.11), the approximate  $\sigma_Y$  is proportional to  $\sigma_X$  and  $|(dg/dx)_0|$  represents a measure of sensitivity of the function to the variation in X.

Now let's consider a random vector  $\mathbf{X} = [X_1, X_2, \dots, X_n]^T$  having the mean vector  $\boldsymbol{\mu}_{\mathbf{X}} = [\mu_1, \mu_2, \dots, \mu_n]^T$  and variance-covariance matrix VC[**X**]. Let's consider the functions  $Y = g(\mathbf{X}), Y_1 = g(X_1, \mu_2, \mu_3, \dots, \mu_n), Y_2 = g(\mu_1, X_2, \mu_3, \dots, \mu_n), \dots$ , and  $Y_n = g(\mu_1, \mu_2, \dots, \mu_{n-1}, X_n)$ . The first and the second moment approximations of  $Y, Y_1, Y_2, \dots, Y_n$  by the FOSM method are

$$\mu_Y = \mu_{Y_i} \approx g(\mu_1, \mu_2, \cdots, \mu_n) = g(\boldsymbol{\mu}_{\boldsymbol{X}}), \quad i = 1, 2, \cdots, n$$
 (3.12)

$$\sigma_Y^2 \approx \nabla^T g(\mathbf{X}) \operatorname{VC}[\mathbf{X}] \nabla g(\mathbf{X})$$
 (3.13)

$$\sigma_{Y_i}^2 \approx \left(\frac{\partial g}{\partial x_i}\right)_0^2 \sigma_i^2, \quad i = 1, 2, \cdots, n$$
(3.14)

where  $\nabla g(\mathbf{X}) = [\partial g/\partial x_1, \partial g/\partial x_2, \cdots, \partial g/\partial x_n]^T$ , the gradient vector of  $g(\mathbf{X})$  with respect to  $\mathbf{X}$  and diag(VC[ $\mathbf{X}$ ]) =  $[\sigma_1^2, \sigma_2^2, \cdots, \sigma_n^2]^T$ .  $\sigma_{Y_i}^2$  can be interpreted as a measure of sensitivity of Y with respect to  $X_i$ . Using these quantities, one can compare the effect of different random variables to the output of the function. In that regard,  $\sigma_Y^2$  is a comparable measure of sensitivity of Y with respect to  $\mathbf{X}$ . Note that  $\sigma_Y^2$  and  $\sigma_{Y_i}^2$  are all scalar quantities where only  $\sigma_Y^2$  takes into account the correlations between the components of the random vector  $\mathbf{X}$ .

In this study, a finite element model (FEM) is used as the method to develop the function g in the above derivation to evaluate the EDPs. Moreover, the gradients of g are numerically evaluated using the finite difference approach, i.e.,

$$\frac{\partial g}{\partial x_i} = \frac{g(\mu_i + \Delta x_i) - g(\mu_i - \Delta x_i)}{2\Delta x_i}, \quad i = 1, 2, \cdots, n$$
(3.15)

where it is assumed that  $\Delta x_i = a_p \sigma_i$ ,  $\sigma_i$  is the standard deviation of the *i*th random variable, and  $a_p$  is a coefficient of proportionality that is determined by a convergence test. Let's assume that the mean of the modulus of elasticity of reinforcing steel is 29,000 ksi, while that of its yield strain is 0.002. A fixed perturbation, e.g.,  $\Delta x_i = 10$ , might be reasonable for the random variable representing the modulus of elasticity, but it is not acceptable for that of the yield strain. In this way, a fraction of  $\sigma_i$  is used as the perturbation  $\Delta x_i$  in (3.15) to handle different scales of random variables. Further discussion on the convergence test for determining  $a_p$  is presented in Section 3.4.5.

## 3.2.2 Tornado Diagram Analysis

The tornado diagram, commonly used in decision analysis (Clemen 1996), has been recently used in sensitivity analysis in earthquake engineering (Porter et al. 2002). This tornado diagram consists of a set of horizontal bars, referred to as swings, one for each random variable. The length of each swing represents the variation in the output due to the variation in the respective random variable. Thus, a variable with larger effect on the output has larger swing than those with lesser effect. In a tornado diagram, swings are displayed in the descending order of the swing size from the top to the bottom. This wide-to-narrow arrangement of swings eventually resembles a tornado.

In this study, the output (EDP in this case) is assumed to be a known deterministic function (developed using FEM) of a set of input random variables whose probability distributions are assumed by the analyst. For each input variable, two extreme values corresponding to pre-defined upper and lower bounds of its probability distribution (e.g., 10th and 90th percentiles) are selected. For each input random variable, the deterministic function is evaluated twice, using the two extreme values of the selected input random variable, while the other input random variables are set to their best estimates such as the medians. Figure 3.1 schematically shows a procedure of developing a swing. This process yields two bounding values of the output for each input random variable. The absolute difference of these two values is the swing of the output corresponding to the selected input random variable. This process is repeated for all other input random variables to compute the swings of the output. Finally one builds the tornado diagram by arranging the obtained swings in a descending order as mentioned above. The procedure of developing a tornado diagram is illustrated in Figure 3.2.



Fig. 3.1 Procedure of developing a swing in tornado diagram.

# 3.3 UC SCIENCE BUILDING

The presented methods for the sensitivity analysis in the previous section are applied to a laboratory (science) building located on the campus of the University of California, Berkeley (referred to as the UCS building in this study). This building represents one of two building test-beds, namely the UCS building and the Van Nuys building, for the PBEE design methodology developed by the PEER Center. The Van Nuys building, a seven-story building whose structural system is a RC moment-frame with flat-plate slabs, was built in 1966 at San Fernando Valley, California, and was strongly shaken and damaged in the 1971 San Fernando and 1994 Northridge earthquakes. PEER researchers working on the Van Nuys building are focusing on estimating structural and architectural damage, collapse potential, repair cost, and repair duration. On the other hand, the UCS building, a seven-story RC



Fig. 3.2 Procedure of developing a tornado diagram.

shear-wall building, is built in 1988 to provide high-technology research laboratories for organic biology. The focus of this test-bed is on estimating contents and equipment damage, and the life safety and operational consequences of such damage.

## 3.3.1 UCS Building Description

The UCS building is designed to meet the 1982 Uniform Building Code (International Conference of Building Officials (ICBO) 1982) for research laboratories, offices, and related support spaces. A RC space frame carries the gravity load of the building, and coupled shear-walls and perforated shear-walls resist lateral loads in the transverse and the longitudinal direction, respectively, as shown in Figure 3.3. The floors consist of waffle slab systems with solid parts acting as integral beams between the columns. The waffle slab is composed of a 4.5 inch-thick RC slab supported on 20 inch-deep joists in each direction. The foundation consists of a 38 inch-thick mat foundation. According to the design specifications, the concrete of the shear-walls (including the boundary columns) and the coupling beams has



Fig. 3.3 Plan view of the UCS building.

nominal 28-day compressive strength  $f'_c = 5$  ksi, while that of all other columns and the waffle slab systems has  $f'_c = 3$  ksi. The reinforcing steel is scheduled as ASTM A-615 Grade 40 for #4 and smaller bars and Grade 60 for #5 and larger bars. The middle frame in the transverse direction (labeled 8 in Figure 3.3 and referred to as Frame 8) is analyzed in the present study. Figure 3.4(a) presents the structural elevation view of Frame 8 and indicates the story heights and the labels of the building levels. The cross sections of the shear-walls and coupling beams, together with their reinforcement schedules of Frame 8 are shown in Tables 3.1 and 3.2, respectively.

# 3.3.2 Structural Modeling

The computational model of the UCS building is developed using the modeling capabilities of the open system for earthquake engineering simulation (OpenSees) software (McKenna and Fenves 2001). Two-dimensional (2D) idealization is considered in the modeling. It is noted that the longitudinal shear-walls are not considered as integrated parts of Frame 8. The complete OpenSees model of Frame 8 is illustrated in Figures 3.4(b).



Fig. 3.4 Elevation view and OpenSees structural model of Frame 8 of the UCS building.

				Column					/all
Story	Depth (in.)			Longitudinal reinf.			Transverse	t (in )	Reinf *
	$b_1$	$b_2$	$b_3$	$WC_1$	$WC_2$	$WC_3$	reinf.	0 (111.)	10.1111.
6th	24	25	24	8#8	8#8	8#8	#4@8''	14	#5@6''
5th	24	25	24	8#8	8#8	8#8	#4@8''	14	#5@6''
$4 \mathrm{th}$	24	31	24	8#8	12#8	8#8	#4@8''	14	#6@6''
3rd	24	37	24	8#8	12#9	8#8	#4@8''	14	#6@6''
2nd	24	44	24	8#8	16#10	8#8	#4@8''	16	#7@6''
1 st	33	49	30	12#9	22 # 10	12#9	#4@4''	18	#7@6''
Basement	33	49	30	14#11	26 # 11	12#9	#4@4''	18	#7@6''
* Same rein	force	ment	in b	oth the he	orizontal a	and verti	ical directions.	•	
-	$  \bullet_1 \rangle   \bullet_1 \rangle   \bullet_2 \rangle   \bullet_$								
$[7] WC_1 WC_1 [7] WC_2 WC_3 WC_2$									$WC_2$
									 ▶
(D) (E) (F) (G) (H)									H
	$\bigcirc$			$\bigcirc$	(	Č	B		A

Table 3.1Geometrical properties and reinforcement schedule of shear-wall crosssections of Frame 8 of the UCS building.

 Table 3.2
 Geometrical properties and reinforcement schedule of coupling beam

Level	Type "a"	Type "b"	
Roof	9#10 top and bottom	3#7 each face	Type "a"
6	9#10 top and bottom	3#7 each face	
5	10#11 top and bottom	3#8 each face	C Type "b"_2 24 5
4	10#11 top and bottom	3#8 each face	
3	11#11 top and bottom	3#8 each face	
2	10#11 top and bottom	3#8 each face	48"
1	10#11 top and bottom	3#8 each face	

cross sections of Frame 8 of the UCS building.

## 3.3.2.1 Element Types

Most elements in the building model are based on flexibility formulation of beam-column elements (referred to as nonlinearBeamColumn in the OpenSees element library). Each beam-column element has two nodes with two translations and one rotation per node. The beam-column element has four monitoring sections with fiber element discretization. In this discretization, a distinction is made between the constitutive model of the reinforcing bars, unconfined concrete, and confined concrete.

Shear-wall members of Frame 8 are modeled using beam-column elements aligned with the centerline of the shear-wall (Chaallal and Ghlamallah 1996). For proper idealization of the geometry, the node at the shear-wall centerline and the node at the boundary of the shear-wall (representing one end of a coupling beam) are connected by rigid elements, as shown in Figure 3.5. It should be noted that one element per story for columns and shearwalls, and one element per span for beams are used in the model illustrated in Figure 3.4(b).



Fig. 3.5 Modeling of shear-walls.

Figure 3.6 shows the shear force-distortion relationship of the coupling beam section at the sixth floor indicating the shear force capacity as well as the shear force demand due to KB-kobj. The shear force-distortion relationship is computed by the modified compression field theory using the software Response-2000 (Bentz 2000). It should be noted that the shear distortion in Figure 3.6 represents an average strain of concrete and steel reinforcement over the cross section. Considering the intensity level of KB-kobj ( $S_a = 2.4$ g), it is concluded that the shear force demand is well below their shear capacity of the cross section. Knowing that the shear force demands on the other cross sections are also below the capacities, it is decided that shear failure is not expected prior to flexural failure at the critical sections. Therefore, conventional fiber element modeling (Spacone, Filippou, and Taucer 1996) is considered for the UCS building.



Fig. 3.6 Shear capacity (based on the modified compression field theory by Response-2000 (Bentz 2000)) and demand (due to KB-kobj) of the coupling beam at the sixth floor (Element 55 in Figure 3.4(b)) of Frame 8 (refer to Table 3.2 for its design parameters).

# 3.3.2.2 Constitutive Models

In OpenSees, steel and concrete are modeled using uniaxial stress-strain relationships. In this study, cover and core concrete materials are defined separately using the model Concrete01 that is based on the modified Kent-Park stress-strain relationship (Scott et al. 1982) with degraded linear unloading/reloading stiffness according to the work of Karsan-Jirsa (1969) and no tensile strength as shown in Figure 3.7(a). The behavior of the ascending branch of the model is expressed as

$$f_c = f_o \left[ \frac{2\epsilon}{\epsilon_0} - \left( \frac{\epsilon}{\epsilon_0} \right)^2 \right] \text{ for } \epsilon \le \epsilon_0$$
(3.16)

where  $f_c$  is the stress,  $\epsilon$  is the corresponding strain,  $f_o$  is the compressive strength of the concrete expressed as  $f_{cc}$  and  $f_{co}$  for confined and unconfined concrete, respectively, in Figure 3.7(a), and  $\epsilon_0$  is the strain corresponding to  $f_o$  expressed as  $\epsilon_{cc}$  and  $\epsilon_{co}$  for confined and unconfined concrete, respectively, in Figure 3.7(a). The expression in (3.16) is valid up to the peak strength, beyond which the stress-strain relationships are approximated as linear functions. A residual stress of the confined concrete,  $f_{cc}^{resid}$ , is assumed as  $0.2f_{cc}$ , while it is assumed zero for the unconfined concrete. The compressive strength and the correspond-



Fig. 3.7 Stress-strain relationships of concrete and steel adopted from the OpenSees material library.

ing strain, and the ultimate strain of confined concrete ( $f_{cc}$ ,  $\epsilon_{cc}$ , and  $\epsilon_{cu}$ , respectively) are estimated using the Mander's model (Mander et al. 1988).

Steel reinforcing bars are modeled using a bilinear stress-strain relationship (referred to as Steel01 in the OpenSees material library) as shown in Figure 3.7(b) with a schematic cyclic behavior. Parameters defining this relationship are the yield strength  $f_y$ , the modulus of elasticity  $E_s$ , and the hardening ratio  $\alpha$ . In this study, all material parameters for concrete and steel are defined in Section 3.4.1 as random variables, except for  $\alpha$  which is specialized as 0.01.

## 3.3.2.3 Gravity Load and Mass Idealization

The dead load accounts for the self-weight of the waffle slab system and the supporting elements, i.e., shear-walls and columns. The assumed unit weight of the concrete is 145 pcf. Accordingly, the computed dead load is 183 psf, which is a relatively high value due to the large depth of the waffle slab system. Moreover, 25 psf representing building contents are included as a superimposed dead load. The live load of 100 psf is assumed according to the original design of the building. The mass of the building is modeled using lumped masses at the nodes. Nodal masses are directly computed from the total dead load including the self-weight and the superimposed dead load. The 2D model of Frame 8 has a tributary area with 101'-6" width as shown in Figure 3.3.

## 3.3.2.4 Viscous Damping Idealization

The damping characteristics of the building are modeled using mass and stiffness proportional damping with 5% of the critical damping for the first two modes of vibration. The periods of these two modes estimated from the eigen solution using the initial elastic stiffness matrix are 0.38 and 0.15 seconds.

## 3.3.2.5 Boundary Conditions

A flexible soil-structure interface at the foundation level is modeled using spring-type elements (referred to as zeroLength in the OpenSees element library) in the vertical direction. These elements represent soil with a modulus of subgrade reaction of 100 lb/in.<sup>3</sup> obtained using the soil properties described in Section 3.4.2.2 and the procedure presented in FEMA 273 (Applied Technology Council (ATC) 1997). The same tributary areas as those of building mass are used to consider the flexible supports of the 2D model of Frame 8. To simulate the characteristics of soil behavior, ENT material in the OpenSees material library is adopted. An element with ENT material has elastic properties in compression and zero tensile strength as shown in Figure 3.8.



Fig. 3.8 Constitutive model for the soil spring.

# 3.3.2.6 Solution Strategy

The Newmark  $\beta$ -method is used as the time integrator with typical coefficients  $\gamma = 0.50$  and  $\beta = 0.25$ . In general, a time step of 1/2 the ground motion time discretization (0.0025 to 0.01 seconds for the considered ground motions, refer to Section 3.4.2) is used for the analyses in the present study. The modified Newton-Raphson solution algorithm that updates the

stiffness matrix at the beginning of each time step only is utilized for solving the nonlinear equilibrium equations.

## 3.4 EDP SENSITIVITY OF THE UCS BUILDING

## 3.4.1 Uncertainties in Structural Properties

Sources of uncertainties considered in this part of the study are mass, viscous damping, stiffness, and strength representing uncertainties in the structural properties. Assumed statistical data related to uncertainties in structural properties are summarized in Table 3.3. These data are mainly adopted from various literatures as discussed in Section 2.5 due to the lack of data specifically related to the variability of the structural properties of the UCS building. It is noted that the nominal compressive strength of the shear-walls and the coupling beams is 5 ksi, while that of all interior columns and waffle slab is 3 ksi. However, all elements of Frame 8 of the UCS building have a nominal compressive strength of 5 ksi as listed in Table 3.3. It should be also noted that some random variables representing material parameters discussed in Section 3.3.2.2, e.g.,  $f_{cc}$ , are not explicitly defined, but derived from another random variable such as Fc and Ec using, e.g., the Mander model (Mander et al. 1988).

Table 3.3Statistical data of the structural properties treated as random vari-<br/>ables for Frame 8 of the UCS building.

Source of uncertainty	Variable	Dist'n	Mean	COV (%)
Mass (per unit floor area)	Ms	Normal	$0.27^b \text{ lb/ft}^2$	10.0
Damping ratio	Dp	Normal	0.05	40.0
Compressive strength of concrete	$Fc^a$	Normal	5  ksi	17.5
Yield strength of steel	Fy	Logn'l	60  ksi	10.0
Initial modulus of elasticity, concrete	$Ec^{a}$	Normal	$4,271 { m \ ksi}$	8.0
Initial modulus of elasticity, steel	Es	Normal	29,000 ksi	3.3

<sup>*a*</sup> Correlation coefficient of Fc and Ec is 0.8.

<sup>b</sup> Computed from the self-weight and the superimposed dead load.

## 3.4.2 Uncertainties in Ground Motion

The intensity measure and the profile of ground motion are considered as two sources of uncertainties in ground motion. This section describes information related to the chosen seismic hazard of the UCS building site to be used for the definition of the intensity measure. Moreover, selected different ground motions to address uncertainties in the ground motion profile are discussed.

#### 3.4.2.1 Seismic Hazard Curve

Frankel and Leyendecker (2001) provide a seismic hazard curve in terms of the mean annual exceedance frequency ( $\lambda$ ) of a specified spectral acceleration  $S_a$  for the location of the UCS building, at the fundamental period ( $T_1$ ) of 0.2, 0.3, and 0.5 seconds, and B-C soil boundary as defined by the International Building Code (International Code Council 2000). The seismic hazard curve for Frame 8 ( $T_1 = 0.38$  second) is interpolated from those of  $T_1 = 0.3$ and 0.5 seconds without any modification for the site class. The seismic hazard curves for Frame 8 is shown in Figure 3.9.

The temporal occurrence of an earthquake is most commonly described by a Poisson model (Kramer 1996). According to the Poisson assumption, the probability that no earthquake with a spectral acceleration in excess of  $S_a$  will occur in period t is

J

$$P_0 = e^{-\lambda t} = e^{-H(S_a)t}$$
(3.17)

where  $\lambda = H(S_a)$  denotes the mean rate of exceeding  $S_a$  (as given in Figure 3.9). From (3.17), one can compute percentiles of Sa for a given t, e.g., the 10th, 50th (i.e., the median), and 90th percentiles of Sa for t = 50 years are indicated by circles in Figure 3.9 and by numerical values in Table 3.4 where Sa is the random variable representing uncertainty in  $S_a$ . These three percentiles are used in developing tornado diagrams as discussed in Sections 3.4.4.

For the FOSM method, the mean and the standard deviation of a random variable are required rather than its percentiles. Therefore, the probability distribution of Sa is



Fig. 3.9 UCS building site seismic hazard curve for Frame 8.

Table 3.4 Percentiles of Sa used in the tornado diagrams of the UCS building.

Percentile	10th	50th (median)	90th
$S_a$ (g)	0.18	0.47	1.39

estimated by fitting the relationship between  $P_0$  and Sa in (3.17). The random variable Sa is assumed to have a lognormal distribution. For t = 50 years, (3.17) is well-fitted by the lognormal assumption of Sa with the mean of 0.633g and standard deviation of 0.526g (COV=83%) as shown in Figure 3.9. These values of the mean and standard deviation of Sa for Frame 8 are used for the FOSM method in Section 3.4.5.

## 3.4.2.2 Selected Ground Motions

The UCS building is located at a site consisting of stiff soil of thickness in the range of 20' to 52' (6 to 16 m), with an estimated average of about 39' (12 m) above Franciscan bedrock assumed to be not pervasively sheared and assumed to have a shear wave velocity of about 2953 ft/sec (900 m/sec). Older alluvium overlies the Franciscan rocks at the site.

The alluvium typically comprises very stiff sandy clay, with average standard penetration resistance values of 50 or greater and estimated shear wave velocity of about 1214 ft/sec (370 m/sec). The site is thus classified as NEHRP category  $S_C$  according to the site classification scheme in the NEHRP provision reproduced in Table 3.5.

	<b>0</b> <i>i</i>	• ( )
NEHRP	Description	Mean shear wave
Category	Description	velocity to $30 \mathrm{m}^a$
А	Hard rock	> 1500  m/sec.
В	Firm to hard rock	760 - 1500  m/sec.
$\mathbf{C}$	Dense soil, soft rock	180 - 360  m/sec.
D	Stiff soil	$< 180 { m m/sec.}$
${ m E}$	Special study soils, e.g., liquefiable soils,	
	sensitive clays, organic soils, soft clays	
	> 36 m thick	
$^{a}$ Mean sh	ear wave velocity from the surface to 30m	depth of the ground.
Note: 1 m	= 3.28 ft.	

Table 3.5NEHRP site categories, after Dobry et al. (2000).

The Hayward Fault, a strike-slip fault, traverses the campus of the University of California, Berkeley, with a trace within 2900 ft (900 m) of the UCS building. The PEER Testbeds Program (http://peertestbeds.net) provides a set of 20 recorded ground accelerations to be used for the site of the UCS building. These ground motions are selected to satisfy the distance and soil conditions of the building site for a strike-slip earthquake on the NEHRP category  $S_C$  site. Selected ground motion recordings are listed in Table 3.6. In general, it is not easy to satisfy the intended distance and soil condition requirements. However, these 20 ground motion records satisfy the requirements to the possible extent. For example, all records are within about 6.2 miles (10 km) from the fault (all strike-slip fault), and all but a few are from the  $S_C$  site. Response spectra of these 20 ground accelerations with 5% damping are plotted as well as the median response spectrum in Figure 3.10. Spectral acceleration values of each of the 20 ground accelerations at the fundamental period of Frame 8 of the UCS building are listed in Table 3.7 together with the median value.

