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Stochastic Simulator-based Uncertainty Quantification for Seismic Responses of Bridges

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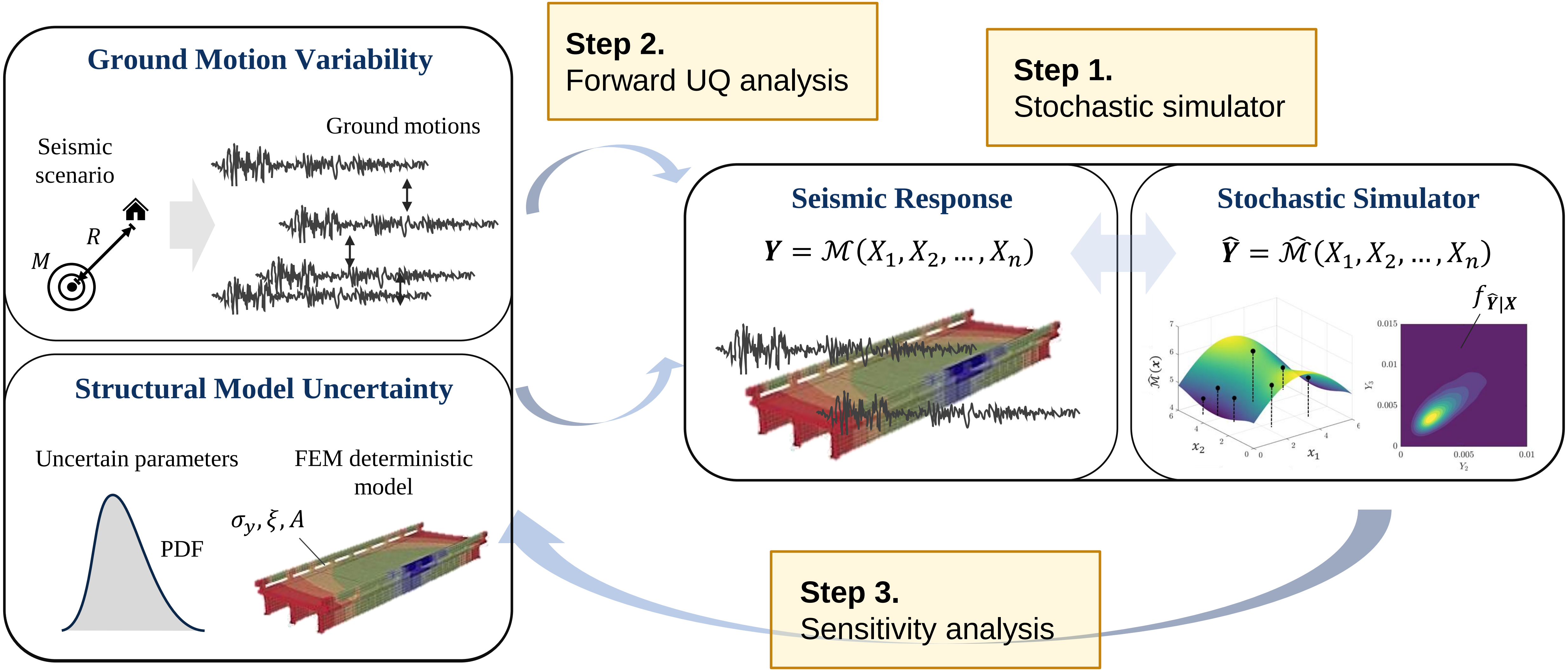
PEER funded project (#NCTRZW)

- **Title**: Stochastic simulator-based uncertainty quantification for seismic responses of bridges
- **PI**: Ziqi Wang (UC Berkeley), **Co-PI**: Marco Broccardo (University of Trento)