Earthquake	$Mw^a$	Station name	$\text{Dist.}^{b}$	$\operatorname{Site}^{c}$	Name	
Coyote Lake	57	Coyote Lake Dam abutment	4.0	С	CL-clyd	
Jun 8, 1979	0.7	Gilroy #6	1.2	С	CL-gil6	
Parkorfield		Temblor	4.4	С	PF-temb	
Jup 27 1066	6.0	Array $\#5$	3.7	D	PF-cs05	
Juli 27, 1900		Array #8	nameDist. $^b$ Site $^c$ NameJake Dam abutment4.0CCL-clyd61.2CCL-gld64.4CPF-tem53.7DPF-cs0588.0DPF-cs0588.0DPF-cs0688.1CLV-fgmTerritory Park8.1CLV-mgmJake Dam abutment0.1CMH-clydAbam Downstream4.5CMH-andIley2.5CMH-halos Presentation Ctr.3.5CLP-lgpdAloha Ave8.3CLP-srtgss3.4CLP-corCollege9.5CLP-gavstoricN/ACLP-glbn Dam abutment6.3CLP-lexIA0.5CKB-kob1.0CTO-ttro1.8CEZ-erzi			
Livermore	55	Fagundes Ranch	4.1	D	LV-fgnr	
Jan 27, 1980	0.0	Morgan Territory Park	8.1	С	LV-mgnp	
Morgan Hill		Coyote Lake Dam abutment	0.1	С	MH-clyd	
$\frac{1000}{1000}$	6.2	Anderson Dam Downstream	4.5	С	MH-andd	
Api 24, 1964		Halls Valley	2.5	С	MH-hall	
		Los Gatos Presentation Ctr.	3.5	С	LP-lgpc	
		Saratoga Aloha Ave	8.3	С	LP-srtg	
Loma Prieta	7.0	Corralitos	3.4	С	LP-cor	
Oct 17, 1989	1.0	Gavilan College	9.5	С	LP-gav	
		Gilroy historic	N/A	С	LP-gilb	
		Lexington Dam abutment	6.3	С	LP-lex	
Kobe, Japan	6.0	Kobo IMA	0.5	С	KB kohi	
Jan 17, 1995	0.9	Robe JMA	0.5	U	KD-KODJ	
Tottori, Japan	6.6	Kofu	10.0	С	TO-ttr007	
Oct 6, 2000	0.0	Hino	1.0	С	TO-ttrh02	
Erzincan, Turkey Mar 13, 1992	6.7	Erzincan	1.8	С	EZ-erzi	

 Table 3.6
 Ground motion recordings selected for the UCS building case study.

 $^{a}$  The moment magnitude (Mw) is a measure that characterizes the relative size of an earthquake, that is based on measurement of the maximum motion records by a seismograph.

<sup>b</sup> Distance in km (1 km = 0.621 mile).

 $^{c}$  NEHRP site category (cf. Table 3.5).

Ground motion record	$S_a(T_1 = 0.38 \text{ sec.}), \text{ g}$
CL-clyd	0.79
CL-gil6	1.17
EZ-erzi	0.78
KB-kobj	2.45
LP-cor	0.97
LP-gav	1.14
LP-gilb	0.62
LP-lex	0.85
LP-lgpc	1.49
LP-srtg	0.73
LV-fgnr	0.60
LV-mgnp	0.26
MH-andd	0.82
MH-clyd	1.63
MH-hall	0.49
PF-cs05	1.14
PF-cs08	0.25
$\operatorname{PF-temb}$	1.18
TO-ttr007	1.56
TO-ttrh02	2.15
Median	0.91

Table 3.7Spectral accelerations at the fundamental period of Frame 8 of the<br/>UCS building.



Fig. 3.10 Response spectra of the 20 selected earthquake records for the UCS building analyses.

## 3.4.3 Selected EDPs

In general, laboratory buildings contain heavy and/or sensitive equipments, and valuable contents. Therefore, in addition to the common interest in the seismic response of structural components, the response of non-structural components and building contents in laboratory building, e.g., the UCS building, is of particular interest. In that regard, the peak absolute floor acceleration and displacement, and the peak inter-story drift ratio (IDR) are selected as the *global* EDPs. Here, the absolute response is the relative response to the fixed base algebraically added to the ground motion. The peak absolute floor acceleration and displacement provide a basis for estimating damage to acceleration-sensitive and displacement-sensitive building contents such as computers on bench-tops and shelves, hazardous chemicals on shelves, and large refrigerators that are important in terms of value and life safety. More-

over, IDR at each story provides a way to estimate the damage to structural components and some of the non-structural components such as windows and claddings. Among global EDPs, only the peak absolute roof acceleration (referred to as PRA), the peak absolute roof displacement (referred to as PRD), and the maximum IDR (referred to as MIDR) among the seven stories of the UCS building are presented in this report.

In addition to the global EDPs, the rotational demands of each structural component are selected as the *local* EDPs. However, monitoring the response history of all structural components may not be feasible depending on the size of the computational model and the type of the adopted finite elements to represent these components. Therefore, only relatively important structural components, determined such that their deformations significantly contribute to the global displacement of the building are selected for the current sensitivity study. For this purpose, a deterministic transient analysis of the UCS building with all random variables assigned their mean values is performed first. In this analysis, the loading is the unscaled ground acceleration record denoted as KB-kobj in Table 3.6 with  $S_a = 2.4$ g as the fundamental period of Frame 8 (0.38 sec.) of the UCS building.

To determine the contribution of each component to the floor displacement, momentcurvature relationships at both ends of all components are monitored (solid lines in Figure 3.11) and compared with those obtained from section analyses under monotonically increasing curvature (dashed lines in Figure 3.11). From Figures 3.11(a) and (c), the bottom of the shear-wall in the first story shows severe nonlinear behavior including significant deformation, while shear-walls in the other stories remain in the elastic range. On the other hand, from Figures 3.11(b) and (d), severe nonlinear behavior is observed in all coupling beams. Accordingly, the three shear-walls in the first story and the coupling beams in all floors are considered as important components with respect to the floor displacement. Therefore, in the presented sensitivity study, the curvatures of the critical cross sections where the curvature is largest are monitored as local EDPs.



Fig. 3.11 Moment-curvature relationships at various cross sections of Frame 8 subjected to KB-kobj (solid lines) and from monotonic section analyses (dashed lines); (a) the bottom of element 47; (b) the left of element 55; (c) the bottom of element 2; (d) the left of element 8. Element numbers are designated in Figure 3.4(b).

## 3.4.4 Tornado Diagram Analysis

Tornado diagrams of different EDPs are developed according to the procedure described in Section 3.2.2. Unlike the other sources of uncertainties, uncertainty in ground motion profile cannot be explicitly expressed as a random variable defined by a probability distribution. In other words, one cannot define the 10th, 50th, and 90th percentiles of ground motion profile. Therefore, the following procedure is developed to determine medians and bounds of the ground motion profile with respect to the EDP of interest.

- 1. Compute the elastic spectral acceleration  $S_{ai}$  at the fundamental period of the structure with 5% viscous damping ratio (recall that this is the adopted definition of the IM of a particular earthquake profile) for the *i*th ground motion profile.
- 2. Determine the scale factor  $(\alpha_i)$  by  $\alpha_i = \text{target } S_a/S_{ai}$  where the target  $S_a$  is the median  $S_a$  obtained from the seismic hazard curve.
- 3. Perform a nonlinear time history analysis of the structure using the *i*th scaled ground motion ( $\alpha_i$  times the original ground motion profile) to obtain EDP corresponding to the *i*th ground motion, EDP<sub>i</sub>.
- 4. Repeat steps 1, 2, and 3 above for another ground motion profile and sort the ground motion profiles by their EDPs.
- 5. Find the 10th, 50th, and 90th percentiles of EDP from the set of EDPs obtained from step 4 above. The corresponding ground motion profiles are respectively the 10th, 50th, and 90th percentiles of the ground motion profile.

Random variables Fc and Fy in Table 3.3 are combined to represent the *strength*, and Ec and Es are also combined to represent the *stiffness* such that each pair of random variables is perfectly dependent. In other words, if Ec increases by one unit, Es also increases by one unit. This assumption is adopted to investigate the combined effect of the concrete and reinforcement in terms of the stiffness and strength of the RC building, not the effect of individual materials. In this chapter, this is the case unless otherwise noted.

Figure 3.12 shows tornado diagrams of different EDPs. The vertical lines in the middle of these tornado diagrams indicate EDPs corresponding to the median values of all random variables. Line-type plots are the results of using the FOSM method described in the following section. According to this figure, Sa is the most significant random variable for PRA, PRD, and MIDR of Frame 8 of the UCS building in a probabilistic sense. Moreover, one can observe that uncertainty in EDPs is more sensitive to uncertainty in ground motion (Sa and GM) than that in structural properties (Ms, Dp, Stiffness and Strength). This implies that a better understanding of characteristics of ground motion profiles and hazard information will greatly reduce uncertainty of the seismic demand of buildings such as the UCS building. On the contrary, the rankings of random variables in structural properties are different EDPs.



Fig. 3.12 Tornado diagrams and FOSM results of Frame 8 of the UCS building.

These tornado diagrams also suggest skew of the EDP distributions. Especially, from swings of PRA induced by GM, PRD induced by Sa and GM, and MIDR induced by Sa, one can estimate rather strong skewness of probability distributions of the considered EDPs. Skewness of PRD and MIDR distributions is partly caused by skew of the Sa distribution itself, since the probability distribution of Sa can be estimated by lognormal as discussed in Section 3.4.2.1. This suggests that the relationship between each of PRD and MIDR, and Sa is close to linear. On the other hand, the swing of PRA induced by Sa does not show a strong skewness. This suggests that the relationship between PRA and Sa is not linear. Further investigation into the relationship between EDP and IM is not conducted in this report because it is not within the scope of the present study. Such investigation can be pursued by, for example, IDA (Vamvatsikos and Cornell 2002).

## 3.4.5 Analysis Using FOSM Method

Using the means and COVs of random variables given in Table 3.3, sensitivity of each EDP to a random variable is estimated. To determine the perturbation size for the finite difference approach (3.15) discussed in Section 3.2.1, sensitivity estimates of each EDP in terms of COV are obtained using different perturbation sizes as summarized in Table 3.8. In this table, the median ground motion profile denotes the ground motion profile that produces the median EDP among the considered 20 ground motion profiles discussed in Section 3.4.2. It should be noted that these ground motions are individually scaled to achieve the mean Sa ( $S_a = 0.633$ g) according to the lognormal fit of the seismic hazard curve of the UCS building site (Figure 3.9). The median ground motion profiles are used to estimate the sensitivity of EDPs to any other random variable than GM that denotes the random variable representing uncertainty in the ground motion profile itself. Table 3.8 clearly shows that the finite difference approach adopted in this study using  $a_p = 0.001$  guarantees convergence of the results by the FOSM method.

EDP sensitivity to combined uncertainties in structural properties in Table 3.3 are estimated for each of the 20 ground motion profiles at the median IM level (0.91g according to Table 3.7) using different perturbation sizes, namely  $a_p = 1.0$  and 0.001. Consequently, 20 COV values are computed for each EDP and each perturbation size. Statistics of each of these 20 COV values, namely the mean and the COV, are compared in Table 3.9. The difference in the levels of scatter (in terms of COV) for the different perturbation sizes is negligible. This is another justification of the selected perturbation size, i.e., for  $a_p = 0.001$ .

of Frame 8 of the UCS building.										
Random		PRA			PRD			MIDR		
variable	*	**	***	*	**	***	*	**	***	$Og/Ox_i$
Sa	36.4	53.0	53.0	67.3	67.3	67.3	81.6	80.3	80.3	+ve
GM		14.7			64.0			9.5		$N/A^a$
Dp	9.0	11.7	11.7	8.0	8.0	8.0	5.8	5.4	5.4	-ve
Ms	10.5	7.3	7.3	6.8	6.3	6.3	8.2	7.5	7.5	+ve
Strength	4.5	4.4	4.4	3.3	3.3	3.3	2.7	4.7	4.7	-ve
Stiffness	1.8	1.8	1.8	1.9	1.9	1.9	3.1	2.8	2.8	-ve
Median GMP TO-ttrh02 TO-ttrh02 CL-clyd										
*, **, and *** for $a_p = 1.0, 0.1$ , and 0.001, respectively. Refer to Section 3.2.1 for $a_p$ .										
$^{a}$ Monte Carlo s	<sup>a</sup> Monte Carlo simulation is used.									

Table 3.8COV (%) of EDPs corresponding to the individual random variables

 Table 3.9
 Statistics of measure of EDP sensitivities to combined uncertainties

in structural properties using different perturbation sizes.

Statistics	COV	of PRA	COV	of PRD	COV of MIDR			
	*	**	*	**	*	**		
Median $(\%)$	16.5	16.6	12.0	12.6	19.5	19.4		
COV (%)	34.7	30.9	37.2	38.3	49.0	41.2		
* and ** for $a_p = 1.0$ and 0.001, respectively.								

From Table 3.8, one observes that the effect of Sa on variability of the selected three global EDPs is the dominant one amongst all considered random variables. However, this result is not surprising because the variability of Sa itself (COV = 83% as discussed in Section 3.4.2) is even larger than any of the EDP variability. It is also observed that random variables in ground motion (Sa and GM) are more significant than those in the structural properties.

It is noted that a random variable shows different effects on different EDPs. For example, COVs of PRA and PRD due to the random variable Dp are 11.7% and 8.0%, respectively. This is attributed to the fact that the selected EDPs are all *peak* responses and they do not necessarily occur simultaneously. For example, Figure 3.13 shows time histories

of absolute and relative accelerations, and absolute and relative displacements at the roof, as well as the respective peak responses (including PRA and PRD) that are indicated by circles due to TO-ttrh02 scaled to the mean Sa, i.e., 0.633g. Clearly, PRA and PRD do not occur simultaneously. Moreover, uncertainty in a peak *absolute* response due to a random variable depends on contributions from the *relative* response and the ground motion. In general, the contribution of the relative response (e.g., displacement) to the corresponding peak absolute response does not necessarily agree with that of another relative response (e.g., acceleration). For example, in Figure 3.13, the contribution of ground acceleration to PRA is very small, while that of ground displacement to PRD is much more significant. Both



Fig. 3.13 Time histories of various EDPs due to TO-ttrh02.
the non-simultaneous occurrence of different EDPs and different contributions by different relative responses to the corresponding peak absolute responses depend on the specific choice of the ground motion profile.

COVs of EDPs due to GM are obtained using Monte Carlo simulation. In this case, all random variables are assigned their mean values to obtain 20 samples (due to 20 ground motions that are individually scaled to the mean Sa, i.e., 0.633g) for each EDP. It is notable that COV of PRD is considerably larger than those of PRA and MIDR (refer to Table 3.8). This is mostly due to uncertainty in the ground displacement itself. In fact, the COV of the 20 peak ground displacements is 84%. On the other hand, COV of the 20 peak ground accelerations is 40%, and its effect on PRA is less significant than that of uncertainty in the ground displacement on PRD.

It is worth mentioning that the sign of the gradient of an EDP with respect to a random variable (expressed as  $\partial g/\partial x_i$  in (3.15)) reflects if the random variable is a *demand* variable or a *capacity* variable. Note that the output of the FOSM method is in terms of statistics of the "demand" of the structural system in this study. In this context, a random variable with a positive gradient of an EDP can be viewed as a "demand" variable. On the other hand, a random variable with a negative gradient of an EDP can be viewed as a "demand" variable. On the other hand, a random variable with a negative gradient of an EDP can be viewed as a "demand" variable. On the other hand, a random variable with a negative gradient of an EDP can be viewed as a "capacity" variable. In this study and according to Table 3.8, gradients with respect to *Sa* and *Ms* are positive, which suggests that those are demand variables. However, gradients with respect to the other random variables are negative, suggesting that they are capacity variables.

### 3.4.6 Comparison of Analyses Using Tornado Diagram and FOSM Method, and Suggested New Approach

It is not straightforward to compare the tornado diagram and the result of the FOSM method directly, because a tornado diagram does not contain any statistical information on EDP (unless EDP is a linear function of random variables), while the only outcome of the FOSM method is EDP statistics. One way of comparing the tornado diagram and the result of the FOSM method is to compare the order of importance of random variables to each EDP. For a "better" comparison than just listing random variables in an order, the results of the FOSM method are presented in the same format as the tornado diagram. For this purpose, the EDPs are assumed in most cases to have the same distributions as the corresponding random variables with estimated means and standard deviations obtained from the FOSM method. For example, lognormal distribution is assumed for EDP distributions induced by Sa because Sa has lognormal distribution. However, lognormal distribution is assumed for EDP distribution induced by GM because skewness of the EDP distributions is observed from tornado diagrams in Figure 3.12. Assumed EDP distributions corresponding to random variables and bases of assumptions are listed in Table 3.10.

516 <b>6</b> 120 1265 dim		Scales of case campero
Random variable	Assumed EDP distribution	Basis of assumption
Sa	Lognormal	Sa distribution
GM	Lognormal	Tornado diagram
Ms	Normal	Ms distribution
Dp	Normal	Dp distribution
Stiffness	Normal	Tornado diagram
Strength	Normal	Tornado diagram

 Table 3.10
 Assumed EDP distributions and bases of assumptions.

Then, the 10th and 90th percentiles of each EDP are computed for each random variable and plotted on the same tornado diagram as shown in Figure 3.12. If an EDP is a linear function of random variables, the envelopes obtained by the FOSM method should exactly match the outlines of the tornado diagram because in this hypothetical case (1) the EDPs corresponding to the 10th and 90th percentiles of a random variable are exactly the 10th and 90th percentiles of a random variable are exactly the 10th and 90th percentiles of the EDP distribution; (2) the FOSM method gives the exact solutions of the mean and standard deviation of the EDP; and (3) the distribution type of EDP is identical to that of the random variable. However, if the function is nonlinear, none of the above is true, and the 10th and 90th percentiles of an EDP by the FOSM method do not have to match their counterparts of the tornado diagram. However, Figure 3.12 shows

a reasonable match between the tornado diagrams and envelopes obtained by the FOSM method for the rankings of random variables. Note that these rankings are slightly different between the tornado diagram and the outcome of the FOSM method. This is particularly the case for the structural properties that have small effect (small swings) on the variability of the EDPs anyway.

As simple methods (in comparison to Monte Carlo simulation) for the sensitivity study, the tornado diagram analysis and the FOSM method are compared in a general sense. A tornado diagram does not provide any statistics on EDP. However, one can have a rough idea of the skew of the EDP distribution. For example, Figures 3.12(b) and (c) suggest strong skew of PRD and MIDR distributions induced by Sa, respectively, while the distribution of PRA induced by Sa shows weaker skew. The FOSM method estimates the two most important statistics of EDP, namely the mean and standard deviation. Unlike the tornado diagram analysis, the sensitivity of an EDP to a combination of correlated random variables can be investigated by the FOSM method. This feature is explicitly utilized in Chapter 5 where EDP uncertainty induced by correlated uncertain parameters in the capacity of a structural component is estimated, while no strong correlation is considered in the case study of the UCS building in this chapter. However, the FOSM method does not take into account the distribution type of the random variable and, consequently, no information on the distribution type of the EDP can be inferred from the results of the FOSM method.

Knowing the pros and cons of both the tornado diagram and the FOSM methods, one may think of combining these two methods so that one benefits from both merits. Obviously, a combined method should be able to estimate the means and standard deviations of EDP with an idea of the skewness of its probability distribution. One possible way of achieving this is to compute the gradient of an EDP with respect to a random variable (required for the FOSM method) using its corresponding swing from the tornado diagram analysis. For example, if the swing of an EDP is obtained by the mean  $\pm$  standard deviation of the corresponding random variable (X), the gradient of the EDP with respect to the given random variable (X) can be estimated by

$$\frac{d\text{EDP}}{dx} \approx \frac{\text{Swing of EDP}}{2 \times \text{Standard deviation}}.$$
(3.18)

Note that (3.18) is equivalent to (3.15) with  $a_p = 1.0$ . The outcome of this approach is a tornado diagram with estimated mean and standard deviations (obtained from FOSM with the help of (3.18)) of EDP. Figure 3.14 illustrates the suggested new approach of combining the tornado diagram and the FOSM method for estimating EDP uncertainty.



# Fig. 3.14 Suggested new approach of combining the tornado diagram and the FOSM method.

#### 3.4.7 Sensitivity of Local EDPs by FOSM Method

Sensitivity of the local EDPs is studied by the FOSM method in terms of the peak curvature at critical cross sections to individual random variables described in Table 3.3. Only the FOSM method is used here assuming the sensitivities of the local EDPs estimated by both the FOSM method and the tornado diagram analysis are close to each other as is the case for the global EDPs. Unlike the sensitivity study of global EDPs, random variables Fc, Fy, Ec, and Es are considered independently. Moreover, the results are presented in an analogous format to the FOSM envelopes of the tornado diagrams in Figure 3.12. The median ground motion profile for PRA and PRD, namely TO-ttrh02 as given in Table 3.8, is used for sensitivity of the local EDPs to all random variables but GM. Figure 3.15 shows the results for some critical cross sections of Frame 8 of the UCS building using the FOSM method. Similar to the results of global EDPs, Sa is the dominant random variable for all critical cross sections. The second significant random variable is Dp for all cross sections where a relatively large COV (40%) assigned to Dp may have led to this high ranking of Dp. The subsequent significant random variables differ from one cross section to the other.

#### 3.4.8 Conditional Sensitivity of EDP Given IM by FOSM Method

In Sections 3.4.4 through 3.4.7, a measure of EDP sensitivities by the tornado diagram analysis or analysis using the FOSM method is estimated at only one IM level, i.e., the median or mean Sa, respectively. However, from the perspective of PBEE, it is desirable to investigate the propagation of uncertainty at various levels of earthquake hazard. In that regard, the conditional sensitivity of EDPs to random variables given IM is investigated where Sa is treated as a deterministic variable at different levels. Nine levels of IM in terms of Sa are selected for this purpose, namely the 10th, 20th, ..., and 90th percentiles of Saaccording to (3.17) as listed in Table 3.11. The range of Sa, bounded by the 10th and the 90th percentiles, is indicated in Figure 3.9. In this part of the study, all 20 ground motion profiles,



Fig. 3.15 Sensitivity of the peak curvatures at critical cross sections of Frame 8 of the UCS building; (a) the bottom of element 1; (b) the left of element 8; (c) the bottom of element 2; (d) the left of element 55. Element numbers are designated in Figure 3.4(b).

not only the median ground motion profile as was conducted in Sections 3.4.4 through 3.4.7, are used to estimate EDPs sensitivity to each random variable at each IM level. It is to be noted that all results in this section are presented with respect to Sa on a semi-log scale due to the wide range of the considered Sa (0.18g to 1.39g).

#### 3.4.8.1 Global EDPs

For each IM level, a deterministic analysis where all random variables are kept at their mean values is conducted for each ground motion that is individually scaled to achieve a given IM.

Table 3.11Various percentiles of Sa for sensitivity of EDP given IM for Frame 8of the UCS building.

			0-						
Percentile	10th	20th	30th	40th	50th	60th	70th	80th	90th
$S_a$ (g)	0.18	0.25	0.32	0.39	0.47	0.57	0.71	0.90	1.39

Figure 3.16 shows scatters of global EDPs reflecting the uncertainty in GM. In this figure, the solid lines represent the median EDPs. It is observed that these medians tend to increase as the IM level increases.



Fig. 3.16 Scatters of global EDPs induced by the uncertainty in *GM* for Frame 8 of the UCS building.

Uncertainties of global EDPs induced by those in structural properties are estimated by using the FOSM method and presented in Figure 3.17 in terms of COV. In this figure, each circle represents the COV of the EDP induced by uncertainties in all structural properties namely, mass, viscous damping, strength, and stiffness for a specific ground motion profile and IM level. The variance is obtained according to (3.13) considering correlations of random variables to derive the COV. Figure 3.17(d) is for the peak relative roof displacement (PRRD) that is the peak roof displacement relative to the fixed base to compare with PRD (Figure 3.17(b)). Note that the scatter of COV values at each IM level is induced by the inherent record-to-record variability of ground motions.

The COVs of global EDPs due to GM only (obtained from the data of Figure 3.16)



Fig. 3.17 Comparison of uncertainties in global EDPs for Frame 8 of the UCS building induced by the uncertainty in GM only (solid line) and in structural properties (dashed line, median quantity).

are also plotted in Figure 3.17 as solid lines for comparison. Dashed lines in Figure 3.17 connect median COVs (the median of 20 COV values from the scaled 20 records for a certain IM in terms of  $S_a$ ) at each IM level induced by combined uncertainties in structural properties. From Figures 3.17(a) and (c), at lower IM levels, uncertainty in structural properties is more significant than that in GM on PRA and MIDR, while at higher IM levels, the opposite is true. However, PRD uncertainty is primarily dependent on GM. This is mostly due to the uncertainty of ground displacement itself because the COV of the 20 peak ground displacements is 84% as mentioned in Section 3.4.5. On the other hand, GM does not dominate the PRRD uncertainty (Figure 3.17(d)) where the uncertainty of ground displacement is canceled by the definition of PRRD. Instead, GM becomes more significant

than the other random variables of structural properties only at much higher IM levels. Finally, one observes that the COV of an EDP induced by GM increases as the IM level increases, while that induced by structural properties does not have an obvious trend.