Research goals

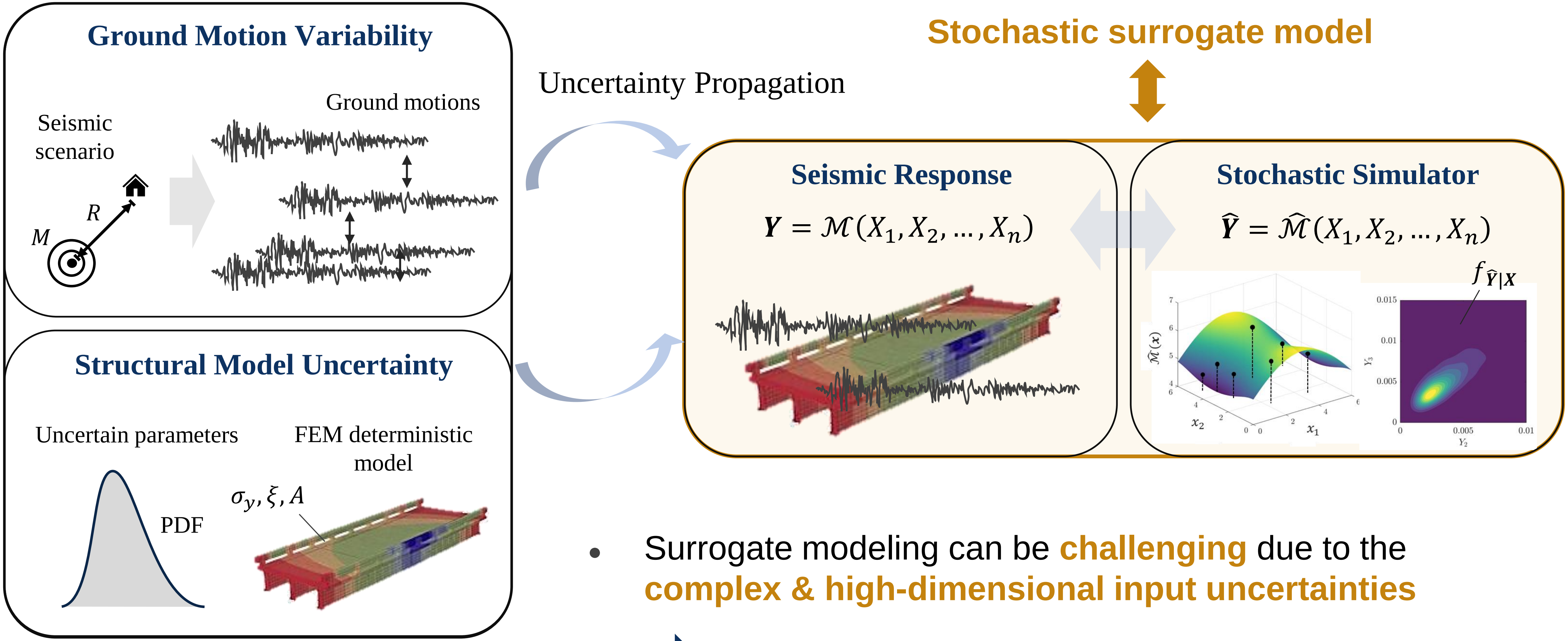
- The goal is to **develop** an efficient **stochastic simulator-based approach** for seismic UQ analysis of bridge responses
- Technical aims are:
  - Develop **stochastic surrogate models for the stochastic simulator**
  - Perform **UQ analysis for seismic responses** using stochastic surrogate models
  - Analyze **sensitivity indices** leveraging stochastic surrogate models

Research overview



1. Development of stochastic surrogate model

Challenges in surrogate modeling



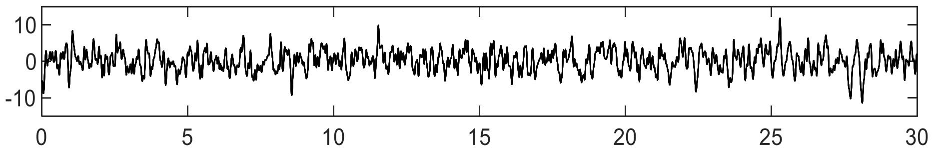
- Surrogate modeling can be **challenging** due to the **complex & high-dimensional input uncertainties**

➡ **Dimensionality reduction** can be useful

Dimensionality reduction-based stochastic surrogate model

- **Main idea**: Perform dimensionality reduction in the input-output space

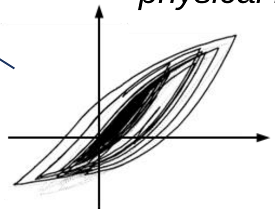
High-dimensional input $\boldsymbol{X}$



Physical model

$$\boldsymbol{Y} = \mathcal{M}(\boldsymbol{X})$$

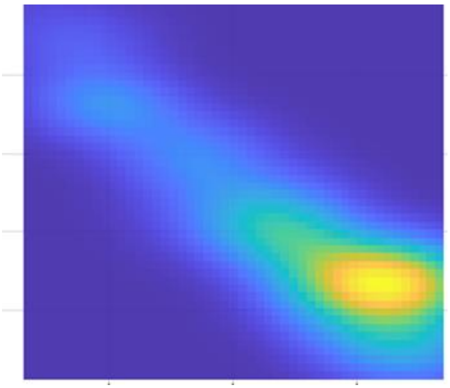
Constrained by physical laws



$$\mathbf{A}\ddot{\mathbf{Y}}(\boldsymbol{X},t) + \mathbf{R}\mathbf{Y}(\boldsymbol{X},t) = q$$

Low-dimensional representation via working with $\{(\boldsymbol{x},\boldsymbol{y}) \in \mathbb{R}^{n+m}: \boldsymbol{y} - \mathcal{M}(\boldsymbol{x}) = 0\}$

$$\mathcal{H}: \boldsymbol{z} \equiv (\boldsymbol{x},\boldsymbol{y}) \mapsto \boldsymbol{\psi}_z :$$



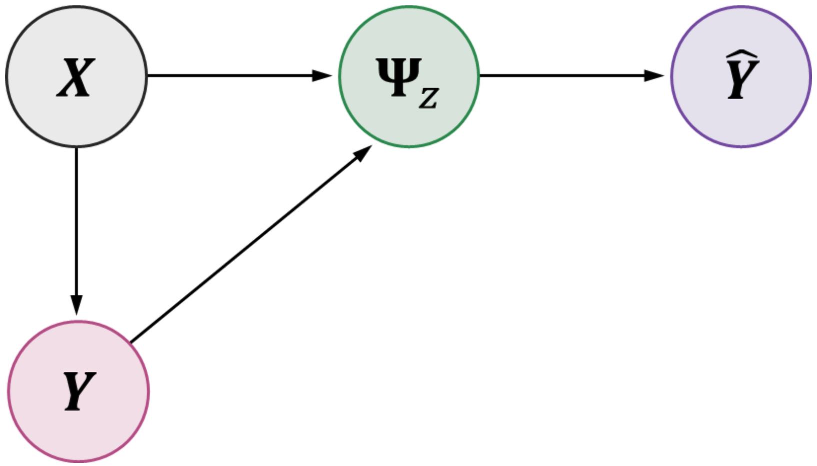
➡ **“Extract” stochastic surrogate model** from results of **dimensionality reduction**

Kim, J., Yi, S. R., & Wang, Z. (2024). Dimensionality reduction can be used as a surrogate model for high-dimensional forward uncertainty quantification. *arXiv preprint*:2402.04582.

Procedures of the proposed stochastic surrogate model

- Dimensionality reduction in the input-output space–construct $\mathcal{H}:\ \boldsymbol{z} \equiv (\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}^{n+m} \mapsto \boldsymbol{\psi}_z \in \mathbb{R}^d$
- Construct a conditional distribution $f_{\widehat{Y}|\boldsymbol{\Psi}_{\mathrm{z}}}(\widehat{\boldsymbol{y}}|\boldsymbol{\psi}_{\mathrm{z}})$ to predict $\boldsymbol{y}$ given $\boldsymbol{\psi}_{\mathrm{z}}$
- "Extract" a surrogate model $f_{\widehat{Y}|\boldsymbol{X}}(\widehat{\boldsymbol{y}}|\boldsymbol{x})$ from $\mathcal{H}$ and $f_{\widehat{Y}|\boldsymbol{\Psi}_{\mathrm{z}}}$

training stage



"True" surrogate model:

$$f_{\widehat{Y}|\boldsymbol{X}}(\widehat{\boldsymbol{y}}|\boldsymbol{x}) = \int\int f_{\widehat{Y}|\boldsymbol{\Psi}_{\mathrm{z}}}(\widehat{\boldsymbol{y}}|\boldsymbol{\psi}_{\mathrm{z}}) f_{\boldsymbol{\Psi}_{\mathrm{z}}|\boldsymbol{XY}}(\boldsymbol{\psi}_{\mathrm{z}}|\boldsymbol{x},\boldsymbol{y}) f_{\boldsymbol{Y}|\boldsymbol{X}}(\boldsymbol{y}|\boldsymbol{x})\, d\boldsymbol{\psi}_{\mathrm{z}} d\boldsymbol{y}$$

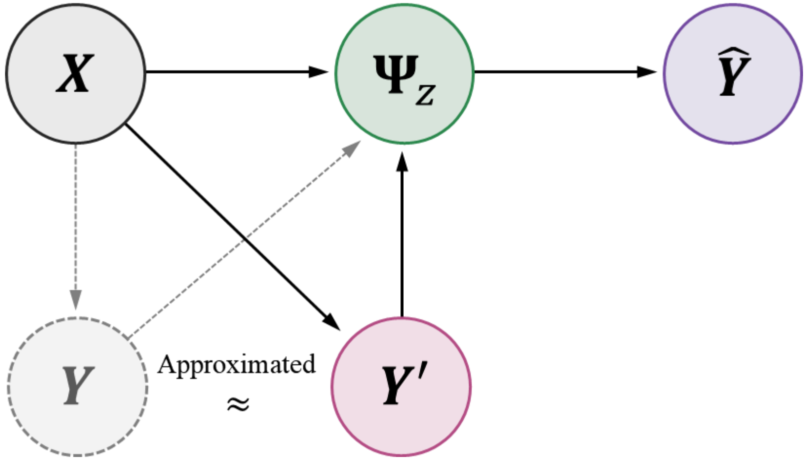
Condi. distribution *Dimensionality reduction* *Original model*