#### 3.4.8.2 Local EDPs

Similar to the previous section, EDP uncertainty induced by GM only is investigated first. Figure 3.18 shows scatters of the peak curvature reflecting the uncertainty in GM only where the solid lines represent the median EDPs. Similar to global EDPs, the medians of the peak curvatures at all critical cross sections tend to increase as the IM level increases.

Sensitivity of the local EDPs conditioned on IM induced by uncertainties in structural properties is studied in terms of the peak curvature demands at critical cross sections, as shown in Figure 3.19. As in the previous section, COVs of EDPs induced by uncertainties in all structural properties are derived from variances of EDPs obtained according to (3.13). Similar to that of global EDPs, the scatter of COV values at each IM level is due to the inherent record-to-record variability of ground motions. Moreover, uncertainty of the peak curvatures depends more on uncertainty in structural properties at lower IM levels and on uncertainty in GM at higher IM levels for all critical cross sections.

#### 3.5 CONCLUDING REMARKS

The propagation of basic uncertainty to the structural system with respect to its seismic demand (referred to as EDP) due to possible future earthquakes is studied in this chapter. An approach of estimating uncertainties in EDP and identifying significant sources of basic uncertainties is demonstrated using a case study RC shear-wall building (referred to as the UCS building). Sensitivity of EDP to uncertainties in structural properties and ground motion is estimated using the tornado diagram analysis and the FOSM method. From the sensitivity measure of an EDP, the relative significance of each basic uncertainty to the given EDP is identified and ranked.



Fig. 3.18 Scatters of the peak curvature at critical cross sections of Frame 8 of the UCS building induced by the uncertainty in GM; (a) the bottom of element 1; (b) the left of element 8; (c) the bottom of element 2; (d) the left of element 55. Element numbers are designated in Figure 3.4(b).



Fig. 3.19 Comparison of local EDPs uncertainty induced by the uncertainty in GM only (solid line) and in structural properties (dashed line, median quantity) at critical cross sections of Frame 8 of the UCS building; (a) the bottom of element 1; (b) the left of element 8; (c) the bottom of element 2; (d) the left of element 55. Element numbers are designated in Figure 3.4(b).

The FOSM method estimates the mean and the standard deviation of an EDP given means and standard deviations of various random variables. The estimated standard deviation of an EDP is its measure of sensitivity to a given random variable. The FOSM method is simple and efficient in estimating EDP sensitivity, in comparison with Monte Carlo simulations in terms of computing the mean and standard deviation of EDP. The tornado diagram analysis, one of the methods of the deterministic sensitivity study, is also simple and efficient in identifying and ranking relatively significant random variables to EDPs. The pros and cons of tornado diagram analysis and the FOSM method are discussed in a general sense, and an approach of combining the two methods is suggested. The peak absolute roof acceleration (PRA), peak absolute roof displacement (PRD), and maximum inter-story drift ratio (MIDR) are selected as global EDPs, while the peak curvatures at critical cross sections are selected as local EDPs. Moreover, several random variables representing uncertainty in ground motion and structural properties are considered.

Sensitivity of global EDPs indicates that the intensity measure of earthquakes is the dominant source of uncertainty to all global EDPs. Moreover, uncertainties in ground motion are more significant than those in structural properties. A sensitivity of local EDPs indicates that the intensity measure of earthquakes is the dominant source of uncertainty to all local EDPs, while the second significant source of uncertainty is the viscous damping where a relatively large COV (40%) assigned to the probability distribution of viscous damping may have been responsible for its high ranking.

The conditional sensitivity of EDPs to random variables given IM is investigated considering uncertainty in the ground motion profile and the combined effect of all uncertainties on structural properties. For all local and global EDPs but PRD, uncertainty in the ground motion profile is more significant than that on structural properties at higher levels of earthquake intensity but less significant at lower levels of earthquake intensity. For PRD, uncertainty in the ground motion profile is dominant, regardless of the level of earthquake intensity.

## 4 Uncertainty in the Capacity of Structural Components

#### 4.1 INTRODUCTION

The quantification of uncertainty in the response of RC structural components in terms of the deformation and strength capacity is necessary for the implementation of the PBEE design methodology. The probabilistic analysis of RC structural components has been the focus of a number of research efforts. One of the earliest works is that of Shinozuka (1972) where he pointed out the importance of considering the spatial variability of the material properties in estimating the strength of plain concrete structures. Several studies were concentrated on RC frame members such as columns or beams using computational approaches such as the Monte Carlo simulation. Knappe et al. (1975) studied the reliability of a RC beam. Grant et al. (1978), Mirza and MacGregor (1989), and Frangopol et al. (1996) conducted strength analyses of RC beam-column members by considering the uncertainty of material properties and of cross-sectional dimensions. However, few research efforts considered the spatial variability of random variables of the RC frame members, which requires the discretization of the random field and the identification of the correlation characteristics of random variables. Because of the nature of RC construction, the spatial variability of material and geometrical properties should be considered for reliable estimates of the nonlinear structural behavior.

In this chapter, a computational model for structural analysis considering the spatial variability of material and geometrical properties of the RC structural members using the Monte Carlo simulation method is developed. This model combines the conventional fiber element formulation and one of the random field representation methods. The fiber element model is selected for computing the structural response because it is a powerful tool in estimating the inelastic behavior of RC framed structures. Among various random field representation methods, the midpoint method (Der Kiureghian and Ke 1988) is selected in the present study. Assumptions and formulations of the adopted fiber element model are described in the next section. Subsequently, the stochastic fiber element model including the random field representation method is presented. The stochastic fiber element model is applied to several structural components to develop probabilistic section models, namely the probabilistic axial force-bending moment diagram and the probabilistic moment-curvature relationship.

As a part of demonstrating the systematic procedure of evaluating a structural system, namely component evaluation phase (cf. Section 2.6.1), typical structural components of a ductile RC frame are identified and evaluated to develop their probabilistic section models. The procedure of the system evaluation phase using these probabilistic section models of typical structural components is demonstrated in Chapter 5.

The main objective of the developed computational model is the probabilistic evaluation of RC structural members such as columns and beams. Although the developed computational procedure can be applied for probabilistic evaluation of any framed structure, it is not practical to use it for a complete structural system due to the large computational demand of the Monte Carlo simulation method combined with the fiber element model. A demonstration of a systematic approach for the probabilistic evaluation of a structural system using various probabilistic section models is presented in Chapter 5.

#### 4.2 FIBER ELEMENT MODEL

Material nonlinearity in a frame element is commonly described by either a lumped (D'Ambrisi and Filippou 1999) or distributed (Spacone et al. 1996) plasticity model. In the lumped plasticity model, a frame element consists of two zero-length nonlinear rotational spring elements and an elastic element connecting them. The nonlinear behavior of a structure is captured by the nonlinear moment-rotation relationships of these spring elements. Due to the simplicity of the formulation, the lumped plasticity model is widely used when the computational cost of the analysis is high, e.g., in the case of nonlinear time-history analysis of a large structure. An example of a structural model using the concentrated plasticity formulation is discussed in Section 5.3.1. On the other hand, material nonlinearity of a structure can develop anywhere in the element using the distributed plasticity model. Due to its capacity for describing nonlinear structural behavior, the distributed plasticity model is widely used for more accurate estimation of the structural response. In this chapter, only the distributed plasticity model is employed for the nonlinear frame element with the fiber section discretization. It is one of the best models to accommodate random fields of structural properties of RC frame members.

The formulation of a nonlinear frame element is categorized by the flexibility (forcebased) method or the stiffness (displacement-based) method. The flexibility method uses assumed force interpolation functions along the element, and a smaller number of elements than that of the stiffness method may be required. The stiffness method uses assumed displacement interpolation functions along the element, and this feature requires the use of a sufficient number of elements for a member to model an accurate structural response. The element formulation in the flexibility method is more complex than that of the stiffness method, because material constitutive models are usually given in the form  $\sigma = \sigma(\epsilon)$ , where  $\sigma$  and  $\epsilon$  are stress and strain measures, respectively, which is suitable for the stiffness method. On the other hand, the element formulation in the stiffness methods.

In this chapter, the stiffness method is used to formulate the distributed plasticity under the assumptions of the Bernoulli beam theory. An element is represented by several cross sections located at the numerical integration points. Each section is subdivided into a number of fibers where each fiber is under a uniaxial state of stress. This discretization process is shown in Figure 4.1 for the special case of an RC structural member.



Fig. 4.1 Element and section discretization.

#### 4.2.1 Element Formulation

Force and deformation variables at the element and section levels are shown in Figure 4.2. From this figure, the element force and deformation vectors are given by

Force 
$$\equiv \mathbf{p} = [p_1, p_2, \dots, p_6]^T$$
 (4.1)

Deformation 
$$\equiv \mathbf{u} = \begin{bmatrix} u_1, \ u_2, \ \dots, \ u_6 \end{bmatrix}^T$$
 (4.2)

On the other hand, the section force and deformation vectors are given by



Fig. 4.2 Force and deformation variables at the element and section levels.

Force 
$$\equiv \mathbf{q}(\mathbf{x}) = [N(x), \ M(x)]^T$$
 (4.3)

Deformation 
$$\equiv \mathbf{v}_{\mathbf{s}}(\mathbf{x}) = [\epsilon_0(x), \ \varphi(x)]^T$$
(4.4)

The normal force N, bending moment M, axial strain at the reference axis  $\epsilon_0$ , and curvature  $\varphi$  are functions of the section position x.

The strain increment in the  $i^{th}$  fiber is defined by

$$d\epsilon_i = d\epsilon_0(x) - y_i d\varphi(x)$$
  
=  $\mathbf{a}_{\mathbf{s}}(y) d\mathbf{v}_{\mathbf{s}}(x)$  (4.5)

where  $\mathbf{a}_{\mathbf{s}}(y) = [1, -y_i], d\mathbf{v}_{\mathbf{s}}(x) = [d\epsilon_0(x), d\varphi(x)]^T$ , and  $y_i$  is the distance between the  $i^{th}$  fiber and the reference axis. Section deformations  $\mathbf{v}_{\mathbf{s}}(x)$  are determined from the straindeformation relationship such that

$$\mathbf{v_s}(x) = \left[\mathbf{B}(x) + \frac{1}{2}\mathbf{G}(x)\right] \mathbf{u_{n+1}}$$
(4.6)

where  $\mathbf{u_{n+1}} = \mathbf{u_n} + \Delta \mathbf{u}$  is the element deformation vector at the load step n + 1,  $\mathbf{B}(x)$  is the first-order strain-deformation transformation matrix which consists of the well-known first and second derivatives of the displacement interpolation matrix assuming small deformations, and  $\mathbf{G}(x)$  is another strain-deformation transformation matrix such that  $\frac{1}{2}\mathbf{G}(x)$  represents the second-order term of the strain-deformation relationship.  $\mathbf{G}(x)$  can be expressed as

$$\mathbf{G}(x) = \begin{bmatrix} 1\\ 0 \end{bmatrix} \left\{ \mathbf{C}(x)\mathbf{u_{n+1}} \right\}^T \mathbf{C}(x)$$
(4.7)

where  $\mathbf{C}(x)$  is a strain-deformation transformation matrix which consists of the first derivatives of displacement interpolation matrix. Explicit forms of  $\mathbf{B}(x)$  and  $\mathbf{C}(x)$  are provided in Section A.1 of Appendix A.

Tangent modulus  $E_{ti}$  and stress  $\sigma_i$  are determined from the strain  $\epsilon_i$  using a particular constitutive relationship for the material of the  $i^{th}$  fiber. In this way, the section stiffness

 $\mathbf{k}_{\mathbf{s}}(x)$  and resisting force  $\mathbf{r}_{\mathbf{s}}(x)$  are determined using the principle of virtual work such that

$$\mathbf{k}_{\mathbf{s}}(x) = \int_{A(x)} \mathbf{a}_{\mathbf{s}}^{T}(y) E_{t}(x, y) \mathbf{a}_{\mathbf{s}}(y) dA$$
(4.8)

$$\mathbf{r}_{\mathbf{s}}(x) = \int_{A(x)} \mathbf{a}_{\mathbf{s}}^{T}(y) E_{t}(x, y) dA$$
(4.9)

These integrals are evaluated by the midpoint rule with n fibers. Thus,  $\mathbf{k}_{\mathbf{s}}(x)$  and  $\mathbf{r}_{\mathbf{s}}(x)$  are numerically obtained as follows

$$\mathbf{k}_{\mathbf{s}}(x) = \sum_{i=1}^{n} \mathbf{a}_{\mathbf{s}_{i}}^{T} E_{ti} \mathbf{a}_{\mathbf{s}_{i}} a_{i}$$
(4.10)

$$\mathbf{r}_{\mathbf{s}}(x) = \sum_{i=1}^{n} \mathbf{a}_{\mathbf{s}_{i}}^{T} E_{ti} a_{i}$$
(4.11)

where the cross-sectional area  $A(x) = \sum_{i=1}^{n} a_i$ .

For nonlinear analysis, the force-displacement relationship at the element level is commonly expressed in an incremental form such that  $\Delta \mathbf{p} = \mathbf{k}_{\mathbf{e}} \Delta \mathbf{u}$  where  $\mathbf{k}_{\mathbf{e}}$  is the element tangent stiffness matrix. Once  $\mathbf{v}_{\mathbf{s}}(x)$  is determined, the section stiffness  $\mathbf{k}_{\mathbf{s}}(x)$  and resisting forces  $\mathbf{r}_{\mathbf{s}}(x)$  are evaluated. Subsequently, the element stiffness  $\mathbf{k}_{\mathbf{e}}$  and resisting forces  $\mathbf{r}_{\mathbf{e}}$  are derived from the principle of virtual work and can be expressed as follows

$$\mathbf{k}_{\mathbf{e}} = \int_{L} \mathbf{T}^{T}(x) \mathbf{k}_{\mathbf{s}}(x) \mathbf{T}(x) dx + \int_{L} \mathbf{C}^{T}(x) \mathbf{C}(x) N_{s}(x) dx$$
(4.12)

$$\mathbf{r}_{\mathbf{e}} = \int_{L} \mathbf{T}^{T}(x) \mathbf{r}_{\mathbf{s}}(x) dx \tag{4.13}$$

where  $\mathbf{T}(x) = \mathbf{B}(x) + \mathbf{G}(x)$ ,  $N_s(x)$  is a component of  $\mathbf{r}_s(x)$  representing the axial force resultant and L is the element length. Derivation of this element stiffness matrix is provided in Section A.1 of Appendix A. In the present study, the Gauss-Lobatto integration scheme is adopted to evaluate these integrations. Four integration points per element are used in this study. Thus, x's are selected at the Gauss integration points. The element stiffness and resisting forces are then assembled by the conventional finite element method procedure to determine the global stiffness and resisting forces. For nonlinear conditions, equilibrium between the applied forces and the resisting forces is usually not satisfied in one iteration. Therefore, an incremental-iterative numerical technique should be utilized to enforce the equilibrium conditions. The adopted nonlinear iterative solution scheme is described in the following section.

#### 4.2.2 Nonlinear Analysis Procedure

An incremental-iterative solution procedure is utilized to solve the nonlinear equilibrium equations obtained from the fiber element model. By the conventional Newton-type analysis method, it is not possible to capture the post-critical response of the structure, which is essential for the performance-based design of an RC structure. This drawback is due to the numerical feature that holds the load parameter constant throughout the iterations within each load step. Passing limit points is therefore impossible due to the singular nature of the tangent stiffness matrix in the vicinity of a limit point. Various techniques to overcome this drawback have been developed. A detailed description and summary of these schemes are found in reference (Clarke and Hancock 1990). Among these methods, the minimum unbalanced displacement norm method is selected in the present study and described in the subsequent paragraphs.

In the incremental-iterative solution method, each load step starts with the application of a load increment and subsequent iterations for equilibrium. In the following, subscript k is used to denote the load step number, while superscript i is used to denote the iteration number within each load step.

#### 4.2.2.1 First Iteration

At the first iteration of each load step, the "tangent" displacement  $\mathbf{U}_{\mathbf{t}k}$  is computed by

$$\mathbf{K}_k \mathbf{U}_{\mathbf{t}k} = \mathbf{P}_{\mathbf{ref}} \tag{4.14}$$

where  $\mathbf{K}_k$  is the tangent stiffness matrix of the structure at the end of the previous load step and  $\mathbf{P_{ref}}$  is the reference external force vector. Next, the incremental displacement is evaluated by

$$\Delta \mathbf{U}_k^1 = \Delta \lambda_k^1 \mathbf{U}_{\mathbf{t}k} \tag{4.15}$$

where  $\Delta \lambda_k^1$  is a load step parameter for the first iteration of the  $k^{th}$  load step, which can be determined by the following procedure.

$$l_k = l_{k-1} \left(\frac{J_d}{J_{k-1}}\right)^{\gamma} \tag{4.16}$$

$$\Delta \lambda_k^1 = \frac{\pm l_k}{\sqrt{\mathbf{U}_t^T \mathbf{U}_t}} \tag{4.17}$$

where  $J_d$  is the desired iteration number for convergence, typically 3 to 5,  $J_{k-1}$  is the actual iteration number for convergence in the previous load step, and  $l_1 = \Delta \lambda_1^1 \sqrt{\mathbf{U_{t_1}}^T \mathbf{U_{t_1}}}$ . The exponent  $\gamma$  typically lies between 0.5 to 1.0 (Clarke and Hancock 1990). In (4.17), the sign follows that of the determinant of the stiffness matrix. Then, the total displacement and load parameter are updated from the previous load step by

$$\mathbf{U}_k^1 = \mathbf{U}_{k-1} + \Delta \mathbf{U}_k^1 \tag{4.18}$$

$$\lambda_k^1 = \lambda_{k-1} + \Delta \lambda_k^1 \tag{4.19}$$

#### 4.2.2.2 Equilibrium Iterative Cycles

The incremental change in the displacements can be written as the solution of

$$\mathbf{K}_k \Delta \mathbf{U}_k^i = \Delta \lambda_k^i \mathbf{P}_{ref} - \mathbf{P}_{\mathbf{u}_k}^{i-1}$$
(4.20)

where  $\mathbf{P}_{\mathbf{u}}$  denotes the vector of the unbalanced forces. Since the modified-Newton-Raphson method is adopted in this study,  $\mathbf{K}_k$  doesn't have superscript *i*, i.e., it is not updated at each iteration. In the above equation,

$$\mathbf{P}_{\mathbf{u}k}^{\ i-1} = \mathbf{P}_k^{i-1} - \mathbf{P}_{\mathbf{r}k}^{\ i-1} \tag{4.21}$$

where  $\mathbf{P}_{k}^{i-1} = \lambda_{k}^{i-1} \mathbf{P}_{ref}$  and  $\mathbf{P}_{\mathbf{r}_{k}}^{i-1}$  is the resisting force vector obtained by the assembly of  $\mathbf{r}_{elem}$  vectors given in the previous section. From the above equations, the incremental displacement vector  $\Delta \mathbf{U}_{k}^{i}$  can be obtained by

$$\Delta \mathbf{U}_{k}^{i} = \Delta \lambda_{k}^{i} \mathbf{U}_{\mathbf{t}_{k}}^{i-1} + \Delta \mathbf{U}_{\mathbf{r}_{k}}^{i}$$

$$(4.22)$$

where  $\Delta \mathbf{U}_{\mathbf{r}_{k}}^{i}$  is the residual displacement vector obtained by solving

$$\mathbf{K}_k \Delta \mathbf{U}_{\mathbf{r}_k}^{\ i} = \mathbf{P}_{\mathbf{r}_k}^{\ i-1} \tag{4.23}$$

Determination of the incremental load parameter  $\Delta \lambda_k^i$  is discussed in the next section. The total displacement vector and load parameter are updated from the previous iteration by

$$\mathbf{U}_k^i = \mathbf{U}_k^{i-1} + \Delta \mathbf{U}_k^i \tag{4.24}$$

$$\lambda_k^i = \lambda_k^{i-1} + \Delta \lambda_k^i \tag{4.25}$$

Iterations are continued until a convergence criterion is satisfied. In this study,  $L_2$  norm of the unbalanced force vector ( $\mathbf{P}_{\mathbf{u}_k}^{i-1}$ ) normalized by that of the total force vector ( $\mathbf{P}_k^{i-1}$ ) is used for the convergence criterion and the tolerance is set to  $10^{-4}$ . If divergence is detected or convergence is not achieved within a specified number of iterations (typically selected as 10 iterations), the iterative procedure for the current load step restarts with a reduced initial load increment.

#### 4.2.2.3 Iterative Scheme

The incremental load parameter  $\Delta \lambda_k^i$  can be obtained by various constraint equations. In this study, the minimum unbalanced displacement norm method is selected. Among various iterative schemes such as arc-length method, this method is simple to implement and is verified to work well (Clarke and Hancock 1990).

The constraint equation involving  $\Delta \lambda_k^i$  is

$$\frac{\partial \parallel \Delta \mathbf{U}_k^i \parallel}{\partial \Delta \lambda_k^i} = 0 \tag{4.26}$$

which guarantees a minimum value for the unbalanced displacement norm in each iteration. Accordingly,

$$\Delta \lambda_k^i = -\frac{\mathbf{U}_{\mathbf{t}_k}^T \mathbf{U}_{\mathbf{r}_k}^i}{\mathbf{U}_{\mathbf{t}_k}^T \mathbf{U}_{\mathbf{t}_k}} \tag{4.27}$$

#### 4.2.3 Constitutive Models

In the fiber element analysis, the behavior of each fiber is governed by a specific uniaxial stress-strain relationship. For RC members, three constitutive models are necessary: (1) an unconfined concrete model for cover concrete, (2) a confined concrete model for core concrete, i.e., concrete inside the transverse reinforcement, and (3) a steel model for longitudinal reinforcing bars.

Figure 4.3 depicts typical stress-strain relationships for confined and unconfined concrete. Relevant parameters in the compression regime are the compressive strength  $f_{cc}$ , the corresponding strain  $\epsilon_{cc}$ , and the ultimate strain  $\epsilon_{cu}$  of confined concrete, and the compressive strength  $f_{co}$  and the corresponding strain  $\epsilon_{co}$  of unconfined concrete. The tension regime is defined by the tensile strength  $f'_t$  and the ultimate tensile strain  $\epsilon_{tu}$ . Initial tangent stiffness  $E_c$  is usually assumed to be the same for both the compression and the tension regimes.

In this study, the model proposed by Hoshikuma et al. (1997) is employed for the confined concrete. This model is expressed as

Ascending branch: 
$$f_c = E_c \epsilon_c \left[ 1 - \frac{1}{n} \left( \frac{\epsilon_c}{\epsilon_{cc}} \right)^{n-1} \right]$$
 (4.28)

Descending branch: 
$$f_c = f_{cc} - E_{des} (\epsilon_c - \epsilon_{cc}) \ge f_{cc}^{resid}$$
 (4.29)

where  $f_c$  and  $\epsilon_c$  are stress and corresponding strain in the confined concrete and  $E_{des}$  is the slope of the descending branch. The model parameters n and  $E_{des}$  are given by

$$n = \frac{E_c \epsilon_{cc}}{E_c \epsilon_{cc} - f_{cc}} \tag{4.30}$$

$$E_{des} = \frac{f_{cc} - f_{cc}^{resid}}{\epsilon_{cu} - \epsilon_{cc}}$$
(4.31)



Fig. 4.3 Concrete constitutive models.

where  $\epsilon_{cu}$  can be given by

$$\epsilon_{cu} = 0.004 + \frac{1.4\rho_{sh}f_{yh}\epsilon_{suh}}{f_{cc}} \tag{4.32}$$

according to Mander's model (Mander et al. 1988) where  $\rho_{sh}$ ,  $\epsilon_{suh}$ , and  $f_{yh}$  are respectively the volumetric ratio, the ultimate strain, and the yield strength of the transverse reinforcement. Other material parameters are expressed as

$$f_{cc} = f_{co} + 3.8\alpha\rho_{sh}f_{yh} \tag{4.33}$$

$$\epsilon_{cc} = 0.002 + 0.033\beta \frac{\rho_{sh} f_{yh}}{f_{co}} \tag{4.34}$$

where  $\alpha$  and  $\beta$  are cross-sectional shape factors. For a circular cross section,  $\alpha = \beta = 1.0$ and for a square cross section,  $\alpha = 0.2$  and  $\beta = 0.4$ . To avoid numerical difficulties in the present study,  $0.2f_{cc}$  is assumed as a residual value of the confined concrete strength  $f_{cc}^{resid}$ .