Kim, J., Yi, S. R., & Wang, Z. (2024). Dimensionality reduction can be used as a surrogate model for high-dimensional forward uncertainty quantification. *arXiv preprint*:2402.04582.

Procedures of the proposed stochastic surrogate model

- Dimensionality reduction in the input-output space–construct $\mathcal{H}$: $\boldsymbol{z} \equiv (\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}^{n+m} \mapsto \boldsymbol{\psi}_z \in \mathbb{R}^d$
- Construct a conditional distribution $f_{\widehat{Y}|\boldsymbol{\Psi}_{\mathrm{z}}}(\widehat{\boldsymbol{y}}|\boldsymbol{\psi}_{\mathrm{z}})$ to predict $\boldsymbol{y}$ given $\boldsymbol{\psi}_{\mathrm{z}}$
- "Extract" a surrogate model $f_{\widehat{Y}|X}(\widehat{\boldsymbol{y}}|\boldsymbol{x})$ from $\mathcal{H}$ and $f_{\widehat{Y}|\boldsymbol{\Psi}_{\mathrm{z}}}$

prediction stage



$\rightarrow \ \widehat{\boldsymbol{y}}' \sim f_{\widehat{Y}|X}(\widehat{\boldsymbol{y}}|\boldsymbol{x})$

"Stationary" surrogate model:

$$f^{(\infty)}_{\widehat{Y}|X}(\widehat{\boldsymbol{y}}|\boldsymbol{x}) = \int\int f_{\widehat{Y}|\boldsymbol{\Psi}_{\mathrm{z}}}(\widehat{\boldsymbol{y}}|\boldsymbol{\psi}_{\mathrm{z}}) f_{\boldsymbol{\Psi}_{\mathrm{z}}|XY}(\boldsymbol{\psi}_{\mathrm{z}}|\boldsymbol{x}, \boldsymbol{y}') f^{(\infty)}_{\widehat{Y}|X}(\boldsymbol{y}'|\boldsymbol{x})\, d\boldsymbol{\psi}_{\mathrm{z}} d\boldsymbol{y}'$$

"approximate" surrogate

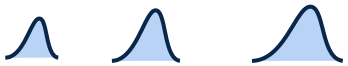
Transition kernel:

$$T\left(\widehat{\boldsymbol{y}}^{(t)}, \widehat{\boldsymbol{y}}^{(t+1)}|\boldsymbol{x}\right) = f_{\widehat{Y}|\boldsymbol{\Psi}_{\mathrm{z}}}\left(\widehat{\boldsymbol{y}}^{(t+1)}\middle|\boldsymbol{\psi}_{\mathrm{z}}\right) f_{\boldsymbol{\Psi}_{\mathrm{z}}|XY}\left(\boldsymbol{\psi}_{\mathrm{z}}\middle|\boldsymbol{x}, \widehat{\boldsymbol{y}}^{(t)}\right)$$

→ Outputs: Stochastic surrogate model, $\widehat{\boldsymbol{y}}^{(t)} \sim f_{\widehat{Y}|X}(\widehat{\boldsymbol{y}}|\boldsymbol{x})$

Kim, J., Yi, S. R., & Wang, Z. (2024). Dimensionality reduction can be used as a surrogate model for high-dimensional forward uncertainty quantification. *arXiv preprint*:2402.04582.

Summary of the proposed stochastic surrogate model



$$\boldsymbol{X} = [\boldsymbol{X}_1, \boldsymbol{X}_2 \dots, \boldsymbol{X}_n] \xrightarrow{\mathcal{M}(\boldsymbol{X})} \boldsymbol{Y} = [Y_1, Y_2, \dots, Y_m]$$

where $n \gg 100$

- We **"extract" a surrogate model** from the results of dimensionality reduction
  - Surrogate model for high-dimensional system
- **Stochastic simulator**: Output predictions are probabilistic distributions
- **Multi-output predictor**: We can quantify interdependencies between multiple outputs

Kim, J., Yi, S. R., & Wang, Z. (2024). Dimensionality reduction can be used as a surrogate model for high-dimensional forward uncertainty quantification. *arXiv preprint*:2402.04582.

2. Uncertainty quantification of seismic response

Sources of uncertainty

Ground motion acceleration time-history $X_{\mathbf{GM}}$

{Synthetic motions by SGMM
or Recorded motions from database}

$$\boldsymbol{X}_{\mathrm{GM}} = \{a_g(t), t = 0, \Delta t, \dots, n\Delta t\}$$

$$n \geq 1{,}000$$

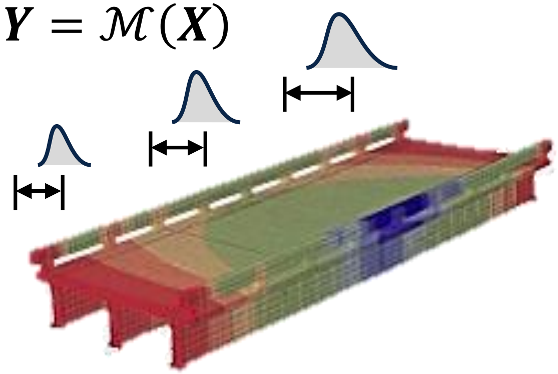
$$\boldsymbol{X} = [\boldsymbol{X}_{\mathbf{GM}}, \boldsymbol{X}_{\mathbf{s}}]$$

Nonlinear response history analysis

Seismic responses Y

{Pier drift ratio, Girder bending moment,…}

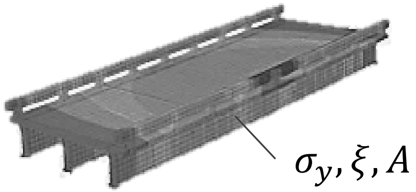
$$\boldsymbol{Y} = \mathcal{M}(\boldsymbol{X})$$



Structural parameters X_s

{Material properties, Damping, …}

$$\boldsymbol{X}_{\mathrm{S}} = \{\sigma_i, A_i, \xi, \dots\}$$



Stochastic simulator

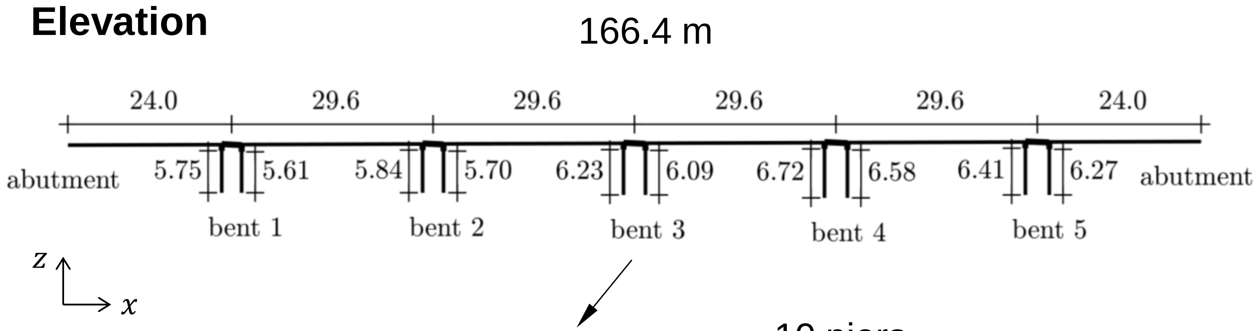
$$Y = \widehat{\mathcal{M}}_s(\boldsymbol{X}_{\mathbf{GM}}, \boldsymbol{X}_{\mathbf{s}})$$

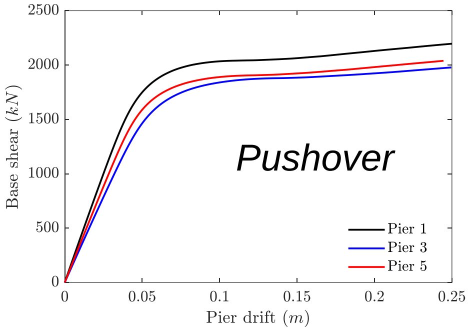
Randomly paired (structural parameter + ground motion) input

Structural model and uncertain parameters

- **Auburn Ravine bridge**: a PSC girder bridge featuring 6 spans and 2 piers per bent (Konaki & ADK, 2011)