The constitutive model of the unconfined concrete in compression consists of a nonlinear ascending branch and a linearly descending branch (Figure 4.3). The expressions for these two branches are the same as those of the confined concrete model (Equations 4.28 to 4.30), except for the following:  $f_{cc}$  and  $\epsilon_{cc}$  are substituted by  $f_{co}$  and  $\epsilon_{co}$ , respectively, and  $E_{des}$  is expressed as  $E_{des} = f_{co}/2\epsilon_{co}$ . In the present study  $f_{co}$  is taken as  $0.85f'_c$  where  $f'_c$  is the compressive strength of concrete from the standard compressive test, and  $\epsilon_{co}$  is taken as 0.002. It is assumed that the strength is zero when  $\epsilon_c \geq 3\epsilon_{co}$  to represent spalling of the cover concrete.

A bilinear constitutive model is adopted for the confined and unconfined concrete in tension as shown in Figure 4.3. The tensile strength  $f'_t$  is computed by  $f'_t = 6\sqrt{f'_c}$  in *psi* units. The ultimate tensile strain  $\epsilon_{tu}$  is assumed to equal  $10\epsilon_t$ , where  $\epsilon_t$  is obtained by  $\epsilon_t = f'_t/E_c$ . It is assumed that a fiber with its tensile strain  $\geq 10\epsilon_t$  is fully cracked and has completely lost its tensile strength.

The constitutive model of the reinforcing steel consists of a bilinear elastic-plastic portion followed by a strain hardening region calculated by the following expression:

$$f_s = f_u - (f_u - f_y) \left(\frac{\epsilon_{su} - \epsilon_s}{\epsilon_{su} - \epsilon_{sh}}\right)^2, \quad \epsilon_{sh} < \epsilon_s < \epsilon_{su}$$
(4.35)

where  $f_s$  is the steel stress corresponding to the steel strain  $\epsilon_s$ ,  $f_y$  is the yield stress,  $f_u$  is the ultimate stress,  $\epsilon_{sh}$  is the strain at the onset of hardening, and  $\epsilon_{su}$  is the ultimate strain as shown in Figure 4.4. Stress is assumed to be zero beyond the ultimate strain. For simplicity, this constitutive model is adopted in both the tension and the compression regimes. Note that only envelopes of the three constitutive models are adopted because only monotonic loading is considered in this chapter.



Fig. 4.4 Reinforcing steel constitutive model.

#### 4.2.4 Verification Examples

A nonlinear static analysis Matlab program (Hanselman and Littlefield 1998) for RC structures is developed based on the nonlinear fiber element formulation described in the previous section. This program is numerically verified using three sets of experiments. The first set of experiments conducted by Mosalam (2002) at the University of California, Berkeley, includes eight identical, simply supported ductile RC beams (referred to as MB) under four-point bending. The second set, also conducted by Mosalam (2002) at the University of California, Berkeley, includes seven identical RC columns (referred to as MC) having square cross section under the effect of axial load. Figures 4.5 and 4.6 show the design parameters of MB



Fig. 4.5 The applied loads and the design parameters of MB.



Fig. 4.6 The applied load and the design parameters of MC.

and MC, respectively. The third set includes the column designated A1 that was tested by Kunnath et al. (1997) under constant axial load and monotonically increasing lateral load at the tip of the column. In the present study, this column is referred to as KC where Figure 4.7 shows its design parameters. The KC specimen was designed such that failure due to flexure preceded that due to shear with ample margin (Kunnath et al. 1997). Nominal values of material properties of MB, MC, and KC are given in Table 4.1.



Fig. 4.7 The applied loads and the design parameters of KC.

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Material property	MB	MC	KC
Compressive strength of concrete, psi	4,216	$5,\!192$	$5,\!149$
Yield stress of longitudinal steel, ksi	71	71	65
Ultimate stress of longitudinal steel, ksi	100	100	100
Strain at on-set of hardening of longitudinal steel	0.01	0.01	0.01
Yield stress of transverse steel, ksi	71	71	65
Modulus of elasticity of steel, ksi	29,000	29,000	29,000

Table 4.1 Material properties of MB, MC, and KC.

Figure 4.8(a) compares load-displacement relationships from the analysis with the experimental results for MB. Load is the sum of the applied two point loads and displacement is measured at mid-span. The analytical results show good agreement with the experimental results. It is interesting to note that the experimental results for the tested eight beams are

scattered even though these beams were identical. This can be explained by the variability of material and geometric properties of the beams, which motivated the present study.

The results from the column tests of MC are compared in Figure 4.8(b). These results are presented in terms of load-displacement relationships in the axial direction of the column. From the comparison, excellent agreement with the experimental results up to the displacement of 0.2 inch is clear. Beyond that the analysis reasonably match the experimental trend. It should be noted that the experiments were conducted under load control where it was rather difficult to control the descending branch of the load-displacement relationship. Similar to the tests of MB, experimental results of MC are scattered in spite of their identical designs and construction.



Fig. 4.8 Load-displacement relationships; (a) MB; (b) MC.

Figure 4.9 shows load-displacement relationships obtained by analysis and experiment of KC. These relationships are for the response in the lateral direction at the column tip. The comparison is shown for two types of analyses. In the first, the effect of  $P-\Delta$  is ignored (dotted line). This analysis did not capture the experimentally recorded descending branch. The second analysis invoked the  $P-\Delta$  capability of the program where an excellent agreement



Fig. 4.9 Load-displacement relationships of KC

with the experiment is observed (dashed line). It should be noted that in the experiment of KC, small initial loading cycles were applied which are not included in the analysis. Due to this simplification, minor adjusting of the material properties was necessary for good match with the experiments at the initial stage of loading. This was conducted by 10% reduction of the concrete compressive strength and the steel yield strength from their specified nominal values.

Figures 4.8 and 4.9 show the capability of the program developed in this study in estimating the softening behavior of RC structural members. The comparisons in Figures 4.8 and 4.9 imply good match between the analytical predictions and the experimental data. Accordingly, the developed computational tool is an accurate one for deterministic nonlinear analysis of RC structures. This tool is subsequently extended to account for the variabilities of material and geometrical properties as discussed in the following section.

It should be noted that shear deformation is ignored in the present fiber element formulation. Accordingly, shear failure is assumed not to occur prior to flexure failure, which is the case in most of the examples presented in this chapter.

#### 4.3 STOCHASTIC FIBER ELEMENT MODEL

In general, stochastic finite element method (SFEM) refers to the finite element method that accounts for spatial variability in the material properties, geometry of the structure, or applied loads. In this context, the *stochastic fiber element model* is developed in this study, such that spatial variability of the material and geometrical properties in the structural model is accounted for in the conventional (deterministic) fiber element model.

#### 4.3.1 Monte Carlo Simulation

Monte Carlo simulation (Rubinstein 1981) is one of the methods widely used to analyze random problems. In a random problem, random outputs are obtained in probabilistic or statistical forms using random inputs. In Monte Carlo simulation, random inputs are represented by sets of deterministic values, often called samples. Then, a random problem is transformed into a set of deterministic problems that can be solved using conventional tools such as the finite element method. From each set of deterministic inputs, one can generate a set of deterministic outputs. Finally, one can construct probabilistic or statistical forms of outputs from the sets of deterministic outputs. Because of its simplicity and robustness, Monte Carlo simulation is frequently used to analyze random problems in engineering applications and to validate other probabilistic analysis methods.

#### 4.3.2 Random Field Representation

In most of SFEM, it is necessary to represent a random field in terms of random variables. This process is usually called representation of the random field. Several methods for representation of random fields have been proposed. These include the midpoint method (Der Kiureghian and Ke 1988), the spatial averaging method (Vanmarcke and Grigoriu 1983), the shape function method (Liu et al. 1986), and the series expansion method (Ghanem and Spanos 1991; Lawrence 1987; Li and Der Kiureghian 1993; Spanos and Ghanem 1989; Zhang and Ellingwood 1994). In this section, existing methods for random field generation are reviewed and issues on random field mesh size are discussed.

Let  $v(\mathbf{x}), \mathbf{x} \in \Omega$ , denote a multidimensional Gaussian random field defined in the domain  $\Omega$ . This field is completely described by its mean function  $\mu(\mathbf{x})$ , variance function  $\sigma^2(\mathbf{x})$ , and autocorrelation coefficient function  $\rho(\mathbf{x}, \mathbf{x}')$  between the two random variables  $\mathbf{x}$  and  $\mathbf{x}'$ .

#### 4.3.2.1 Midpoint Method

The simplest method of discretization is the midpoint method applied to the stochastic problem by Der Kiureghian and Ke (1988). In this method, the random field is represented by a constant value over each element. A representing value for each element is the one evaluated at the centroid  $\mathbf{x}_c$  of the element, i.e.,

$$\hat{v}(\mathbf{x}) = v(\mathbf{x}_c); \quad \mathbf{x} \in \Omega_e$$

$$(4.36)$$

where  $\hat{v}(\mathbf{x})$  is an approximation of  $v(\mathbf{x})$ , and  $\Omega_e$  is an element domain. Mean  $\boldsymbol{\mu}$  and covariance matrix  $\boldsymbol{\Sigma}_{\boldsymbol{v}\boldsymbol{v}}$  of  $\mathbf{v} = \{v(\mathbf{x}_c^1), \dots, v(\mathbf{x}_c^{ne})\}$  are obtained from  $\mu(\mathbf{x})$ ,  $\sigma^2(\mathbf{x})$ , and  $\rho(\mathbf{x}, \mathbf{x}')$  of  $v(\cdot)$ evaluated at centroids of all elements having the number *ne*. It has been shown that this method tends to *over-represent* the variability of the random field within each element (Der Kiureghian and Ke 1988).

#### 4.3.2.2 Spatial Averaging Method

In this method proposed by Vanmarcke and Grigoriu (1983), the random field is also represented by a constant value over each element. A representing value is the spatial average of the field over the element. Mathematically, this can be written as

$$\hat{v}(\mathbf{x}) = \frac{\int_{\Omega_e} v(\mathbf{x}) d\Omega}{\int_{\Omega_e} d\Omega} = \bar{v}_e; \quad \mathbf{x} \in \Omega_e$$
(4.37)

The mean and covariance matrix of  $\mathbf{v}$  are obtained from  $\mu(\mathbf{x})$ ,  $\sigma^2(\mathbf{x})$ , and  $\rho(\mathbf{x}, \mathbf{x}')$  of  $v(\cdot)$  as integrals over the domain  $\Omega_e$ . This averaging process allows us to expect a better estimate on the random field than the midpoint method. Matthies et al. (1997) pointed out the difficulties involved in this method as follows:

- the approximations of the non-rectangular elements may lead to a non-positive definite covariance matrix.
- this method is practically limited to cases where the random field is Gaussian because the distribution function of each random variable  $v_i$  is almost impossible to obtain except for Gaussian random fields.

Moreover, this method is known to *under-represent* the local variance of the random field (Der Kiureghian and Ke 1988).

#### 4.3.2.3 Shape Function Method

The shape function method, proposed by Liu et al. (1986), uses sets of nodal values and corresponding shape functions to describe the random field, i.e.,

$$\hat{v}(\mathbf{x}) = \sum_{i=1}^{nn} N_i(\mathbf{x}) v(\mathbf{x}_i); \quad \mathbf{x} \in \Omega_e$$
(4.38)

where nn is the number of nodes in the element,  $N_i$  is the  $i^{th}$  shape function, and  $\mathbf{x}_i$  is the  $i^{th}$  nodal coordinates. The mean and covariance are approximated by the following:

$$E[\hat{v}(\mathbf{x})] = \sum_{i=1}^{nn} N_i(\mathbf{x})\mu(\mathbf{x}_i)$$
(4.39)

$$\operatorname{Cov}[\hat{v}(\mathbf{x}), \hat{v}(\mathbf{x}')] = \sum_{i,j=1}^{nn} N_i(\mathbf{x}) N_j(\mathbf{x}') \operatorname{Cov}[v(\mathbf{x}_i), v(\mathbf{x}_j)]$$
(4.40)

Here, the random field is described by a continuous function, while the aforementioned two methods use discontinuous functions.

#### 4.3.2.4 Optimal Linear Estimation

In the optimal linear estimation (OLE) method, presented by Li and Der Kiureghian (1993), the random field is estimated by a linear estimator  $\hat{v}$  expressed as

$$\hat{v}(\mathbf{x}) = a(\mathbf{x}) + \sum_{i=1}^{nn} b_i(\mathbf{x})v(\mathbf{x}_i)$$

$$= a(\mathbf{x}) + \mathbf{b}^T(\mathbf{x})\mathbf{v}; \quad \mathbf{x} \in \Omega$$
(4.41)

where, nn is the number of nodes in the domain,  $\mathbf{v} = [v(\mathbf{x}_i)]$  is the nodal vector. Using the linear estimation theory, we determine  $a(\mathbf{x})$  and  $\mathbf{b}(\mathbf{x})$  such that the variance of the error  $v(\mathbf{x}) - \hat{v}(\mathbf{x})$  is minimized, where  $\hat{v}(\mathbf{x})$  is an unbiased estimator of  $v(\mathbf{x})$ , i.e.,

minimize 
$$\operatorname{Var}[v(\mathbf{x}) - \hat{v}(\mathbf{x})]$$
 (4.42)

subject to  $E[v(\mathbf{x}) - \hat{v}(\mathbf{x})] = 0; \quad \mathbf{x} \in \Omega$  (4.43)

According to Li and Der Kiureghian (1993) after finding the solution of this problem,  $\hat{v}(\mathbf{x})$  can be rewritten as

$$\hat{v}(\mathbf{x}) = \mu(\mathbf{x}) + \boldsymbol{\Sigma}_{v(\mathbf{x})\mathbf{v}}^T \boldsymbol{\Sigma}_{\mathbf{vv}}^{-1}(\mathbf{v} - \boldsymbol{\mu}); \quad \mathbf{x} \in \Omega$$
(4.44)

where  $\Sigma_{v(\mathbf{x})\mathbf{v}}$  is the vector containing the covariance of  $v(\mathbf{x})$  with the elements of  $\mathbf{v}$ , and  $\boldsymbol{\mu}$ and  $\Sigma_{\mathbf{vv}}$  are the mean vector and variance-covariance matrix of  $\mathbf{v}$ . It is notable that this method always *under-represents* the variance of the actual random field  $v(\mathbf{x})$  (Li and Der Kiureghian 1993).

#### 4.3.2.5 Karhunen-Loeve Expansion

In the Karhunen-Loeve expansion (KL) method (Spanos and Ghanem 1989), the random field is expressed in terms of its spectral decomposition. The random field can be written as

$$v(\mathbf{x}) = \mu(\mathbf{x}) + \sum_{i=1}^{\infty} \xi_i \sqrt{\lambda_i} \ \phi_i(\mathbf{x}); \quad \mathbf{x} \in \Omega$$
(4.45)

where  $\xi_i$  is statically independent standard normal variables, and  $\lambda_i$  and  $\phi_i$  are eigenvalues and eigenfunctions of  $\rho(\mathbf{x}, \mathbf{x}')$  such that

$$\int_{\Omega} \rho(\mathbf{x}, \mathbf{x}') \phi_i(\mathbf{x}') d\mathbf{x}' = \lambda_i \phi_i(\mathbf{x})$$
(4.46)

and 
$$\int_{\Omega} \phi_i(\mathbf{x}) \phi_j(\mathbf{x}) d\mathbf{x} = \delta_{ij}$$
 (Kronecker delta) (4.47)

However, usually only a few of the terms with the largest eigenvalues are important in the spectral decomposition. Thus, by truncating insignificant terms with relatively small eigenvalues, (4.45) can be approximated by

$$\hat{v}(\mathbf{x}) = \mu(\mathbf{x}) + \sum_{i=1}^{r} \xi_i \sqrt{\lambda_i} \ \phi_i(\mathbf{x}); \quad \mathbf{x} \in \Omega$$
(4.48)

where r is the number of terms included in the series. Some of the interesting properties of this method can be found in (Li and Der Kiureghian 1993; Matthies et al. 1997; Sudret and Der Kiureghian 2000) and briefly summarized here.

• The set of random variables  $\{\xi_i, i = 1, 2, ...\}$  is orthonormal in the sense that

$$E[\xi_i \xi_j] = \delta_{ij} \tag{4.49}$$

- $\{\xi_i, i = 1, 2, ...\}$  form a set of independent standard normal random variables.
- This method always *under-represents* the true variance of the random field.

Provided that the exact solution of the eigenvalue problem given by (4.47), this method is very efficient for simulating a random field, since it requires small number of random variables to describe the random field with a given level of accuracy. However, the eigenvalue problem seldom has an exact solution except for some special cases as discussed by Spanos and Ghanem (1989), and an approximation method should be used. Ghanem and Spanos (1991) suggested a Galerkin-type procedure where the eigenfunctions  $\phi_i$  are approximated by a set of basis functions and a corresponding set of coefficients. Subsequently, the integral eigenvalue problem in (4.46) and (4.47) is converted into a discrete (matrix) eigenvalue problem.

#### 4.3.2.6 Orthogonal Series Expansion

The orthogonal series expansion method is proposed by Zhang and Ellingwood (1994). This method avoids solving the eigenvalue problem in (4.46 and 4.47) by selecting a proper set of orthogonal functions instead of the eigenfunctions. A random field  $v(\mathbf{x})$  with its mean function  $\mu(\mathbf{x})$  and correlation function  $\rho(\mathbf{x}, \mathbf{x}')$  can be expressed as

$$v(\mathbf{x}) = \mu(\mathbf{x}) + \sum_{i=1}^{\infty} \chi_i h_i(\mathbf{x})$$
(4.50)

where  $h_i(\mathbf{x})$  are orthogonal functions and  $\chi_i$  are correlated random variables. It should be noted that Lawrence (1987) proposed a similar method to the orthogonal series expansion called the *basis random variable expansion*. Using the orthogonality of  $h_i$ 's and some basic algebra, it can be shown that

$$\chi_i = \int_{\Omega} [v(\mathbf{x}) - \mu(\mathbf{x})] h_i(\mathbf{x}) d\Omega \qquad (4.51)$$

$$(\Sigma_{\mathbf{vv}})_{ij} \equiv E[\chi_i \chi_j] = \int_{\Omega} \int_{\Omega} \rho(\mathbf{x}, \mathbf{x}') h_i(\mathbf{x}) h_j(\mathbf{x}') d\Omega d\Omega$$
(4.52)

For Gaussian  $v(\mathbf{x})$ , (4.51) proves that  $\chi_i$ 's are zero-mean random variables. A random field in (4.50) can be approximated by selecting a set of proper orthogonal functions and a finite number of terms used in the expression, i.e.,

$$\hat{v}(\mathbf{x}) = \mu(\mathbf{x}) + \sum_{i=1}^{r} \chi_i h_i(\mathbf{x})$$
(4.53)

where r is the number of terms included.

#### 4.3.2.7 Expansion Optimal Linear Estimation

The expansion optimal linear estimation (EOLE) method, proposed by Li and Der Kiureghian (1993), uses a spectral decomposition of the random field vector  $\mathbf{v}$  of size N in OLE method. If  $v(\mathbf{x})$  is assumed to be Gaussian random variable, the spectral decomposition of its covariance matrix  $\Sigma_{vv}$  is expressed as

$$\mathbf{v}(\mathbf{x}) = \boldsymbol{\mu}(\mathbf{x}) + \sum_{i=1}^{N} \sqrt{\lambda_i} \,\xi_i \phi_i \tag{4.54}$$

where  $\xi_i$ 's are independent standard normal variables, and  $\lambda_i$  and  $\phi_i$  are the eigenvalues and eigenvectors of  $\Sigma_{vv}$ , such that

$$\boldsymbol{\Sigma}_{\boldsymbol{v}\boldsymbol{v}}\phi_i = \lambda_i\phi_i \tag{4.55}$$

Substituting (4.54) into (4.44) gives

$$\hat{v}(\mathbf{x}) = \mu(\mathbf{x}) + \sum_{i=1}^{N} \frac{\xi_i}{\sqrt{\lambda_i}} \phi_i^T \Sigma_{v(\mathbf{x})\mathbf{v}}$$
(4.56)

The second term on the right-hand side of (4.56) can be approximated by using only r terms which have significant eigenvalues, i.e.,

$$\hat{v}(\mathbf{x}) = \mu(\mathbf{x}) + \sum_{i=1}^{r} \frac{\xi_i}{\sqrt{\lambda_i}} \phi_i^T \boldsymbol{\Sigma}_{v(\mathbf{x})\mathbf{v}}, \quad r < N$$
(4.57)

This expression is optimal in the sense that it minimizes the error in the variance. It is to be noted that EOLE always *under-represents* the true variance (Li and Der Kiureghian 1993).

Comparison between the above methods mentioned in this chapter for random field representation is presented by Sudret and Der Kiureghian (2000). Even though the midpoint method may exhibit relatively poor performance for representing a given random field, this method is adopted in this study for its simplicity. However, replacing this simple method by a more sophisticated method should neither pose a major difficulty nor alter the conclusions of the present study.

#### 4.3.2.8 Nataf Model

In general,  $v(\mathbf{x})$  is a non-Gaussian random field. In this case,  $\mathbf{v}(\mathbf{x}) = [v_1, \ldots, v_{ne}]$  is a vector of dependent non-Gaussian random variables whose joint distribution function is not known and only the marginal distribution and correlation coefficient matrix ( $\mathbf{R} = \{\rho_{ij}\}$ , i, j = 1, ..., ne) are available in many applications. In this general case, the Nataf model (Liu et al. 1986) can be used to simulate  $\mathbf{v}(\mathbf{x})$ . Using the marginal transformation of  $v_i$ , a standard normal random vector  $\mathbf{Z} = [Z_1, ..., Z_{ne}]$  is defined by

$$Z_i = \Phi^{-1}[F_{v_i}(v_i)], \quad i = 1, 2, \dots, ne$$
(4.58)

where  $\Phi(\cdot)$  is the standard normal cumulative distribution function and  $F_{v_i}(v_i)$  is the marginal cumulative distribution function of  $v_i$ . Nataf's distribution of  $\mathbf{v}(\mathbf{x})$  is obtained by assuming  $\mathbf{Z}$  is jointly normal with the correlation coefficient matrix  $\mathbf{R}_0 = \{\rho_{0,ij}\}, i, j = 1, \ldots, ne$ . Approximate relations of  $\rho_{0,ij}(\rho_{ij})$  for a large number of distribution types are given by Liu and Der Kiureghian (1986). In Monte Carlo simulation, the procedure of simulating  $\mathbf{v}(\mathbf{x})$  is summarized as follows:

- 1. Generate a vector **U** of size *ne*, whose components are independent standard normal random samples.
- 2. Compute  $\mathbf{Z} = \mathbf{L}_0 \mathbf{U}$  where  $\mathbf{L}_0$  is a lower triangle matrix of  $\mathbf{R}_0$  such that  $\mathbf{R}_0 = \mathbf{L}_0 \mathbf{L}_0^T$ .
- 3. Compute  $v_i = F_{v_i}^{-1}[\Phi(Z_i)]$  to simulate  $\mathbf{v}(\mathbf{x})$ .