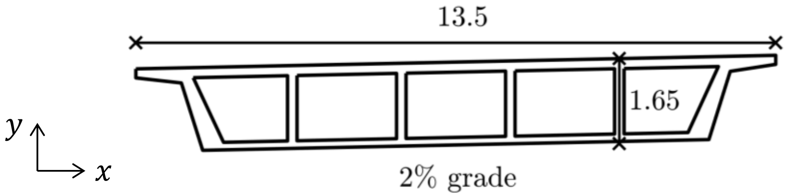
Elevation





- 10 piers
- 25 girder elements

Box-girder cross section



Structural uncertain parameters

$$\boldsymbol{X}_s = \{\text{Damping, Material prop., Geo.}\} \in \mathbb{R}^{12}$$

Modeling parameter	P_1	P_2	Distribution
Rayleigh damping ratio, ξ	0.05	0.35	Lognormal
Cross-sectional area of the girder, A_b (m^2)	4.50	8.50	Uniform
Elastic modulus of the girder, E_b (GPa)	28.3	0.35	Lognormal
Elastic modulus of pier reinforcing steel, E_s (GPa)	200	0.35	Lognormal
Yield strength of pier reinforcing steel, f_s^y (MPa)	475	0.35	Lognormal
Ultimate strength of pier reinforcing steel, f_s^u (MPa)	655	0.35	Lognormal
Onset of strain hardening of pier reinforcing steel, ε_s	0.0115	0.30	Lognormal
Elastic modulus of pier concrete, E_c (GPa)	27.6	0.30	Lognormal
Compressive strength of pier concrete, f_c (MPa)	34.5	0.30	Lognormal
Strain at compressive strength of pier concrete, ε_0	0.002	0.25	Lognormal
Diameter of pier column, D_{col} (m)	0.90	1.90	Uniform
Thickness of concrete cover, c_v (m)	0.02	0.08	Uniform

Bridge EDPs

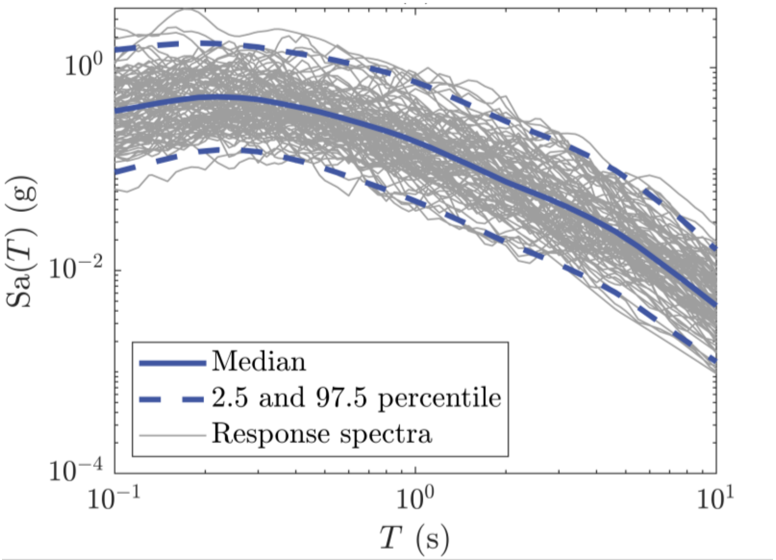
$$\boldsymbol{Y} = [\text{Pier drift ratios}] \in \mathbb{R}^{10}$$
$$\boldsymbol{Y} = [\text{Girder bending moments}] \in \mathbb{R}^{25}$$

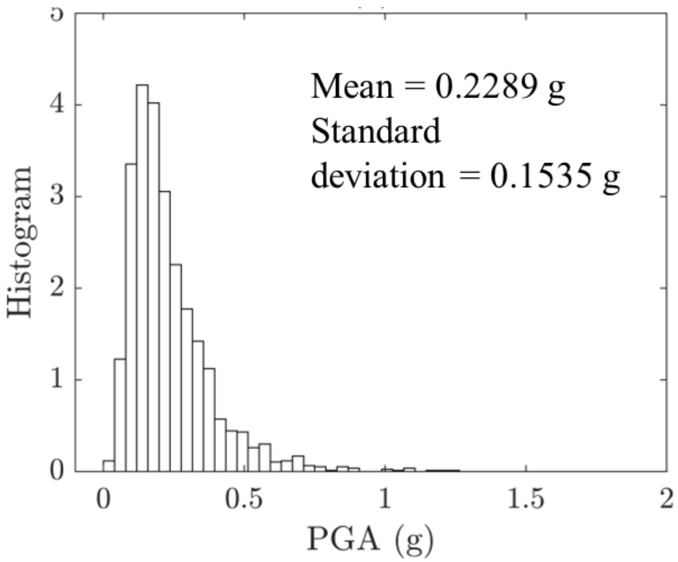
Konakli, K., & Der Kiureghian, A. (2011). Stochastic Dynamic Analysis of Bridges Subjected to Spatially Varying Ground Motions, *PEER Report* 2011/105.