#### 4.3.2.9 Issues on Random Field Mesh Size

One of the important issues in SFEM is selecting the mesh sizes. While the finite element mesh size is governed by the geometry and the expected gradient of the stress field, the random field mesh size is controlled by the rate of fluctuation of the random field. This fact implies that two independent mesh discretizations are acceptable: one for the finite element and the other for the random field.

The rate of fluctuation of the random field is usually measured by the so-called *correlation length*  $\theta$  which is defined as a measure of the distance over which significant loss of correlation occurs. Mathematically, this correlation length can be written as

$$\theta = \int_0^\infty \rho(\Delta x) d\Delta x \tag{4.59}$$
where  $\Delta x = \| \mathbf{x} - \mathbf{x}' \|$  is the Euclidian distance between  $\mathbf{x}$  and  $\mathbf{x}'$ , vectors defining the locations of two random variables, for a random field. In random field discretization, the distance between two adjacent random variables has to be short enough to capture the essential features of the random field. For this, Der Kiureghian and Ke (1988) examined the influence of  $\theta$  on reliability analysis and recommended the use of the random field element size as one quarter to one half of the correlation length. This recommendation has been generally accepted by the SFEM community. It should be noted that the use of excessively fine mesh discretization of a random field produces highly correlated random variables. In addition to the high computational cost, the correlation matrix becomes nearly singular causing numerical difficulties.

### 4.3.3 Stochastic Fiber Element Model for RC Elements

Unlike three-dimensional finite element methods, there is flexibility in the discretization of the cross section in the fiber element method as shown in Figure 4.10. This is one of the motivations of developing the stochastic fiber element method in this investigation. This flexibility is essential in the present study to practically account for construction variability. As shown in Figure 4.10, the material variability in this study distinguishes between unconfined concrete (cover outside the transverse reinforcement), confined concrete (core inside the transverse reinforcement) and reinforcing steel properties. The location of the longitudinal reinforcing bars dictated by the size of the closed transverse reinforcement is also treated as a random variable as an example of the geometrical variability.

Stochastic fiber element analysis is described in the flowchart in Figure 4.11. This procedure starts with characterizing the random fields and the deterministic parameters in the structural model. In this procedure, a random field is defined by a specified distribution with its autocorrelation function. Correlations between two random fields are also defined. Subsequently, a random field mesh is generated for each of the random fields. Once random field meshes are defined, samples of random fields are represented using the midpoint method



Fig. 4.10 Random fields and fiber element meshes.

where samples of material properties are assigned to the proper fiber element. Defining boundary conditions completes the model. Using a nonlinear analysis algorithm, structural responses of interest are obtained and added to the database of the simulation output. This procedure is repeated for the required number (sample size) of simulations such that the variation of the output of interest is less than a specified tolerance for convergence. The sample size and the selected convergence criteria are discussed in Section 4.4.2 using numerical examples.



Fig. 4.11 Analysis procedure using the stochastic fiber element model.

# 4.3.3.1 Variability of Material Properties

Let  $C(\mathbf{x})$  denote the random field representing one of the concrete properties, e.g., compressive strength. Let  $\mu_C(\mathbf{x})$ ,  $\sigma_C^2(\mathbf{x})$ , and  $\rho_{CC}(\boldsymbol{\xi}^{ij})$  denote the mean, variance, and autocorrelation coefficient function of  $C(\mathbf{x})$ , respectively. The separation vector  $\boldsymbol{\xi}^{ij}$  is given by  $\boldsymbol{\xi}^{ij} = \begin{bmatrix} \xi_x^{ij} \ \xi_y^{ij} \ \xi_z^{ij} \end{bmatrix}^T = \mathbf{x}^i - \mathbf{x}^j$  where  $\mathbf{x}^i$  and  $\mathbf{x}^j$  are the position vectors of random variables iand j, respectively. Amongst the commonly used autocorrelation coefficient functions found in Li and Der Kiureghian (1993), the author adopts the following,

$$\rho_{CC}(\boldsymbol{\xi}^{ij}) = \exp\left[-\sqrt{\left(\frac{\xi_p^{ij}}{\theta_p}\right)^2 + \left(\frac{\xi_z^{ij}}{\theta_z}\right)^2}\right]$$
(4.60)

where  $\xi_p^{ij} = \sqrt{(\xi_x^{ij})^2 + (\xi_y^{ij})^2}$  and  $\xi_z^{ij}$  are, respectively, in-plane distance and distance along the length of the element between the two random variables. Using the midpoint method, the random variables are associated with the centroids of the fiber elements as shown in Figure 4.10. The scale parameters  $\theta_p$  and  $\theta_z$  are related to the scale of fluctuation of the random field within the cross section and along the longitudinal axis of the structural component, respectively. Distinction is necessary between  $\theta_p$  and  $\theta_z$  because it is expected that there is a significant difference between the size of the random field mesh within the cross section and along the length of the structural component.

Let  $S(\mathbf{x})$  denote the random field representing one of the reinforcing steel properties, e.g., yield strength. Let  $\mu_S(\mathbf{x})$ ,  $\sigma_S^2(\mathbf{x})$ , and  $\rho_{SS}(\mathbf{x}_i, \mathbf{x}_j)$  denote the mean, variance, and autocorrelation coefficient function of  $S(\mathbf{x})$ , respectively. We assume the autocorrelation coefficient function as

$$\rho_{SS}(\mathbf{x}_i, \mathbf{x}_j) = 1, \quad i = j \quad \text{and} \quad \rho_{SS}(\mathbf{x}_i, \mathbf{x}_j) = \rho_S, \quad i \neq j \quad (4.61)$$

where  $\rho_S$  is a constant correlation coefficient. It should be noted that unlike  $\rho_{CC}(\boldsymbol{\xi}^{ij})$ ,  $\rho_{SS}(\mathbf{x}_i, \mathbf{x}_j)$  is not a function of the separation vector  $\boldsymbol{\xi}^{ij}$ . This implies that the correlation between any two reinforcing bars is constant regardless of their locations.

Figures 4.12(a) and (b) show schematic views of probabilistic constitutive models of unconfined and confined concrete, and reinforcing steel, respectively. In Figure 4.12(a), the random variables are  $f_{co}$  and  $f_{cc}$ , while the random variable in Figure 4.12(b) is  $f_y$ . In these figures, upper and lower bounds of the constitutive model are schematically shown as well as the mean model.

# 4.3.3.2 Variability of Construction Geometry

The variability of the location of the longitudinal reinforcement is accounted for as a variability of construction geometry. For a circular structural component, for example, the location of the longitudinal reinforcement is dictated by the size of the circle representing the trans-



Fig. 4.12 Probabilistic constitutive models; (a) Concrete; (b) Steel.

verse reinforcement, or by the thickness of the concrete cover. In the present study, the cover thickness is treated as a random variable that is assumed to remain constant along the length of the structural component. In other words, spatial variability of the cover thickness is not considered. Before each simulation, a sample of the cover thickness is generated from a particular distribution function. This sample is used as an input parameter to define the fiber element mesh and the corresponding random field meshes as illustrated in Figure 4.11.

# 4.4 STRENGTH ANALYSIS OF RC COLUMNS

As an example of the probabilistic analysis procedure developed in the present study, a probabilistic strength analysis of an RC column is conducted. The strength of the RC column is affected by the strength properties of the concrete and the steel and the construction geometry such as cross-sectional dimensions and locations of the reinforcing bars. Therefore, variation in the overall strength of the RC column depends on variations in those variables of the material properties and the construction geometry. The specimen selected for the probabilistic strength analysis is one of the specimens used for the verification examples, referred to as KC in Section 4.2.4. The variability of the strength of KC is investigated in terms of the axial load-bending moment (P-M) interaction at the column base.

#### 4.4.1 Probabilistic Models and Discretization of Random Fields

In this study, only variables that are considered to have significant effect on the strength of the RC column are selected as random fields. Selected variables are compressive strength and initial modulus of elasticity for material variability of concrete, yield strength and initial modulus of elasticity for material variability of reinforcing steel, and concrete cover thickness to account for variability of construction geometry. Statistical properties of random fields are given in Table 4.2. These parameters are adopted from the works of Mirza et al. (1979, 1979a, 1979b) as discussed in Section 2.5.2. The correlation between Fc and Ec is assumed to be 0.8 (Mirza and MacGregor 1979b), while any other random fields are assumed to be uncorrelated.

Source of uncertainty	Variable	Dist'n	Mean	COV (%)				
Compressive strength of concrete	Fc	Normal	3.5  ksi	17.5				
Initial modulus of elasticity of concrete	Ec	Normal	3,375  ksi	12.0				
Yield strength of steel	Fy	Logn'l	$71 \mathrm{ksi}$	9.3				
Initial modulus of elasticity of steel	Es	Normal	29,000  ksi	3.3				
Cover thickness	Tc	Normal	0.8 in.	14.5				
Note: Correlation coefficient of $Fc$ and $Ec$ is 0.8								

 Table 4.2
 Statistical properties of variables.

Random field mesh for Fc and Ec is identical to the fiber element mesh along the length of the column. However, within the cross section, the random field element size for concrete properties is taken as four times the fiber element size. The scale parameters  $\theta_p$  and  $\theta_z$  in (4.60) are taken as the length of one random field element and the distance between two adjacent random field fibers within the cross section, respectively. For each of the two random fields of Fy and Es, one random variable is used along the length of the column, while within the cross section, different random variables are used for different reinforcing bars. It is assumed that  $\rho_S = 0.8$  in (4.61) for both Fy and Es.

It should be noted that a non-positive sample of Fc, Ec, or Es is rejected to avoid an unrealistic realization. Sample rejection is not recommended when using Monte Carlo simulation because the sample distribution can statistically differ from the assumed one. Nevertheless, the normal distribution assumption with a sample rejection is adopted in this analysis for simplicity. However, the sample rejection rarely occurs in the actual analysis of this study due to the specific values of means and standard deviations. Thus, the distortion of the sample distributions can be ignored. The distribution of Tc is also truncated at the mean  $\pm$  two standard deviations for the same reason.

#### 4.4.2 Analysis Procedure

The process of developing the probabilistic P-M interaction diagram is described in Figure 4.13. At first, random fields are defined and the corresponding random variables are generated using the midpoint method. Incremental axial load ( $P_a$ ) and lateral load ( $P_l$ ) are applied at the tip of the column simultaneously (Figure 4.7) until the extreme concrete fiber strain exceeds a specified value. In each analysis, the  $P_l/P_a$  ratio is kept constant such that the base moment is proportional to the axial load. Nine different ratios of  $P_l/P_a = 0, 0.01, 0.02, 0.04, 0.07, 0.1, 0.2, 0.3, and \infty$  are considered, where the ratio of 0 represents zero lateral load and the ratio of  $\infty$  corresponds to zero axial load. Then, nine pairs of the maximum axial load and the corresponding bending moment at the base of the column corresponding to a specified limit state in terms of the strain at the extreme fibers are obtained to form a P-M interaction diagram as shown in Figure 4.14. This process is repeated up to the required sample size to guarantee the convergence of the estimated quantities, namely, the mean and the standard deviation of axial loads and bending moments.

In this study, 2000 sets of random variables are generated for each of the random fields, e.g., compressive strength of concrete and yield strength of steel. The nine cases with different  $P_l/P_a$  ratios are considered for each of the random field sets. Consequently, 18,000 pairs of maximum axial load and bending moment are obtained at the end of the simulations. Subsequently, nine different means and standard deviations of axial loads and



Fig. 4.13 Procedure of developing a probabilistic P-M interaction diagram.

bending moments corresponding to  $P_l/P_a$  ratios are computed. The minimum required sample size is determined according to a selected tolerance. Figure 4.15 shows the result of a convergence test of the mean and the standard deviation of the column strength for  $P_l/P_a = 0.02$ . Figure 4.15(a) shows the convergence with respect to the sample size of the mean and the standard deviation of axial loads corresponding to extreme (outmost) concrete fiber strain of 0.005. In this plot, mean and standard deviations are respectively normalized by the mean and standard deviation of the simulated 2000 axial loads. Figure 4.15(b) shows the COVs corresponding to the results in Figure 4.15(a). It is evident that the sample size of 2000 is large enough to satisfy the selected tolerance of convergence, namely a COV of 5%. The convergence test is conducted for the other  $P_l/P_a$  ratios and the selected sample size of 2000 is determined to be satisfactory.



Fig. 4.14 Typical P-M diagram and various  $P_l/P_a$  ratios.



Fig. 4.15 The convergence test result of the mean and standard deviation of the column strength for  $P_l/P_a = 0.02$ : (a) normalized mean and standard deviation; (b) COV of the mean and standard deviation.

#### 4.4.3 Strength Variability

For generating P-M interaction diagrams, three different limit states (LS) in terms of the strain of the concrete extreme fiber ( $\epsilon_{c,ext}$ ) are considered: (1) LS 1:  $\epsilon_{c,ext} = 0.003$ , (2) LS 2:  $\epsilon_{c,ext} = 0.004$ , and (3) LS 3:  $\epsilon_{c,ext} = 0.005$ . Figure 4.16 shows mean P-M interaction diagrams for these three cases of LS. Axial forces and bending moments are normalized by the mean pure axial capacity and the mean pure bending moment capacity, respectively, for LS 1. It is noted that the P-M interaction diagram expands as LS increases. This is because while the strength of unconfined concrete, where the extreme concrete fiber is located, decreases as the strain increases from 0.003 to 0.005, the strength of confined concrete and reinforcing bars increase as can be shown in Figures 4.3 and 4.4 when introducing the numerical values of Table 4.1 for the KC specimen. To demonstrate the expansion and contraction of the P-M interaction diagram, higher cases of LS are investigated. For example, Figure 4.17(a) clearly



Fig. 4.16 Mean P-M interaction diagrams for different cases of LS.

shows changes of the deterministic P-M interaction diagrams depending on the adopted LS. The base bending moment-tip displacement relationship shown in Figure 4.17(b) corresponds to the deterministic P-M interaction diagrams shown in Figure 4.17(a) for  $P_l/P_a = 0.015$  where the corresponding limiting points are marked.



Fig. 4.17 Changes of P-M interaction diagram; (a) Deterministic P-M interaction diagrams for various cases of LS; (b) Base bending moment-tip displacement relationship for  $P_l/P_a = 0.015$ .

COVs of column strength for different cases of LS are plotted in Figure 4.18. It is observed that variability of the column strength is higher when  $P_l/P_a \leq 0.01$  and  $P_l/P_a \geq$ 0.2. This result agrees with that of Mirza and MacGregor (1989) where they investigated the strength variability of slender RC columns without considering spatial variability of random variables. One can also observe that the COV increases as the LS increases. When only the axial load is applied to the column, the strength variability of the column depends only on the combination of variability of random fields. As the  $P_l/P_a$  ratio increases, the strength variability begins to be dependent on the geometry of the cross section as well because the curvature is introduced to the cross section due to the applied bending moment. Once the



Fig. 4.18 COV of column strength for different cases of LS.

one fiber to the other. As Figure 4.12 shows, variability of stress changes as the strain level does. This fact causes the changes of strength variability from one  $P_l/P_a$  ratio to the other, and also from one LS to the other. These arguments are theoretically discussed in Section A.2 of Appendix A.

The sensitivity of the column strength to an individual random field is investigated for Fc, Fy, and Tc. The effect of each of these random fields is studied keeping all other variables at their mean values. In this analysis, only three  $P_l/P_a$  ratios are considered, namely 0.01, 0.05, and 0.3 spanning different failure modes from compression to tension as shown in Figures 4.14 and 4.16. COVs of column strength for each sensitivity analysis are summarized in Table 4.3. For the sensitivity of Fc, the variation of the column strength in the compression-failure region ( $P_l/P_a = 0.01$  and 0.05) is larger than that in the tensionfailure region. Conversely, for the sensitivity of Fy, it is obvious that the variation in the tension-failure region is larger than that in the compression-failure region. The strength variability due to variability of the cover thickness is not significant relative to the other sources of variability.

Random field	Fc			Fy			Tc		
$P_l/P_a$	0.01	0.05	0.3	0.01	0.05	0.3	0.01	0.05	0.3
LS 1	3.7	3.4	2.4	2.1	1.1	3.9	0.8	0.2	0.1
LS 2	3.3	3.4	2.4	2.7	2.4	5.1	1.4	0.8	0.5
LS $3$	3.0	3.3	2.3	2.7	2.3	5.5	1.8	1.5	0.7

Table 4.3 Sensitivity of the column strength in terms of COV (%).

#### 4.4.4 The Effect of Spatial Variability

The effect of the spatial variability of Fc on the column strength is studied in this section. For this purpose, the sensitivity analysis of Fc is conducted with and without considering the spatial variability of Fc while keeping all other random variables at their mean values. COVs of the column strength without spatial variability of Fc are significantly larger than those considering spatial variability of Fc, as shown in Figure 4.19. This result justifies the importance (to avoid overestimating COVs when spatial variability is ignored) of considering spatial variability of random variables in the probabilistic evaluation of RC structures.

# 4.5 PROBABILISTIC EVALUATION OF STRUCTURAL COMPONENTS OF AN RC FRAME

This section presents the probabilistic evaluation of typical structural components of a ductile RC frame (referred to as VE), representing the *structural system* in the present study, according to the methodology discussed in Section 2.6. A description of the VE frame is presented, followed by the presentation of a process of identifying *typical structural components*. Probabilistic moment-curvature relationships of typical components of the VE frame are developed using the stochastic fiber element model and OpenSees. Finally, probabilistic shear force-distortion relationships of these components are developed using Response 2000 (Bentz 2000), based on the modified compression field theory, and the FOSM method.



Fig. 4.19 COVs of column strength with and without spatial variability of Fc.

#### 4.5.1 Description of the Frame VE

The VE frame was experimentally tested by Vecchio and Emara (1992). It is a two-story, one-bay RC frame which consists of beams and columns with rectangular cross sections, as shown in Figure 4.20. It was designed with a center-to-center span of 137.8 inches (3,500 mm), a story height of 78.7 inches (2,000 mm) and an overall height of 181.1 inches (4,600 mm) including an integrally constructed large heavily reinforced concrete base beam. All frame members were 11.8-inches (300 mm) wide by 15.7-inches (400 mm) deep and reinforced with four No. 20M deformed bars on both sides of the cross section. The transverse steel reinforcement consists of No. 10M deformed bars in the form of closed stirrups or ties spaced at 4.9 inches (125 mm). Note that the nominal cross-sectional areas of No. 10M and No. 20M bars are 0.166 in.<sup>2</sup> (75 mm<sup>2</sup>) and 0.465 in.<sup>2</sup> (300 mm<sup>2</sup>), respectively. The testing sequence involved initial application of a total axial load of 157.5 kips (700 kN) to each column, which was maintained through the test. The lateral load was then monotonically applied until the ultimate capacity of the frame was achieved. The failure mechanism involved ductile



Fig. 4.20 Design details of the VE test frame (Vecchio and Emara 1992).

hinging at the ends of the beams and at the bases of the columns. The material properties of concrete, and longitudinal and transverse reinforcing steel bars are listed in Table 4.4. The same material properties are used for all beams and columns.

# 4.5.2 Typical Structural Components

According to the methodology proposed in the present study (cf. Section 2.6), a linear elastic analysis of the VE frame is performed to obtain the bending moment, shear force, and axial force diagrams. Following the procedure discussed in Section 2.6, force boundary conditions of all six structural components (referred to as C1, C2, C3, C4, B1, and B2 as shown in Figure 4.21(a)) are identified where the force boundary condition is defined by a constant axial load ( $P_a$ ), monotonic lateral load ( $P_l$ ), and monotonic axial load ( $\alpha P_l$ ) that is proportional to  $P_l$  with the constant of proportionality  $\alpha$  as shown in Figure 4.21(b). Parameters defining force boundary conditions of all six components are listed in part (a) of Table 4.5.

Property	Nominal value
(a) Concret	e
Compressive strength	4,350 psi (30 MPa)
(b) Longitudinal steel	(No. 20M)
Yield strength	61 ksi (418 MPa)
Ultimate strength	86 ksi (596 MPa)
Modulus of elasticity	27,900 ksi (192 GPa)
Strain at the onset of hardening	$9.5 \times 10^{-3}$
Modulus of strain hardening	450 ksi (3,100 MPa)
Ultimate strain	0.07
(c) Transverse steel (	No. 10M)
Yield strength	66 ksi (454 MPa)
Ultimate strength	93 ksi (640 MPa)

Table 4.4 Material properties of the VE test frame.



(a) Structural system model of VE

(b) Structural component model



Component	$P_a$ (kips)	$\alpha^a$	Length (in.)					
(a	a) All six co	mponer	nts					
C1	157.5	-1.46	78.7					
C2	157.5	1.46	78.7					
C3	157.5	-0.63	78.7					
C4	157.5	0.63	78.7					
B1	0.0	$0.0^{b}$	137.8					
B2	0.0	$0.0^{b}$	137.8					
(b)	) Typical co	mpone	nts					
CN	157.5	1.0	78.7					
$\operatorname{CS}$	157.5	-1.0	78.7					
BM 0.0 0.0 137								
<sup>a</sup> Negative indicates tension.								
<sup><math>b</math></sup> Refer to Section 2.6.2.1 for approximation.								

 Table 4.5
 Analysis parameters for structural components
 of the VE frame.

For simplicity, three typical components are identified by grouping components in part (a) of Table 4.5 with similar boundary conditions and modifying analysis parameters. Part (b) of Table 4.5 shows analysis parameters of the identified typical structural components. CN represents north columns (C2 and C4) subjected to an incremental axial load in compression with  $\alpha = -1.0$ , an average of C2 and C4. On the other hand, CS represents south columns (C1 and C3) subjected to an incremental axial load in tension with  $\alpha = 1.0$ , an average of C1 and C3. BM represents beam components (B1 and B2) without any applied axial load.

### 4.5.3 Random Fields

Random fields of the VE frame are defined based on design parameters as given in Table 4.4 and summarized in Table 4.6. Assumed statistical data related to these random fields are mainly adopted from various literatures as discussed in Section 2.5 due to the lack of data specifically related to the variability of the structural properties of the VE frame. It should be noted that properties of some random fields, e.g., initial modulus of elasticity of concrete, are derived from those of another random fields, e.g., compressive strength of concrete.

Uncertain source	Distribution	Mean	COV (%)					
(a)	) Concrete							
Compressive strength <sup><math>a</math></sup>	Normal	4,036 psi	15					
Initial modulus of elasticity <sup><math>b</math></sup>	Normal	$3,984 \text{ ksi}^c$	8					
(b) Longitudinal reinforcing steel								
Yield strength <sup><math>d</math></sup>	Lognormal	60.61  ksi	9					
Ultimate strength <sup><math>e</math></sup>	Lognormal	86.42  ksi	9					
Young's modulus	Normal	27,900 ksi	3.3					
Fracture strain <sup><math>f</math></sup>	Normal	0.07	20					
(c) Transve	rse reinforcing	steel						
Yield strength <sup><math>d</math></sup>	Lognormal	65.83  ksi	9					
Ultimate strength <sup><math>e</math></sup>	Lognormal	93.86 ksi	9					
Young's modulus	Normal	27,900 ksi	3.3					
Fracture strain <sup><math>f</math></sup>	Normal	0.07	20					
Correlation coefficient of $a$ and $b$ , $d$ and $f$ , and $e$ and $f$ are								
0.8, -0.5,  and  -0.55,  respectively (cf. Sections  2.5.2.1  and  2.5.2.2).								

 Table 4.6
 Probability distributions of basic random variables.

<sup>c</sup> Computed by (2.3) given in Section 2.5.2.1.