Ground motions

(Baker & Lee, 2018)

- **2,000 ground motions** are selected from PEER NGA-West2 database
- The target spectrum is derived from a GMM (Boore et al., 2014) and spectral correlation model (Baker & Jayaram, 2008)





Seismic hazard parameters

Parameter	Value
M	6.5
R_{rup}	10 (km)
$V_{s,30}$	450 (m/s)
Fault	Normal
Region	California

Input: structural parameter + ground motion

$$\boldsymbol{X} = [\boldsymbol{X}_{GM}, \boldsymbol{X}_S] \in \mathbb{R}^{\geq 1,000}$$

Output: bridge EDP

$$\boldsymbol{Y} = [\text{Pier drift ratios}] \in \mathbb{R}^{10}$$
$$\boldsymbol{Y} = [\text{Girder bending moments}] \in \mathbb{R}^{25}$$

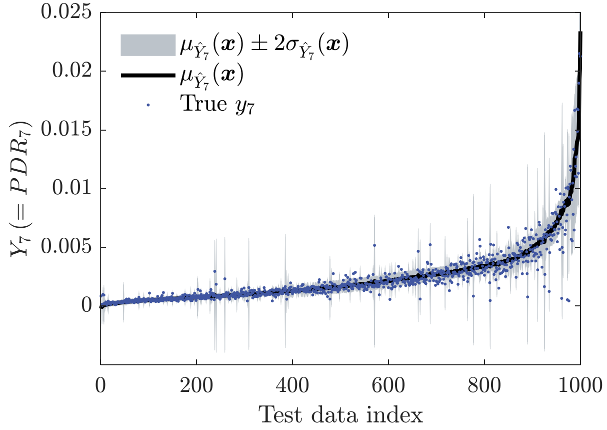
Baker, J.W. and Lee, C., 2018. An improved algorithm for selecting ground motions to match a conditional spectrum. *Journal of Earthquake Engineering*, 22(4), pp.708-723.
Baker, J. W., & Jayaram, N. (2008). Correlation of spectral acceleration values from NGA ground motion models. *Earthquake Spectra*, 24(1), 299-317.
Boore, D. M., Stewart, J. P., Seyhan, E., & Atkinson, G. M. (2014). NGA-West2 equations for predicting PGA, PGV, and 5% damped PSA for shallow crustal earthquakes. *Earthquake Spectra*, 30(3), 1057-1085.

Surrogate modeling of seismic response

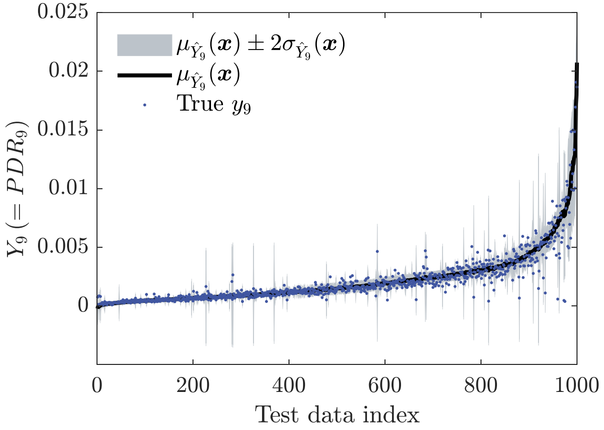
- Inputs: $\mathbf{X} = [\boldsymbol{X}_{GM}, \boldsymbol{X}_S]$
- Outputs: $\boldsymbol{Y} = [\mathrm{PDR}_1, \dots, \mathrm{PDR}_{10}]$

- PDR prediction by stochastic simulator ($N_{Train} = 600$, $N_{Test} = 1400$)

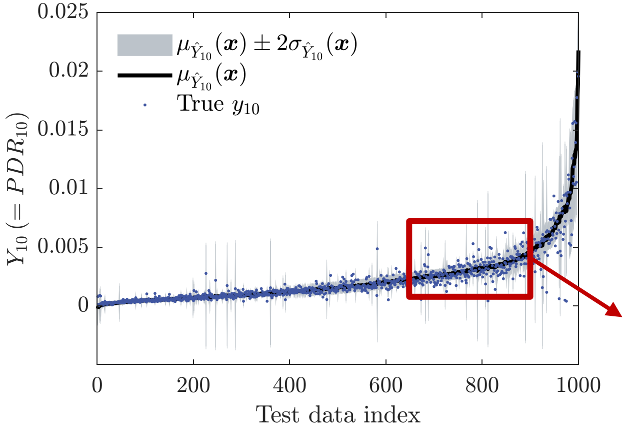
Pier 1 (RMSE=0.168)