Uncertainty in concrete cover thickness is ignored, assuming that its effect on momentcurvature relationships at critical cross sections of the VE frame's components is negligible according to one of the observations in Section 4.4.3. As discussed in Section 4.4.1, nonpositive samples of a normal random variable are rejected to avoid an unrealistic realization in Monte Carlo simulation. However, the sample rejection rarely occurs in the actual analysis of this study due to the specific values of means and standard deviations. Thus, the distortion of the sample distributions can be ignored.

# 4.5.4 Probabilistic Moment-Curvature Relationship

A series of pushover analysis is performed to develop probabilistic moment-curvature relationships at critical cross sections of each typical structural component using OpenSees (McKenna and Fenves 2001) and the stochastic fiber element model described in Section 4.3. OpenSees has its own reliability toolbox that is mainly aimed at estimating the failure probability of a structure for a given limit-state function, rather than EDP statistics. Consequently, the stochastic fiber element model developed in this study is used to generate OpenSees inputs for Monte Carlo simulation considering various random fields to estimate EDP statistics.

## 4.5.4.1 Modeling Assumptions

Random fields of concrete properties along the length of a component are modeled by four random field elements. Within the cross section, random fields are described by sixteen identical rectangular patches (each side of the cross section is divided into four). The scale parameters  $\theta_z$  and  $\theta_p$ , refer to (4.60), are taken as the length of one random field element and the length of the long side of the cross section (15.7 in.), respectively. For each of the random fields of steel properties, one random variable is used along the length of the column, while within the cross section, different random variables are used for different reinforcing bars. It is assumed that  $\rho_S = 0.8$ , refer to (4.61), for random fields of steel properties.

To model a typical component, four nonlinearBeamColumn elements are used along the length of the component. Since nonlinearBeamColumn is formulated by a flexibility method (refer to Section 4.2), only one element is sufficient to capture the nonlinear response of the typical component given force boundary conditions. However, the finite element mesh is dictated by the random field mesh for concrete properties in this case.

Among many constitutive models in OpenSees material library, Steel01 is used for the reinforcement. As for the concrete, Concrete01 material model based on the modified Kent-Park stress-strain relationship (Scott et al. 1982) is used for both confined and unconfined concrete fibers (with zero tensile strength). Refer to Section 3.3.2.2 for descriptions of these material models of OpenSees.

## 4.5.4.2 Probabilistic Moment-Curvature Relationship

Moment-curvature relationships at two critical cross sections (both ends of the component as shown in Figure 4.21(b)) are monitored during the pushover analysis. For each component,

Monte Carlo simulation is performed with the sample size of 1500 to obtain a set of 1500 moment-curvature curves at each critical cross section. Each of the 1500 moment-curvature curves at the cross section close to the fixed base in Figure 4.21(b) (referred to as Cross section 1, while the cross section at the other end of the typical component is referred to as Cross section 2) are idealized as a trilinear moment-curvature relationship according to the procedure described in Section 2.6.2.3.

A convergence test with respect to the sample size of Monte Carlo simulation is necessary to guarantee the accuracy of the estimated quantity as discussed in Section 4.4.2. Figure 4.22 shows the result of a convergence test of the means and the standard deviations of the six moment-curvature parameters of the CS typical component; refer to part (b) of Table 4.5. It is evident that the sample size of 1500 is large enough to satisfy the selected tolerance of convergence, namely a COV of 5%. The convergence test is conducted for all parameters of CN and BM, and the selected sample size of 1500 is determined to be satisfactory.

Figure 4.23 shows the means of the idealized moment-curvature relationships of the three typical components of the VE frame. CN shows softening behaviors in the moment-curvature relationship, while BM and CS show hardening behavior in an average sense. Generated moment-curvature relationships of BM by Monte Carlo simulation are idealized by bilinear relationships because no distinctive *peak* points are observed. Therefore, the means and standard deviations of only four, instead of six for CN and CS, parameters, namely  $M_y$ ,  $M_u$ ,  $\varphi_y$ , and  $\varphi_u$  (cf. Section 2.6.2.3) are estimated for BM. It is noted that definitions of the four parameters remain unchanged.

Estimated means, standard deviations, and correlation coefficients of the momentcurvature parameters for the three typical components of the VE frame are listed in Table 4.7. It is observed that uncertainties related to the yielding point are smaller than those related to the peak or the ultimate points. This can be explained by the fact that uncertainties in the yielding point are only strongly related to the uncertainty in the yield strength of



Fig. 4.22 Convergence test of the means and standard deviations of the six moment-curvature parameters of CS typical component of the VE frame.

reinforcing steel. On the contrary, uncertainties of the peak and the ultimate points are related to all sources of uncertainties. It is also observed that COVs of curvature parameters  $(\varphi_y, \varphi_p, \text{ and } \varphi_u)$  are larger than those of moment parameters  $(M_y, M_p, \text{ and } M_u)$ . This is an expected observation because deformation uncertainty is often larger than force uncertainty according to the stress-strain behavior. For example, uncertainty in the strain corresponding to the compressive strength of concrete is related to both uncertainties of the compressive strength and the initial modulus of elasticity of concrete (cf. Table 4.6), and it also affects uncertainty in the ultimate strain of concrete as shown in Figure 4.3.

From correlation matrices in Table 4.7, one can notice that only negative correlation coefficients are those related to  $\varphi_u$ . This is mostly due to the assigned correlation coefficient to random variables representing yield strength, ultimate strength, and fracture strain of longitudinal reinforcing steel as given in Table 4.6.



Fig. 4.23 Mean trilinear moment-curvature relationships at critical cross sections of the typical components of the VE frame.

To estimate the distribution type of each moment-curvature parameter, histograms of all parameters are drawn as shown in Figures 4.24, 4.25, and 4.26 for CN, CS, and BM, respectively. Observing these histograms, it is assumed that the distribution types of moment-curvature parameters of the typical components are all normal distributions. To check this normality assumption of each parameter, normal probability plots are shown in Figures 4.27, 4.28, and 4.29, for CN, CS, and BM, respectively. From these plots, it was decided that the normality assumption is reasonable for all moment-curvature parameters of the three typical components.

Finally, the correlation of the two critical cross sections, namely 1 and 2, of the same component is estimated in terms of moment-curvature parameters. This piece of information is useful when probabilistic section models are used in a system evaluation to enrich the probabilistic description of a given structural component by correlating two spatially separated critical cross sections.

the V	the VE frame.										
Parameter	Moon	SD	COV		Correlation coefficients						
1 arameter	Mean	SD	COV	$M_y$	$M_p$	$M_u$	$\varphi_y$	$\varphi_p$	$\varphi_u$		
			(a	u) CN							
$M_y$	2536	132	5.2%	1							
$M_p$	2638	143	5.4%	0.92	1						
$M_u$	2107	173	8.2%	0.77	0.85	1					
$arphi_y$	0.317	0.031	9.9%	0.62	0.36	0.27	1				
$arphi_p$	0.474	0.071	14.9%	0.44	0.60	0.54	0.13	1			
$\varphi_u$	3.233	0.436	13.5%	-0.30	-0.24	-0.18	-0.31	-0.04	1		
(b) CS											
$M_y$	1964	102	5.2%	1							
$M_p$	2176	125	5.7%	0.92	1						
$M_u$	2274	183	8.1%	0.78	0.88	1					
$\varphi_y$	0.263	0.022	8.5%	0.87	0.69	0.60	1				
$\varphi_p$	1.562	0.310	19.8%	0.30	0.51	0.40	0.00	1			
$arphi_u$	6.237	0.748	12.0%	-0.18	-0.25	-0.22	-0.19	-0.17	1		
			(c	) BM							
$M_y$	1360	112	8.2%	1	-			-			
$M_u$	2320	190	8.2%	0.73	-	1		-			
$arphi_y$	0.234	0.023	9.7%	0.91	-	0.63	1	-			
$\varphi_u$	12.964	2.379	18.3%	-0.43	-	-0.45	-0.37	-	1		
Note: mome	nt in kip-	in. and	curvature	e in 10 <sup>-</sup>	$^{3}/\text{in. fo}$	r means	and SI	)s.			

Table 4.7Estimates of means, standard deviations, and correlation coefficient<br/>matrices of moment-curvature parameters for typical components of



Fig. 4.24 Histograms of moment-curvature parameters for CN typical compo-



Fig. 4.25 Histograms of moment-curvature parameters for CS typical component of the VE frame.



Fig. 4.26 Histograms of moment-curvature parameters for BM typical compo-



Fig. 4.27 The normality check of the moment-curvature parameters of CN typical component of the VE frame.



Fig. 4.28 The normality check of the moment-curvature parameters of CS



Fig. 4.29 The normality check of the moment-curvature parameters of BM typical component of the VE frame.

Table 4.8 shows the correlation coefficient matrices of the two end cross sections for CN, CS, and BM typical components of the VE frame. Overall trends of CN, CS, and BM are almost the same. For example, three moment parameters, namely  $M_y$ ,  $M_p$ , and  $M_u$  are strongly correlated to each other with the mean correlation coefficient = 0.75, involving all 22 (9 for CN, 9 for CS, and 4 for BM) correlation coefficients related to these three moment parameters for CN, CS, and BM. Correlation coefficients related to  $\varphi_u$  are almost all negative. As discussed earlier in this section, this negative correlation is attributed to the assigned correlation coefficients to random variables representing yield strength, ultimate strength, and fracture strain of the longitudinal steel.

Table 4.8Estimates of correlation coefficient matrices of moment-curvatureparameters at different cross sections for typical components of theVE frame.

Cross section 1	Cross section 2									
CIUSS Section 1	$M_y$	$M_p$	$M_u$	$\varphi_y$	$\varphi_p$	$\varphi_u$				
		(a) C	CN							
$M_y$	0.79	0.80	0.65	0.69	0.41	-0.16				
$M_p$	0.71	0.75	0.56	0.59	0.43	-0.19				
$M_u$	0.62	0.63	0.47	0.55	0.34	-0.28				
$arphi_y$	0.62	0.57	0.58	0.63	0.26	0.01				
$arphi_p$	0.42	0.49	0.32	0.34	0.45	-0.15				
$arphi_u$	-0.38	-0.41	-0.55	-0.32	-0.22	-0.11				
	(b) CS									
$M_y$	0.87	0.86	0.75	0.77	0.34	0.19				
$M_p$	0.82	0.84	0.79	0.72	0.31	0.09				
$M_u$	0.77	0.82	0.79	0.67	0.26	-0.05				
$arphi_y$	0.75	0.74	0.65	0.75	0.30	0.13				
$arphi_p$	0.23	0.19	0.16	0.21	0.07	-0.02				
$arphi_u$	-0.21	-0.37	-0.48	-0.19	-0.15	0.17				
		(c) B	М							
$M_y$	0.85	-	0.75	0.78	-	-0.35				
$M_u$	0.75	-	0.77	0.65	-	-0.58				
$arphi_{m{y}}$	0.78	-	0.65	0.72	-	-0.32				
$\varphi_u$	-0.39	-	-0.69	-0.34	-	0.67				

#### 4.5.5 Probabilistic Shear Force-Distortion Relationship

Similar to the moment-curvature relationship, probabilistic shear force-distortion relationships at critical cross sections of the typical components of the VE frame are developed considering uncertainties in structural properties as given in Table 4.6. To compute the shear force-distortion relationship at a given critical cross section, Response 2000 (Bentz 2000), software based on the modified compression field theory (Vecchio and Collins 1986), is used. Since Response 2000 is only for a deterministic computation, Monte Carlo simulation is not appropriate for developing a probabilistic shear force-distortion relationship. Instead, it is decided to use the FOSM method (cf. Section 3.2.1) to estimate uncertainties in the shear force-distortion relationship.

The mean shear force-distortion relationships of all typical structural components of the VE frame are shown in Figure 4.30. It is to be noted that the incremental axial load ( $\alpha P_l$  as given in Table 4.5) is not considered due to the capability of Response 2000, while the constant axial load is considered. Consequently, CN and CS are identical in terms of the shear force-distortion relationship.

Estimated means and standard deviations of shear force-distortion parameters for the three typical components of the VE frames are given in Table 4.9. Similar to the observation of moment-curvature COVs, force parameters are less sensitive to basic uncertainties than deformation parameters. As previously mentioned, this can be attributed to the fact that the deformation uncertainty is often larger than the force uncertainty at the stress-strain level.

Compared to statistical data for the moment-curvature relationship provided in Section 4.5.4.2, those for the shear force-distortion relationship presented in this section lack sufficient information. For example, estimations of distribution types and correlations of shear force-distortion parameters are not provided. This is attributed to the fact that the FOSM method estimates only the mean and the standard deviation of the output as discussed in Section 3.2.1, while Monte Carlo simulation, used to estimate moment-curvature



Fig. 4.30 Mean shear force-distortion relationships of typical structural components of the VE frame.

parameters in Section 4.5.4.2, provides the basis of estimating all parameters of the probability distribution of the output. In Chapter 5, probabilistic section models developed in this section will be used in estimating EDP sensitivity of the VE frame using the FOSM method. It should be noted that all shear force-distortion parameters will be arbitrarily assumed to be uncorrelated due to lack of information. However, the focus of Chapter 5 is on the demonstration of the methodology of propagating basic uncertainties to structural systems.

### 4.6 CONCLUDING REMARKS

The propagation of basic uncertainties to the capacity of structural components is investigated in this chapter. An effort is made to consider the effect of basic uncertainties including

parameters for typical components of the VE frame.									
Paramotor	(a)	CN and	l CS	(b) BM					
	Mean	SD	COV	Mean	SD	COV			
$V_c$	55.9	2.29	4.1%	29.2	0.93	3.2%			
$V_p$	105.5	5.47	5.2%	98.1	5.70	5.8%			
$V_u$	99.9	3.06	3.1%	65.3	7.84	12.0%			
$\gamma_c$	0.318	0.037	11.6%	0.187	0.038	20.6%			
$\gamma_p$	4.514	0.292	6.5%	8.417	0.547	6.5%			
$\gamma_u$	9.177	0.616	6.7%	15.161	1.802	11.9%			
Note: shear	Note: shear force in kip and distortion in $10^{-3}$ .								

 Table 4.9
 Estimates of means and standard deviations of shear force-distortion

their spatial variability and the nonlinear behavior of RC members in the most accurate and realistic way to the possible extent. For this, a computational tool for probabilistic evaluation of RC structural components is developed using stochastic fiber element formulation. This formulation accounts for the spatial variability of material and geometrical properties of RC components using Monte Carlo simulation. In this stochastic fiber element formulation, conventional fiber element model and the midpoint method for representing random fields are combined. Assumptions and formulations of the fiber element model adopted in this study are described. The verification examples show that the deterministic capability of the developed program for estimating structural behaviors of different experimental studies.

bed program for estimating structural behaviors of different experimental studies. A probabilistic strength analysis of a RC column subjected to combined axial load and lateral load  $(P_l)$  is conducted in terms of the axial load-bending moment (P-M)

 $(P_a)$  and lateral load  $(P_l)$  is conducted in terms of the axial load-bending moment (P-M) interaction. For generating P-M interaction diagrams, different limit states in terms of the strain of the concrete at the extreme fiber  $(\epsilon_{c,ext})$  are considered. The coefficient of variation of column strength ranges from 3.7% to 7.2% depending on the limit state and the  $P_l/P_a$ ratio. Variability of the column strength is higher when  $P_l/P_a \leq 0.01$  and  $P_l/P_a \geq 0.2$ . Variability increases as  $\epsilon_{c,ext}$  increases except for the compression-failure region where change of variability with the  $P_l/P_a$  ratio is insignificant.

According to sensitivity analyses, the variation of the compressive strength of concrete

controls the variation of the column strength in the compression-failure region, while the variation of the steel strength controls that in the tension-failure region. The importance of considering spatial variability of a random variable is also investigated. It is observed that neglecting the spatial variability of concrete strength can lead to overestimating the strength variation of a RC component.

Due to the computational demand, the use of the stochastic fiber element method using Monte Carlo simulation is practically limited to RC structural components. An example of a probabilistic evaluation of a RC structural system using the developed probabilistic P-M interaction diagram will be presented in Chapter 5.

The typical structural components of a ductile case study RC frame (referred to as VE) are identified and evaluated to develop their probabilistic section models. This is conducted as a demonstration of the first phase, namely the component evaluation phase, of the systematic procedure of evaluating a structural system as discussed in Section 2.6.1.

The typical structural components of the VE frame are evaluated using the stochastic fiber element model and OpenSees software to develop probabilistic moment-curvature relationships at critical cross sections of the typical components. On the other hands, probabilistic shear force-distortion relationships at critical cross sections of the typical components are developed using the FOSM method and software Response 2000. Each of the moment-curvature and shear force-distortion relationships are idealized by either a trilinear relationship defined by six parameters (three force and three deformation parameters) or a bilinear curve defined by four parameters (two force and two deformation parameters). Therefore, each of the probabilistic component models is defined by means, standard deviations, and correlation coefficient matrices of these parameters. These probabilistic section models will be used in estimating EDP sensitivity of the VE frame using the FOSM method in Chapter 5.

Normal distributions can be assumed for all parameters defining a multilinear momentcurvature relationship. Variability of curvature parameters at the critical cross section of RC structural components is larger than those of moment parameters because deformation uncertainty is often larger than force uncertainty according to the stress-strain behavior.

# 5 EDP Sensitivity Induced by Component Uncertainty

#### 5.1 INTRODUCTION

Identifying critical structural components to a specific EDP of the structural system is an important component of a PBEE methodology. This identification provides an insight of how the structural system behaves under the seismic loading in terms of a specific EDP. The quantification of the importance of structural components should consider the location of each individual component in the system, the stiffness contribution of each component, and the probabilistic distribution of the strength and deformation capacities of each structural component. This identification can be also useful for the decision-making process, in particular, for the rehabilitation of an existing structure within the framework of the PBEE.

In spite of a large number of publications on the probabilistic evaluation of structural systems (Baker and Cornell 2003; Chryssanthopoulos et al. 2000; Ghobarah and Aly 1998; Singhal and Kiremidjian 1996), the effort of assessing the importance of structural components on the system performance is very rare. Gharaibeh et al. (2002) studied the relative importance of structural members using reliability methods. They used the conventional system reliability formulation for the mixed series-parallel system where the system reliability is an explicit function of a set of known component reliability corresponding to a specified limit-state function that distinguishes the failure and safety of the component. Due to this failure-or-not nature of the description of components, the conventional system reliability method can only be applicable to few types of structures, e.g., truss structures. In addition, the conventional system reliability method requires the identification of all possible  $cutsets^1$ . However, this process tends to be very difficult when the static indeterminacy of the structure is high or the number of structural components is large, because the number of cutsets is large in any of these cases.

In this chapter, the propagation of uncertainty in the strength and deformation capacities of structural components to their structural system with respect to its EDP is investigated. First, a probabilistic strength analysis of a portal frame is performed to demonstrate an application of probabilistic component models to the system evaluation. Then, EDP sensitivity to individual components of the VE frame discussed in Chapter 4 is estimated using the FOSM method. EDP uncertainty induced by uncertainty in each structural component is used to identify significant structural components of these case study frames for a specific EDP. The significance of a structural component is defined in terms of the EDP sensitivity where a more significant component corresponds to a higher EDP sensitivity for this particular component.

# 5.2 PORTAL FRAME APPLICATION

The example presented in this section shows an application of using the stochastic fiber element method (cf. Section 4.3) to evaluate a RC structural system. Rather than selecting a real structural system, a relatively simple one is chosen to demonstrate the methodology. The same approach can be easily extended to more complex structures.

Consider a portal frame consisting of an elastic beam and two inelastic circular columns subjected to two gravity loads (N) and a lateral load (H) as shown in Figure 5.1(a). In this application, the statistics of the failure lateral load  $(H_f$  in Figure 5.1(b)) for fixed gravity loads is determined. It is assumed that the design of the two columns is identical to

<sup>&</sup>lt;sup>1</sup>Cutset is a mixed series-parallel sub-system that is defined such that the system fails when any of the cutsets occurs.



Fig. 5.1 A portal frame subjected to fixed gravity loads and an increasing lateral load; (a) Configurations of the frame; (b) Failure mechanism.

that of the KC specimen in Section 4.4.

The columns are modeled using lumped plasticity with the expected failure mechanism consisting of plastic hinges located at the tops of the columns as shown in Figure 5.1(b). The behavior of each plastic hinge is controlled by the predefined probabilistic P-M curve, i.e., the plastic hinge forms when the axial force and bending moment pair at the section is on the P-M curve.

Various levels of gravity load are considered ranging from N = 0 to N = 400 kips. For each level of gravity load, the lateral load H is incrementally increased until the first plastic hinge forms at one of the two critical sections (Figure 5.1(b)) and axial forces in the two columns are computed. The bending moment at this first plastic hinge is obtained such that the bending moment and axial force pair is located on the P-M curve. Subsequently, the bending moment at the other critical section is computed to satisfy equilibrium. This process is continued with increasing H until the second plastic hinge forms and a failure mechanism (corresponding to failure load  $H_f$ ) is obtained.

#### 5.2.1 Uncertainty in the Strength of the Portal Frame

Using Monte Carlo simulation, 4000 samples of P-M curves are generated for each column and the procedure described in the previous section is repeated for each P-M curve. It should be noted that P-M random curves for these two columns are assumed to be uncorrelated. El-Tawil and Deierlein (2001) suggested an expression of P-M curve for RC columns where the bending moment capacity M corresponding to the axial force P at a section is obtained by

$$M = M_b \left\{ 1 - \left(\frac{\delta P}{\Delta P}\right)^a \right\}$$
(5.1)

where  $M_b$  is the balanced bending moment,  $\delta P = |P - P_b|$ , and  $\Delta P = P_c - P_b$  if  $P \ge P_b$ and  $\Delta P = P_b - P_t$  if  $P < P_b$ . Strength parameters  $P_b$ ,  $P_c$ , and  $P_t$  are the balanced axial force, crushing strength, and tensile strength, respectively. Parameter *a* controls the shape of the curve and is determined by calibration. In this study, it is determined that a = 1.8for  $P \ge P_b$  and, a = 1.9 for  $P < P_b$  for the mean P-M curve of LS 3 (corresponding to  $\epsilon_{c,ext} = 0.005$ ) in Section 4.4.  $P_c$ ,  $P_b$ ,  $P_t$ , and  $M_b$  are selected as random variables, and their statistical properties (mean, COV, and correlation coefficient) are obtained from the results of Section 4.4 as listed in Table 5.1. Figure 5.2 shows the mean P-M curve of LS 3 and the calibrated one where the top bar of a symbol indicates the mean of the random variable represented by this symbol.

Parameter	Dist'n	Mean	COV	Correlation coefficient				
1 arameter	D150 II	wittan	001	$P_c$	$P_b$	$P_t$	$M_b$	
$P_c$	Normal	$507 \mathrm{~kips}$	5.0%	1.0	0.7	-0.04	0.7	
$P_b$	Normal	$115 \mathrm{~kips}$	5.0%		1.0	-0.03	0.9	
$P_t$	Normal	-220 kips	5.0%			1.0	-0.03	
$M_b$	Normal	$955 \mathrm{kip}\text{-in}$	5.0%	Sym	metric		1.0	

 Table 5.1
 Statistical properties of random variables for the P-M relationship.

Figure 5.3(a) shows the mean values of  $H_f$  (solid line) at different gravity load levels (N). In this figure,  $H_f$  is normalized by the shear capacity of the two columns, i.e.,  $2V_n$ where  $V_n$  is the nominal shear capacity of the circular column section (estimated according


Fig. 5.2 Calibration of the analytical P-M curve to the mean P-M curve of LS 3.

to ACI 318-02 (2002) as 42.3 kips), and N is normalized by  $P_c$ . It is obvious that the mean values of  $H_f$  are far below the shear capacity of the frame, which verifies the assumed failure mechanism. Accordingly,  $H_f$  is determined by the bending moment capacity of sections, and the shape of the N- $H_f$  interaction curve is similar to that of P-M interaction curve as shown in Figure 5.3(a). Moreover, figure 5.3(a) shows the mean±2 standard deviation curves (dashed lines) to show the dispersion of  $H_f/2V_n$ . Figure 5.3(b) shows COV of  $H_f$ (solid line labeled as "Total"). Note that the other two curves labeled KC1 and KC2 are explained in the next section. From this figure, one observes that the variability of  $H_f$ increases as N increases for  $N/P_c > 0.1$ . In other words,  $H_f$  is more sensitive to uncertainty of structural properties when the gravity load level is high. This is because  $H_f$  depends only on the bending moment capacity of the sections, and the variability of the bending moment capacity is high when the axial load level is high. This can be shown from (5.1) by taking the variance of M numerically.



Fig. 5.3 Statistics of  $H_f/2V_n$ ; (a) Mean and Mean $\pm 2$  standard deviation; (b) COV due to uncertainty in KC<sub>1</sub> (dashed line), in KC<sub>2</sub> (dotted line), or in both of them (solid line).

#### 5.2.2 Relative Significance of Columns

The sensitivity of  $H_f$  to the strength uncertainty of an individual column is estimated. When KC<sub>1</sub> is considered as the column with uncertain strength, KC<sub>2</sub> is considered as deterministic whose P-M curve is described as the mean P-M curve, and vice versa. The dashed line and the dotted line in Figure 5.3(b) show variability of  $H_f$  (in terms of COV) induced by uncertainty in the strength of KC<sub>1</sub> and KC<sub>2</sub>, respectively. The contributions of each column to  $H_f$  variability are different from one to another and they vary depending on the applied level of axial load. The contribution of KC<sub>2</sub> to  $H_f$  variability is larger than that of KC<sub>1</sub> when  $N/P_c > 0.1$ . In other words, KC<sub>2</sub> is more significant to  $H_f$  variability than KC<sub>1</sub> when  $N/P_c > 0.1$ . However, the difference in the relative significance of these two columns decreases as  $N/P_c$  increases and one can ignore the difference in the relative significance of the two columns for  $N/P_c > 0.7$ .

#### 5.3 DUCTILE RC FRAME: VE

The method of evaluating the propagation of uncertainty in the strength and deformation capacities of structural components to the structural system and identifying significant structural components with respect to EDP is demonstrated using a ductile RC frame, VE, as introduced in Section 4.5. The FOSM method is used to estimate the mean and standard deviation of an EDP due to the given uncertainty in the capacity of each typical structural component. Uncertainty in the capacity of the structural component is described by the probabilistic moment-curvature relationship or the probabilistic shear force-distortion relationship as discussed in Sections 4.5.4 and 4.5.5, respectively. COV is used as a measure of sensitivity of EDP to individual structural components. A structural component with larger corresponding COV of EDP is considered as more significant than that with smaller corresponding COV of EDP. The peak absolute floor acceleration (PFA), the peak absolute floor displacement (PFD), and the peak inter-story drift ratio (IDR) are selected as EDPs in this study because they are commonly used EDPs of structures in earthquake engineering.

# 5.3.1 Structural Modeling

The description of the VE frame is presented in Section 4.5.1 including material properties, dimensions, and reinforcement of beams and columns. The 2D computational model of the VE frame is developed using OpenSees as illustrated in Figure 5.4.

# 5.3.1.1 Element Type

All structural components in the VE frame are modeled by a plastic hinge model (referred to as beamWithHinges element in the OpenSees element library). Figure 5.5 illustrates the configuration of the beamWithHinges element. Each element has two nodes with two translations and one rotation per node. The beamWithHinges element consists of one elastic beam and two plastic hinge regions at both ends of the elastic beam as shown in Figure 5.5.



Fig. 5.4 The OpenSees model of the VE frame.

Each plastic hinge region has one monitoring section whose behavior is dictated by the assigned moment-curvature relationship and/or shear force-distortion relationship to capture a possible nonlinear behavior of the section and eventually a nonlinear behavior of the element. The monitoring section is located at the center of the plastic hinge region for performing the required numerical integration along the length of the plastic hinge region. The lengths of the two plastic hinge regions ( $h_i$  and  $h_j$  in Figure 5.5) are not necessarily identical. According to the evaluation of the components of the VE frame using the fiber element model (cf. Section 4.5.4.2), it is decided that 1/8 of the total element length is used as the plastic hinge length on each side of the element. They are 62% and 110% of the depth of the cross section (15.7 inches), respectively, for the column and the beam. It should be noted that the plastic hinge length is not considered as a random variable in this study because modeling uncertainty is not within the scope of this study. Certainly, this plastic hinge length is an important candidate to address modeling uncertainty in a future study.



Fig. 5.5 Plastic hinge model in OpenSees.

# 5.3.1.2 Mass and Viscous Damping

Mass is derived from the applied gravity loads for the pushover test and lumped at each node. Accordingly, 203 lb of mass is assigned to each node in each translational direction. The damping characteristics of the VE frame are modeled using mass and stiffness proportional damping with 5% of the critical damping for the first two models of vibration. The periods of these two modes estimated from the eigen solution using the initial elastic stiffness matrix are 0.55 and 0.15 seconds.

# 5.3.1.3 Solution Strategy

The Newmark  $\beta$ -method with typical coefficients  $\gamma = 0.50$  and  $\beta = 0.25$  is used as the time integrator. A half the ground motion time discretization is used for the analysis time step. Nonlinear equilibrium equations are solved using the modified Newton-Raphson solution algorithm.

# 5.3.2 Ground Motions

It is assumed that the VE frame is located at the same site as the UCS building, the case study building in Chapter 3. This assumption is made for convenience because the seismic hazard curve and a set of ground motion profiles are readily available. The seismic hazard curves for the fundamental period  $(T_1)$  of 0.3 and 0.5 seconds, provided by Frankel and Leyendecker (2001), are used to obtain the seismic hazard curve for  $T_1 = 0.55$  second for the VE frame. Figure 5.6 shows the seismic hazard curve for the VE frame in terms of the mean annual exceedance frequency of a specified spectral acceleration  $S_a$ . Assuming the Poisson occurrence of an earthquake and lognormal distribution of the random variable Sadenoting uncertainty in  $S_a$ , the mean and the standard deviation of Sa are estimated as 0.54g and 0.50g (COV=93%). The seismic hazard curve derived from the assumed distribution of Sa fits well with the seismic hazard curve given by Frankel and Leyendecker, as shown in Figure 5.6.



Fig. 5.6 Seismic hazard curve for the VE frame.

#### 5.3.3 Verification Analyses

In this section, the developed plastic hinge computational model of the VE frame is verified. For the first verification, another computational model of the VE frame is developed using the fiber element model and nominal material properties as given in Table 4.4. Pushover analyses with applied gravity loads of 157 kips (700 kN) at each column are performed by both the fiber model and the plastic hinge model. The relationships between the lateral load and the lateral displacement at Level 2 (Figure 5.4) obtained by the experiment, the fiber model, and the plastic hinge model are compared in Figure 5.7. The load-displacement relationship of the fiber model shows an excellent agreement with the experiment, while that of the plastic hinge model shows some disagreement with the experiment particularly in the displacement range of 1 to 2 inches. However, the estimations of the initial stiffness and the shear strength are considered accurate enough considering the simplicity of the element formulation of the plastic hinge model. Moreover, both the fiber model and the plastic hinge model successfully identify the experimentally identified failure mechanism that involves plastic hinges at both ends of the two beams and the two column bases.

The second verification is performed by the fiber model and the plastic hinge model



Fig. 5.7 Comparison of load-displacement relationships at Level 2 of the VE frame by the experiment (Vecchio and Emara 1992) and present analyses.

under a seismic loading. Figure 5.8 shows the time histories of the relative floor displacement at Level 2 with respect to the base due to the TO-ttrh02 earthquake scaled such that  $S_a =$ 0.54g for the VE frame by the fiber model (solid line) and by the plastic hinge model (dashed line). It is clear that the two time histories show a good overall agreement except for the prediction of the residual displacement. However, predictions of the peak displacement, which is of major interest, by the two models are very close as indicated by the circle in Figure 5.8 (2.42 inches and 2.52 inches by the plastic hinge model and the fiber model, respectively).



Fig. 5.8 Comparison of floor displacement time histories at Level 2 of the VE frame due to the TO-ttrh02 earthquake scaled to  $S_a = 0.54$ g.

#### 5.3.4 Convergence Test for FOSM Method

Since the finite difference approach is used in computing the gradient of EDP for the FOSM method, the convergence test with respect to the perturbation size is necessary to obtain a stable and reliable solution. According to Section 3.2.1, the perturbation size  $\Delta x_i$  is

expressed as  $\Delta x_i = a_p \sigma_i$  where  $\sigma_i$  is the standard deviation of the random variable and  $a_p$ is the coefficient of proportionality that is to be determined by the convergence test. In this study,  $a_p = 1.0, 0.1, 0.01$ , and 0.001 are considered in the convergence test where the convergence of COV of EDPs induced by individual random variables is tested. Among 20 ground motion profiles (cf. Table 3.6), TO-ttrh02 scaled to correspond to the mean IM level  $(S_a = 0.54g)$  is chosen for this convergence test.

Figure 5.9 shows COVs of PFA and PFD at Level 2 (referred to as PFA<sub>2</sub> and PFD<sub>2</sub>, respectively), and IDR at the second story (referred to as IDR<sub>2</sub>). It is noted that the data labeled as ET represent the uncertainty in EDPs induced by uncertainty in the capacity of all structural components. From Figure 5.9, it is observed that COVs of all EDPs due to Sa, Dp, and Ms converge when  $a_p \leq 0.1$ . On the other hand, COVs of all EDPs for the ET case start to diverge from  $a_p = 0.01$  and smaller. This divergence is attributed to a numerical error, e.g., round-off error, caused by using an unnecessarily small perturbation size with respect to the solution (the EDP gradient with respect to a random variable in this case) in the finite difference approach. Based on these observations, it is decided to use  $a_p = 0.1$  to compute the perturbation size for the subsequent analyses.



Fig. 5.9 Convergence of COV of various EDPs of the VE frame.

#### 5.3.5 Significant Components

In this section, only sensitivity of EDPs to the strength and deformation capacity of different structural components is studied. Each structural component is considered as a random variable and COVs of an EDP induced by each random variable are compared to identify the relative importance of each random variable or each structural component. The relative importance of a structural component is expressed as the ratio of its contribution due to EDP uncertainty to the contribution of all components due to EDP uncertainty. This investigation is conducted excluding the effects of the random variables Ms, Dp, and Sa. Mathematically, the contribution of *i*th structural component is  $\sigma_i^2/\sigma_T^2$  where  $\sigma_T^2 = \sum_{j=1}^n \sigma_j^2$  and *n* is the number of structural components that is specialized to six in this study. However, the effect of uncertainty in the ground motion profile is accounted for in identifying rankings of structural components using Monte Carlo simulation. Figure 5.10 shows the relative contributions of different structural components to uncertainty in  $PFA_2$  for 20 different earthquakes. In this figure, structural components are plotted in a descending order of relative significance to PFA<sub>2</sub>, similar to the tornado diagram. It is notable that rankings of structural components and their corresponding contributions are different for different earthquakes. For example, the beam on Level 2 (referred to as Element 6 in Figure 5.4) is the most significant component to  $PFA_2$  among all six components of the VE frame for 13 out of 20 earthquakes, while the beam on Level 1 (referred to as Element 3 in Figure 5.4) is the most significant component to  $PFA_2$  for 6 out of 20 earthquakes.

To take into account the effect of uncertainty in the ground motion profile, the contributions of each structural component to EDP uncertainty are compared in an average sense. Figure 5.11 shows *mean* relative contributions from the 20 earthquakes of different structural components to uncertainties in various EDPs. It is observed that the two columns in the second story (referred to as Elements 4 and 5 in Figure 5.4) have the least contribution to all EDPs. It is clear that Element 6 is the most significant structural component for PFA at Level 1 (referred to as PFA<sub>1</sub>) and PFA<sub>2</sub>. Element 3 is also significant to PFA<sub>2</sub>, while its



Fig. 5.10 Relative contributions of components of the VE frame to uncertainty in  $PFA_2$  for various earthquakes.

contribution to PFA<sub>1</sub> is not that significant. Contributions of structural components with respect to PFD at Level 1 (referred to as PFD<sub>1</sub>) and PFD<sub>2</sub> are similar even though their rankings are slightly different. Elements 3 and 6, and the two columns in the first story (referred to as Elements 1 and 2 in Figure 5.4) are almost equally significant to PFD<sub>1</sub> and PFD<sub>2</sub>. Elements 1, 6, 2, and 3 are almost equally significant to IDR at the first story (referred to as IDR<sub>1</sub>), while Elements 3, 2, and 1 are almost equally significant to IDR<sub>2</sub> where their contributions to IDR<sub>2</sub> is next to that of Element 6. Figure 5.11 suggests that structural



Relative contribution to EDP uncertainty

# Fig. 5.11 Mean relative contributions of components of the VE frame to EDPs uncertainty.

components that are involved in the failure mechanism (identified through the experiment) are more significant than the other components.

#### 5.3.6 Important Cross Sections

The contribution of a structural component is a combination of the contributions of the two cross sections of the component accounting for their correlation. Accordingly, it is important to investigate the relative significance of each cross section to different EDPs. Figure 5.12 shows the relative contributions of different cross sections to  $PFA_2$  uncertainty for the considered 20 earthquakes. In this figure, the section label Sij denotes the Section



Fig. 5.12 Relative contributions of cross sections of different components in the VE frame to uncertainty in  $PFA_2$  for various earthquakes.

j of component i (cf. Figure 5.4). It is notable that rankings of cross sections and their corresponding contributions to PFA<sub>2</sub> are different for different earthquakes.

To take into account the effect of uncertainty in the ground motion profile, the contributions of each cross section to EDP uncertainty are compared in an average sense. Figure 5.13 shows *mean* relative contributions from the 20 earthquakes of different cross sections to uncertainties in various EDPs. It is observed that the six least significant ones among the 12 cross sections to all EDPs are the four cross sections of Elements 4 and 5 (referred to as



Fig. 5.13 Mean relative contributions of cross sections of the VE frame to EDP uncertainty.

S41, S42, S51, and S52), and the upper cross sections of Elements 1 and 2 (referred to as S12 and S22). Moreover, the contributions of these 6 cross sections are distinctively smaller than those of the other 6 cross sections (S11, S21, S31, S32, S61, and S62) for all EDPs. This observation confirms the failure mechanism of the VE frame reported by Vecchio and Emara (1992).

It is noted that rankings of cross sections with respect to  $PFA_1$  are different from those with respect to  $PFA_2$ . On the other hand, rankings of cross sections with respect to  $PFD_1$ are identical to those with respect to  $PFD_2$ , and those with respect to  $IDR_1$  are identical to those with respect to  $IDR_2$ . In particular, S11 and S21 are the two most significant cross sections with respect to  $PFD_1$ ,  $PFD_2$ ,  $IDR_1$ , and  $IDR_2$  with distinctive contributions to these EDPs. It is interesting to note that  $PFD_2$  and  $IDR_2$  are most sensitive to S11 and S21 even though the locations of these cross sections are far from Level 2 or the second story. This is partly attributed to the fact that the relative floor displacement at Level 2 implicitly includes that at Level 1 and the relative floor displacement at Level 1 is obviously sensitive to sections S11 and S21.

#### 5.3.7 Conditional Sensitivity of EDPs Given IM

Similar to the conditional sensitivity study discussed in Section 3.4.8, the conditional sensitivity of EDPs of the VE frame to random variables given IM is investigated in this section. Selected IM levels in this section, bounded by the 10th and the 90th percentiles as shown in Figure 5.6, are listed in Table 5.2. All 20 ground motion profiles are used to estimate EDP sensitivity to each random variable at each IM level. As before, all results in this section are presented as semi-log plots due to the wide range of the considered  $S_a$  (0.14g to 1.10g).

Table 5.2Various percentiles of Sa for sensitivity of EDP of the VE frame given

IM.									
Percentile	10th	20th	30th	40th	50th	60th	70th	80th	90th
$S_a$ (g)	0.14	0.20	0.25	0.32	0.39	0.48	0.59	0.77	1.10

#### 5.3.7.1 Significant Components

The procedure of identifying significant components described in Section 5.3.5 is repeated, but at various IM levels. Figure 5.14 shows mean relative contributions from the 20 earthquakes of different structural components to uncertainties in various EDPs at various IM levels.

It is observed that the relative contribution of each structural component to an EDP varies depending on the IM level. The significance of Elements 4 and 5 is the least among all



Fig. 5.14 Mean contributions of components of the VE frame to uncertainty in various EDPs.

components to all EDPs at almost all IM levels. It is interesting to note that the significance of Element 6 to PFA<sub>1</sub> and PFA<sub>2</sub> increases from the least to the most as IM level increases. On the other hand, the significance of Element 6 to PFD<sub>1</sub>, PFD<sub>2</sub>, IDR<sub>1</sub>, and IDR<sub>2</sub> increases as IM level increases up to  $S_a = 0.5$ g, and then decreases as IM level increases thereafter. In general, contributions of components to PFA<sub>1</sub> and PFA<sub>2</sub> vary with IM level more than those to the other EDPs.

#### 5.3.7.2 Significant Cross Sections

The significance of each cross section to various EDPs at various IM levels is investigated in this section. Figures 5.15 to 5.20 show mean relative contributions from the 20 earthquakes of different cross sections to uncertainties in PFA, PFD, and IDR.

From Figure 5.15, it is observed that S11 and S21 are the most significant cross sections to PFA<sub>1</sub> when  $S_a \leq 0.39$ g. The significance of S61 and S62 increases as the IM level increases, while that of S31 and S32 stays at relatively low level as the IM level increases. From Figure 5.16, it is observed that S11 and S21 are the most significant cross sections to PFA<sub>2</sub> when  $S_a \leq 0.25$ g. Similar to PFA<sub>1</sub>, the significance of S61 and S62 increases as the IM level increases. Unlike PFA<sub>1</sub>, the significance of S31 and S32 to PFA<sub>2</sub> is not negligible and varies with the IM level.

Overall observations of Figures 5.17 and 5.18 are almost identical. For all IM levels, S11 and S21 are the most significant cross sections to PFD uncertainty, while S31 and S32 are the next significant ones, except for the case when  $S_a = 0.48$ g where S61 and S62 are equally significant as S31 and S32.

From Figures 5.19 and 5.20, it is observed that S11 and S21 are the most significant cross sections to IDR uncertainty for all IM levels. The significance of S31 and S32 to IDR uncertainty does not change appreciably as the IM level increases. Moreover, S31 and S32 are the second significant cross sections to IDR<sub>1</sub> for all IM levels. The significance of S61 and S62 to IDR uncertainty increases as the IM level increases up to  $S_a = 0.48$ g and then decreases thereafter.

#### 5.3.7.3 Discussions

Several conclusions can be drawn from observations related to the conditional sensitivity of EDPs to individual components or cross sections of the VE frame. These conclusions can be summarized as follows:



Fig. 5.15 Mean contributions of cross sections of the VE frame to  $PFA_1$  uncertainty.



Fig. 5.16 Mean contributions of cross sections of the VE frame to  $PFA_2$  uncertainty.



Fig. 5.17 Mean contributions of cross sections of the VE frame to  $PFD_1$  uncertainty.



Fig. 5.18 Mean contributions of cross sections of the VE frame to  $PFD_2$  uncertainty.



Fig. 5.19 Mean contributions of cross sections of the VE frame to  $IDR_1$  uncertainty.



Fig. 5.20 Mean contributions of cross sections of the VE frame to  $IDR_2$  uncertainty.

- The sensitivity of EDP to cross sections provides clearer trends than the sensitivity of EDP to structural components.
- For a wide range of IM levels, upper story columns are negligible in terms of their contributions to uncertainties of all EDPs.
- The significance of structural components to all EDPs varies depending on the IM level. In particular, contributions of the top level beam to all EDPs vary within the widest range of all structural components.
- At lower IM levels (i.e.,  $S_a \leq 0.2$ g), only first story columns and first floor beam are significant to all EDPs implying that the first yielding may occur in one or some of these structural components.
- The significance of the top level beam to all EDPs at low IM levels is negligible. This implies that such component only yields at high level of earthquake intensity.
- Both end-sections of the two beams and the bases of the first story columns are significant cross sections to all EDPs at all IM levels. This confirms the failure mechanism of the VE frame reported by Vecchio and Emara (1992).
- The bases of the first story columns are the most significant cross sections to all EDPs at all IM levels except for PFA at higher IM levels where the contributions of the two cross sections of the top level beam are also significant.

# 5.4 CONCLUDING REMARKS

The quantification of the propagation of basic uncertainty in structural components to the structural system with respect to its EDP is an important component of PBEE methodology. In this chapter, the propagation of uncertainty in the capacities of structural components to the structural system with respect to its EDP is investigated. First, an example of

a probabilistic evaluation of an RC portal frame using the probabilistic P-M interaction diagram is presented. It demonstrates an application of probabilistic component models to the evaluation of the structural system.

A systematic approach of identifying significant structural component or cross sections using the FOSM method is demonstrated by a ductile RC frame (referred to as VE). Sensitivity of EDPs (i.e., the peak absolute floor acceleration and displacement, and the peak inter-story drift ratio) to individual structural components is estimated using the FOSM method. Uncertainty in the strength and deformation capacities of the component is expressed as probabilistic moment-curvature and shear force-distortion relationships at critical cross sections of the component located at its ends. EDP uncertainty induced by each structural component is used to determine which components are most significant to the corresponding EDP. To consider the effect of uncertainty in the ground motion profile, a set of 20 ground motion records are selected and scaled according to specified IM levels.

The significance of an individual structural component or a cross section to EDP uncertainty is different for different earthquakes. Therefore, contributions of different structural components to EDP uncertainty are compared in an average sense to identify significant components. A conditional sensitivity of EDPs to uncertainty in the capacity of the structural component is estimated where the sensitivity study is performed at specified IM levels.

For the VE frame, the two beams and the two first-story columns are significant to EDPs at almost all IM levels. In particular, the beam at Level 1 and the two first-story columns are more significant than any other structural components to all EDPs at lower IM levels implying that the first yielding may occur in one or more of these three components. Both end-sections of the two beams and the first-story column bases are significant cross sections to all EDPs at all IM levels.

As an extension of the present study, it is recommended to consider uncertainties in the mass and the viscous damping at each element, not at the system level where only a single random variable for each of the two variables is considered for a whole structural system. In this way, more realistic evaluation of the significance of each structural component to EDP uncertainties can be achieved. Investigations on generic structural systems with another modes of failure such as non-ductile frame are also recommended for completeness.

# 6 Summary, Conclusions, and Future Extensions

# 6.1 SUMMARY

A systematic way of understanding propagation of uncertainties in ground motion and structural properties (referred to as *basic* uncertainties) to the structural system, and identifying significant sources of basic uncertainties and structural components with respect to seismic demand (referred to as engineering demand parameter, EDP) of a RC structural system is developed. A structural system is defined as an assembly of a number of typical structural components such as beams and columns. The developed procedure consists of three phases. The details of each of these phases are described in a separate chapter of this report where the method is demonstrated using case study structures. Procedures and observations of these case studies of each phase are summarized in this section.

#### 6.1.1 EDP Uncertainty Induced by Basic Uncertainty

The propagation of basic uncertainty to the structural system with respect to its EDPs due to possible future earthquakes is investigated using a case study RC shear-wall building (referred to as the UCS building). The peak absolute roof acceleration, peak absolute roof displacement, and maximum inter-story drift ratio are selected as global EDPs, while the peak curvatures at critical cross sections are selected as local EDPs. On the other hand, several random variables representing uncertainty in ground motion and structural properties are considered.

At first, sensitivity of global and local EDPs to the basic uncertainties is estimated by a FOSM method and a tornado diagram analysis at the mean and the median IM levels, respectively. The pros and cons of the FOSM method and the tornado diagram analysis are discussed and an approach of combining the two methods is suggested. Then, conditional sensitivity of EDPs to the basic uncertainties except for the random variable representing IM is estimated using the FOSM method at a wide range of IM levels. From the sensitivity measure of an EDP, relative significance of each basic uncertainty to the given EDP is identified.

#### 6.1.2 Uncertainty in the Capacity of Structural Components

The propagation of basic uncertainties to structural components with respect to their strength and deformation capacities is investigated. For this purpose, a computational tool for probabilistic evaluation of RC structural components is developed using stochastic fiber element formulation. This formulation accounts for the spatial variability of material and geometrical properties of RC members using the midpoint method within the framework of Monte Carlo simulation. Assumptions and formulations of the adopted fiber element model are described. The verification examples show the capability of the developed computer program to estimate the structural behaviors of several tested specimens.

A probabilistic strength analysis of a RC column, referred to as the KC column, subjected to combined axial load  $(P_a)$  and lateral load  $(P_l)$  is conducted in terms of the axial load-bending moment (P-M) interaction. For generating P-M interaction diagrams, different limit states in terms of the strain of the concrete at the extreme fiber  $(\epsilon_{c,ext})$  are considered. The sensitivity of the column strength to an individual random field is investigated for concrete compressive strength, steel yield strength, and concrete cover thickness. The importance of considering the spatial variability of a random variable is also investigated. The typical structural components of a ductile RC frame (referred to as VE) are identified and evaluated using the stochastic fiber element model and OpenSees software. In this way, probabilistic moment-curvature relationships at critical cross sections of the typical components are developed. Probabilistic shear force-distortion relationships at critical cross sections of the typical components are also developed using the FOSM method and Response 2000 software. These probabilistic section models of typical structural components of the VE frames are used in estimating EDP sensitivity of these frames in the third phase of the developed methodology using the FOSM method.

#### 6.1.3 EDP Uncertainty Induced by Component Uncertainty

The propagation of uncertainty in the strength and the deformation capacities of structural components to the structural system with respect to its EDP is investigated. In this case, the peak absolute horizontal acceleration and displacement at each floor, and the maximum inter-story drift ratio of each story are selected as EDPs. First, a probabilistic evaluation of a RC portal frame using the probabilistic P-M interaction diagram is presented. This example demonstrates an application of probabilistic component models to the evaluation of a simple structural system.

Sensitivity of EDPs to individual structural components and cross sections of the VE frames is estimated using the FOSM method where EDP uncertainty is used to determine which component is most significant to the corresponding EDP. Conditional sensitivity of EDPs at various IM levels is also estimated.

#### 6.2 CONCLUSIONS

Several conclusions can be drawn from the three chapters where the developed procedure of propagating basic uncertainties to the structural system is demonstrated. These conclusions can be summarized as follows:

- The FOSM method and the tornado diagram analysis are simple and efficient in identifying and ranking significant sources of basic uncertainties with respect to EDP of the structural system.
- A new approach where the tornado diagram analysis is used in conjunction with the FOSM method is suggested to better characterize EDP uncertainties induced by basic uncertainties. In this approach, the tornado diagram facilitates assuming the EDP distribution based on the swings and their skew, while the FOSM method is used to assess the EDP statistics.
- The intensity measure (IM) of the earthquakes is the dominant source of uncertainty to all global and local EDPs of the UCS building. Moreover, uncertainties in ground motion are more significant than those in structural properties to global EDPs.
- For most EDPs of the UCS building, uncertainty in the ground motion profile is more significant than those in structural properties at higher IM levels but less significant at lower IM levels.
- The developed stochastic fiber element model combining Monte Carlo simulation and fiber element model is robust and accurate in estimating the probabilistic distributions of the strength and deformation capacities of structural components.
- The strength uncertainty of the KC column varies depending on the limit state and the  $P_l/P_a$  ratio. Moreover, uncertainty of the compressive strength of concrete controls uncertainty of the column strength in the compression-failure region, while uncertainty of the steel strength controls that in the tension-failure region.
- Neglecting the spatial variability of concrete strength can lead to overestimating the strength uncertainty of a RC member.
- Uncertainties of curvature parameters at the critical cross section of structural components of the VE frame are larger than those of moment parameters because defor-

mation uncertainty is often larger than force uncertainty according to the stress-strain behavior. Moreover, normal distributions can be assumed for all parameters defining a multilinear moment-curvature relationship.

- The developed procedure of identifying and ranking significant structural components with respect to the EDP of the structural system using the FOSM method is simple, yet efficient and robust.
- For the VE frame, the level of significance of each of the structural components and cross sections to an EDP changes as the IM level does. Moreover, significant structural components and cross sections identified by the developed procedure at a given IM level indicate the failure mechanism of the structural system at the IM level.

# 6.3 FUTURE EXTENSIONS

During the course of the present study, several issues are determined to be worthy of future investigations. These issues are summarized in the following sections.

**Uncertainty** Several sources of basic uncertainties considered as significant to selected EDPs are selected and taken into account in the course of the present study. Among excluded sources of uncertainties are soil-foundation interface modeling, three-dimensional effect, and non-structural components. These excluded ones may affect uncertainties in the selected EDPs significantly. Moreover, the spatial distribution of structural properties may affect uncertainties in the EDPs of the structural system. Therefore, it is recommended to examine the effect of these aforementioned uncertainties and the spatial distribution of structural properties to EDPs.

In the procedure of identifying significant structural components to a given EDP, the dynamic properties of structural components, namely mass and viscous damping are not taken into account in this study. As an extension of the present study, it is recommended to consider uncertainties in the mass and the viscous damping at each element, not at the system level where only a single random variable for each of the two variables is considered for a whole structural system. In this way, more realistic evaluation of the significance of each structural component to EDP uncertainties can be achieved.

**Class of Structures** This study adopts several case study structural components and systems to demonstrate the procedure of propagating uncertainties and identifying significant sources of uncertainties to a given EDP in the structural analysis. Specially, a seven-story ductile shear-wall building and a two-story ductile frame are used as the main case study structural systems in this study. Consequently, conclusions drawn from the investigation of these structures should be evaluated for other classes of structures. It is recommended to examine other generic types of structures to confirm the generality of the conclusions discussed in Section 6.1.

**Structural Modeling** Most of the fiber element formulations including the stochastic fiber element model developed in this study and the formulations of nonlinear beam-column elements in OpenSees do not have the capability of describing the axial force-bending momentshear force interaction at the monitoring cross section of the beam-column element. It is recommended to develop and implement a beam-column element with such a capability in software such as OpenSees for a more accurate and realistic evaluation of structural components to eventually develop probabilistic component models. Moreover, modeling reinforcing bar buckling in the framework of the fiber element model is recommended for a possible inclusion in a stochastic setting, eventually.

Most of the lumped plasticity elements including beam with hinges element in OpenSees do not have the capability of describing the interaction between the axial force and the bending moment at the plastic hinge of the beam-column element. The formulation of a nonlinear beam-column element using a *bounding surface* plasticity model as in reference (El-Tawil and Deierlein 2001) can be adopted. A stress-resultant bounding surface plasticity model, analogous to yield surface plasticity model, describes the interaction between the axial force and the bending moment. This type of strength model defined at the section level of the element can be useful for more accurate evaluation of structures using the lumped plasticity elements.

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# **A** Derivations

### A.1 ELEMENT STIFFNESS

Consider the element displacement vector  $\mathbf{u}$ , element force vector  $\mathbf{p}$ , section deformations  $\epsilon(x)$  and  $\varphi(x)$ , and section force resultants N(x) and M(x) as specified in Section 4.2.1. Here  $\epsilon(x)$  is the axial strain considering the second order effect, such that  $\epsilon(x) = \epsilon_0(x) + \frac{1}{2}v(x)'^2 = u(x)' + \frac{1}{2}v(x)'^2$  where u(x) and v(x) are the axial and transverse displacements at x, respectively, and ' denotes a partial derivative with respect to the coordinate x. In the subsequent derivations, the argument x will be dropped for convenience. The principle of virtual work implies

$$\delta \mathbf{u}^T \mathbf{p} = \int_L \begin{bmatrix} \delta \epsilon & \delta \varphi \end{bmatrix} \begin{bmatrix} N \\ M \end{bmatrix} dx \tag{A.1}$$

where L is the element length. Since  $\delta \epsilon = \delta u' + v' \delta v'$ , (A.1) can be rewritten as

$$\delta \mathbf{u}^{T} \mathbf{p} = \int_{L} \begin{bmatrix} \delta u' + v' \delta v' & \delta \varphi \end{bmatrix} \begin{bmatrix} N \\ M \end{bmatrix} dx$$

$$= \int_{L} \begin{bmatrix} \delta u' & \delta \varphi \end{bmatrix} \begin{bmatrix} N \\ M \end{bmatrix} dx + \int_{L} v' \delta v' N dx$$
(A.2)

Now consider two interpolation functions

$$\mathbf{B}(x) = \begin{bmatrix} -\frac{1}{L} & 0 & 0 & \frac{1}{L} & 0 & 0\\ 0 & -6\frac{1}{L^2} + 12\frac{x}{L^3} & \left(-4\frac{1}{L^2} + 6\frac{x}{L^3}\right)L & 0 & 6\frac{1}{L^2} - 12\frac{x}{L^3} & \left(-2\frac{1}{L^2} + 6\frac{x}{L^3}\right)L \end{bmatrix}$$
(A.3)

and

$$\mathbf{C}(x) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -6\frac{x}{L^2} + 6\frac{x^2}{L^3} & \left(\frac{1}{L} - 4\frac{x}{L^2} + 3\frac{x^2}{L^3}\right)L & 0 & 6\frac{x}{L^2} - 6\frac{x^2}{L^3} & \left(-2\frac{x}{L^2} + 3\frac{x^2}{L^3}\right)L \end{bmatrix}$$
(A.4)

such that  $\begin{bmatrix} u'\\ \varphi \end{bmatrix} = \mathbf{B}\mathbf{u}$  and  $\begin{bmatrix} 0\\ v' \end{bmatrix} = \mathbf{C}\mathbf{u}$ . Substituting variational forms of these equations into (A.2) gives

$$\delta \mathbf{u}^T \mathbf{p} = \int_L \left( \mathbf{B} \delta \mathbf{u} \right)^T \begin{bmatrix} N \\ M \end{bmatrix} dx + \int_L \left( \mathbf{C} \delta \mathbf{u} \right)^T \mathbf{C} \mathbf{u} N dx$$
(A.5)

Then we get the weak form of equilibrium as

$$\mathbf{p} = \int_{L} \mathbf{B}^{T} \mathbf{q} dx + \int_{L} \mathbf{C}^{T} \mathbf{C} \mathbf{u} N dx$$
(A.6)

where  $\mathbf{q} = \mathbf{q}(x) = \begin{bmatrix} N & M \end{bmatrix}^T$ . To obtain the element stiffness matrix  $\mathbf{k}_{\mathbf{e}}$ , take the derivative of  $\mathbf{p}$  with respect to  $\mathbf{u}$  as

$$\mathbf{k}_{\mathbf{e}} = \frac{\partial \mathbf{p}}{\partial \mathbf{u}} = \int_{L} \mathbf{B}^{T} \frac{\partial \mathbf{q}}{\partial \mathbf{u}} dx + \int_{L} \mathbf{C}^{T} \mathbf{C} \mathbf{u} \frac{\partial N}{\partial \mathbf{u}} dx + \int_{L} \mathbf{C}^{T} \mathbf{C} N dx$$
(A.7)

From the section equilibrium,

$$\mathbf{q} = \mathbf{k}_{\mathbf{s}} \begin{bmatrix} \epsilon \\ \varphi \end{bmatrix} = \mathbf{k}_{\mathbf{s}} \begin{bmatrix} u' + \frac{1}{2}v'^2 \\ \varphi \end{bmatrix}$$

$$= \mathbf{k}_{\mathbf{s}} \left\{ \mathbf{B} + \frac{1}{2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} (\mathbf{C}\mathbf{u})^T \mathbf{C} \right\} \mathbf{u}$$
(A.8)

where  $\mathbf{k_s} = \mathbf{k_s}(x)$  is the section stiffness matrix. Therefore,

$$\frac{\partial \mathbf{q}}{\partial \mathbf{u}} = \mathbf{k}_{\mathbf{s}} \left\{ \mathbf{B} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} (\mathbf{C}\mathbf{u})^T \mathbf{C} \right\}$$

$$= \mathbf{k}_{\mathbf{s}} \left( \mathbf{B} + \mathbf{G} \right)$$
(A.9)

where 
$$\mathbf{G} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} (\mathbf{C}\mathbf{u})^T \mathbf{C}$$
. Moreover,

$$\frac{\partial N}{\partial \mathbf{u}} = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{k}_{\mathbf{s}} \left( \mathbf{B} + \mathbf{G} \right)$$
(A.10)

Consequently,

$$\begin{aligned} \mathbf{k}_{\mathbf{e}} &= \int_{L} \mathbf{B}^{T} \mathbf{k}_{\mathbf{s}} (\mathbf{B} + \mathbf{G}) dx + \int_{L} \mathbf{C}^{T} \mathbf{C} \mathbf{u} \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{k}_{\mathbf{s}} (\mathbf{B} + \mathbf{G}) dx + \int_{L} \mathbf{C}^{T} \mathbf{C} N dx \\ &= \int_{L} \mathbf{B}^{T} \mathbf{k}_{\mathbf{s}} (\mathbf{B} + \mathbf{G}) dx + \int_{L} \mathbf{G}^{T} \mathbf{k}_{\mathbf{s}} (\mathbf{B} + \mathbf{G}) dx + \int_{L} \mathbf{C}^{T} \mathbf{C} N dx \\ &= \int_{L} (\mathbf{B} + \mathbf{G})^{T} \mathbf{k}_{\mathbf{s}} (\mathbf{B} + \mathbf{G}) dx + \int_{L} \mathbf{C}^{T} \mathbf{C} N dx \\ &= \int_{L} \mathbf{T}^{T} \mathbf{k}_{\mathbf{s}} \mathbf{T} dx + \int_{L} \mathbf{C}^{T} \mathbf{C} N dx \end{aligned}$$
(A.11)

where  $\mathbf{T} = \mathbf{B} + \mathbf{G}$ .

#### A.2 VARIATIONS IN THE STRENGTH OF RC COLUMN

#### A.2.1 Model Assumptions

Consider a RC component with a rectangular cross section. As shown in Figure A.1, this cross section is discretized into  $n_b \times n_h$  concrete fibers and  $n_s$  steel fibers. The area of a steel fiber is  $a_s$  and that of a concrete fiber is  $a_c$ . Let  $n_c = n_b \times n_h$  be the total number of concrete fibers. It is assumed that all concrete fibers have the identical probabilistic constitutive model dictated by the distribution of the compressive strength  $f'_c$ . The correlation coefficient between the *i*th and the *j*th concrete fibers is  $\rho_{ij}$ , which is a function of the distance between the two fibers such that  $\rho_{ij}$  decreases as the distance increases. It is assumed that all steel fibers have the identical probabilistic constitutive model dictated by the distribution of the constitutive model dictated by the distribution of the steel fibers is  $\rho_{ij}$ , which is a function of the distance between the two fibers such that  $\rho_{ij}$  decreases as the distance increases. It is assumed that all steel fibers have the identical probabilistic constitutive model dictated by the distribution of the constitutive model dictated by the distribution of the steel fibers have the identical probabilistic constitutive model dictated by the distribution of the vield strength  $f_y$ . The correlation coefficient between the *i*th and the *j*th steel fibers is constant,  $\rho_s$ . It is assumed that  $f'_c$  and  $f_y$  are uncorrelated.

Let  $\sigma_c$  be the standard deviation of  $f'_c$  and  $\sigma_{ci}$  be that of  $f_{ci}$ , where  $f_{ci}$  is the stress in the *i*th concrete fiber at any strain level. Similarly, let  $\sigma_s$  be the standard deviation of  $f_y$ 



Fig. A.1 Discretization of the cross section.



Fig. A.2 Distributions of elastic constitutive models.

and  $\sigma_{si}$  be that of  $f_{si}$ , where  $f_{si}$  is the stress in the *i*th steel fiber at any strain level. In the elastic case of concrete and steel,  $\sigma_{c1} < \sigma_{c2} < \sigma_c$  and  $\sigma_{s1} < \sigma_{s2} < \sigma_s$  hold for  $f_{c1} < f_{c2} < f'_c$  and  $f_{s1} < f_{s2} < f_y$ , respectively, as shown in Figure A.2.

#### A.2.2 Variance of Axial Force

The axial force P at the cross section can be written as

$$P = \sum_{i=1}^{n_c} a_c f_{ci} + \sum_{i=1}^{n_s} a_s f_{si}$$
  
=  $a_c \sum_{i=1}^{n_c} f_{ci} + a_s \sum_{i=1}^{n_s} f_{si}$  (A.12)

Then the variance of the axial force is

$$\operatorname{Var}[P] = a_c^2 \operatorname{Var}\left[\sum_{i=1}^{n_c} f_{ci}\right] + a_s^2 \operatorname{Var}\left[\sum_{i=1}^{n_s} f_{si}\right]$$
(A.13)

It can be shown that

$$\operatorname{Var}\left[\sum_{i=1}^{n_{c}} f_{ci}\right] = \sum_{i=1}^{n_{c}} \sigma_{ci}^{2} + 2 \sum_{i=1}^{n_{c}-1} \sum_{j=i+1}^{n_{c}} \rho_{ij} \sigma_{ci} \sigma_{cj}$$
(A.14)

and similarly,

$$\operatorname{Var}\left[\sum_{i=1}^{n_{s}} f_{si}\right] = \sum_{i=1}^{n_{s}} \sigma_{si}^{2} + 2\rho_{s} \sum_{i=1}^{n_{s}-1} \sum_{j=i+1}^{n_{s}} \sigma_{si}\sigma_{sj}$$
(A.15)

Thus,

$$\operatorname{Var}[P] = a_c^2 \left[ \sum_{i=1}^{n_c} \sigma_{ci}^2 + 2 \sum_{i=1}^{n_c-1} \sum_{j=i+1}^{n_c} \rho_{ij} \sigma_{ci} \sigma_{cj} \right] + a_s^2 \left[ \sum_{i=1}^{n_s} \sigma_{si}^2 + 2\rho_s \sum_{i=1}^{n_s-1} \sum_{j=i+1}^{n_s} \sigma_{si} \sigma_{sj} \right]$$
(A.16)

From (A.16), it is obvious that the variance of the axial force increases as the limit state increases in the elastic case.

If  $\rho_{ij} = \rho_s = 1$ , the variance of the axial force becomes

$$\operatorname{Var}[P] = a_c^2 \left[ \sum_{i=1}^{n_c} \sigma_{ci}^2 + 2 \sum_{i=1}^{n_c-1} \sum_{j=i+1}^{n_c} \sigma_{ci} \sigma_{cj} \right] + a_s^2 \left[ \sum_{i=1}^{n_s} \sigma_{si}^2 + 2 \sum_{i=1}^{n_s-1} \sum_{j=i+1}^{n_s} \sigma_{si} \sigma_{sj} \right]$$
(A.17)

This is the maximum variance of the axial force since  $0 \le \rho_{ij}$ ,  $\rho_s \le 1$ . This is the case when the spatial variability of random variables is not considered. Consequently, the variance of the axial force with no spatial variability of random variables is always greater than that with spatial variability of random variables. If only the axial load is applied, all fibers have the same stress levels, in the cross section,

$$\sigma_{c1} = \sigma_{c2} = \ldots = \sigma_{cn_c} \quad \text{and} \quad \sigma_{s1} = \sigma_{s2} = \ldots = \sigma_{sn_s} \tag{A.18}$$

Accordingly, the variance of the axial force becomes

$$\operatorname{Var}[P] = a_c^2 \sigma_{c1}^2 \left[ n_c + 2 \sum_{i=1}^{n_c-1} \sum_{j=i+1}^{n_c} \rho_{ij} \right] + a_s^2 \sigma_{s1}^2 \left[ n_s + 2\rho_s \sum_{i=1}^{n_s-1} \sum_{j=i+1}^{n_s} 1 \right]$$
$$= a_c^2 \sigma_{c1}^2 \left[ n_c + 2 \sum_{i=1}^{n_c-1} \sum_{j=i+1}^{n_c} \rho_{ij} \right] + a_s^2 \sigma_{s1}^2 \left[ n_s + \rho_s n_s (n_s - 1) \right]$$
$$= a_c^2 \sigma_{c1}^2 \left[ n_c + 2 \sum_{i=1}^{n_c-1} \sum_{j=i+1}^{n_c} \rho_{ij} \right] + a_s^2 \sigma_{s1}^2 \left[ \rho_s n_s^2 + (1 - \rho_s) n_s \right]$$
(A.19)

If  $\rho_{ij} = \rho_s = 1$  under the pure axial load condition, the variance of the axial force becomes

$$\operatorname{Var}[P] = a_c^2 n_c^2 \sigma_{c1}^2 + a_s^2 n_s^2 \sigma_{s1}^2$$
(A.20)

## A.2.3 Variance of Bending Moment

The bending moment M at the cross section can be written as

$$M = \sum_{i=1}^{n_c} a_c f_{ci} y_{ci} + \sum_{i=1}^{n_s} a_s f_{si} y_{si}$$
  
=  $a_c \sum_{i=1}^{n_c} f_{ci} y_{ci} + a_s \sum_{i=1}^{n_s} f_{si} y_{si}$  (A.21)

It can be shown that

$$\operatorname{Var}\left[\sum_{i=1}^{n_{c}} f_{ci} y_{ci}\right] = \sum_{i=1}^{n_{c}} y_{ci}^{2} \sigma_{ci}^{2} + 2 \sum_{i=1}^{n_{c}-1} \sum_{j=i+1}^{n_{c}} y_{ci} y_{cj} \rho_{ij} \sigma_{ci} \sigma_{cj}$$
(A.22)

and similarly,

$$\operatorname{Var}\left[\sum_{i=1}^{n_s} f_{si} y_{si}\right] = \sum_{i=1}^{n_s} y_{si}^2 \sigma_{si}^2 + 2\rho_s \sum_{i=1}^{n_s-1} \sum_{j=i+1}^{n_s} y_{si} y_{sj} \sigma_{si} \sigma_{sj}$$
(A.23)

Thus,

$$\operatorname{Var}[M] = a_{c}^{2} \left[ \sum_{i=1}^{n_{c}} y_{ci}^{2} \sigma_{ci}^{2} + 2 \sum_{i=1}^{n_{c}-1} \sum_{j=i+1}^{n_{c}} y_{ci} y_{cj} \rho_{ij} \sigma_{ci} \sigma_{cj} \right] + a_{s}^{2} \left[ \sum_{i=1}^{n_{s}} y_{si}^{2} \sigma_{si}^{2} + 2\rho_{s} \sum_{i=1}^{n_{s}-1} \sum_{j=i+1}^{n_{s}} y_{si} y_{sj} \sigma_{si} \sigma_{sj} \right]$$
(A.24)

From (A.24), it is obvious that variance of the bending moment increases as the limit state increases in the elastic case.

If  $\rho_{ij} = \rho_s = 1$ , the variance of the axial force becomes

$$\operatorname{Var}[M] = a_c^2 \left[ \sum_{i=1}^{n_c} y_{ci}^2 \sigma_{ci}^2 + 2 \sum_{i=1}^{n_c} \sum_{j=i+1}^{n_c} y_{ci} y_{cj} \sigma_{ci} \sigma_{cj} \right] + a_s^2 \left[ \sum_{i=1}^{n_s} y_{si}^2 \sigma_{si}^2 + 2 \sum_{i=1}^{n_s-1} \sum_{j=i+1}^{n_s} y_{si} y_{sj} \sigma_{si} \sigma_{sj} \right]$$
(A.25)

This value is also the maximum variance of the bending moment. Consequently, the variance of the bending moment with no spatial variability of random variables is always greater than that with spatial variability of random variables.

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