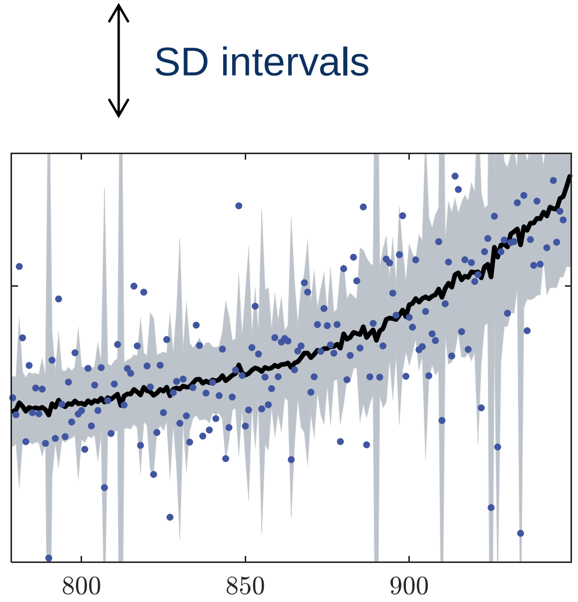


Pier 3 (RMSE=0.167)



Pier 5 (RMSE=0.168)





- Mean peaks of PDRs

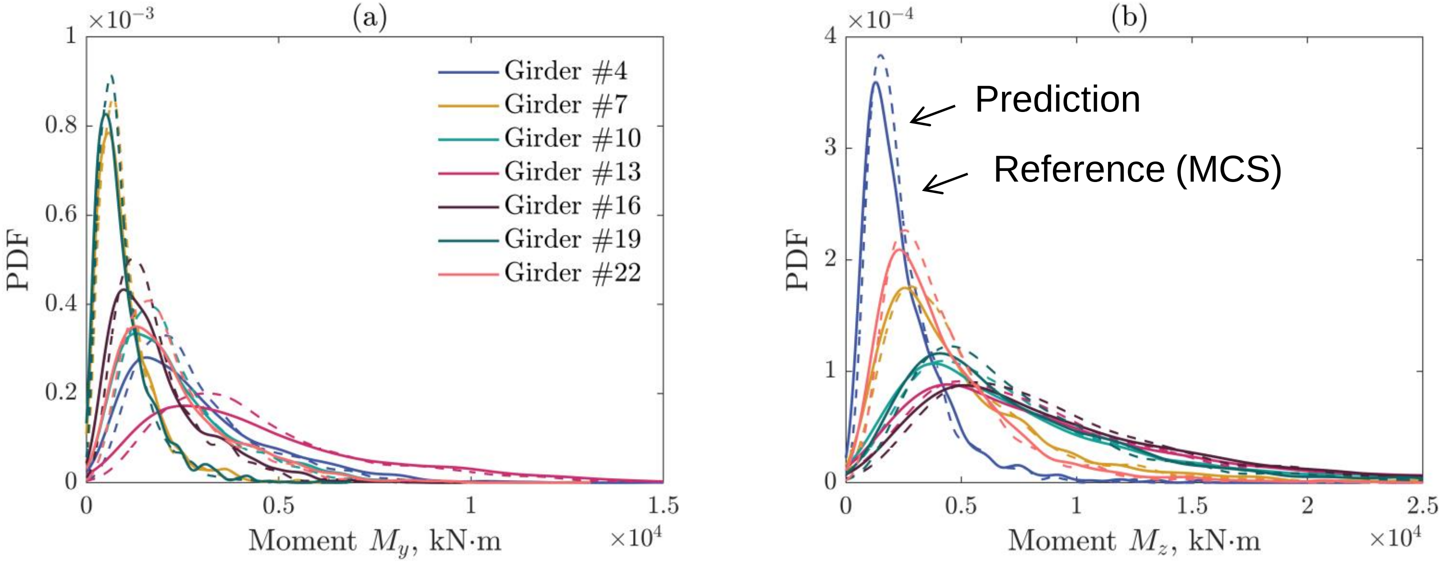
Pier location	NLRHA with OpenSees ($\times 10^{-3}$) Left	Right	Stochastic simulator ($\times 10^{-3}$) Left	Right	Error (%) Left	Right
Bent 1	3.96	3.76	3.87	3.68	2.23	2.20
Bent 2	3.90	3.76	3.81	3.67	2.22	2.21
Bent 3	3.63	3.57	3.55	3.50	2.22	2.21
Bent 4	3.34	3.37	3.27	3.30	2.20	2.22
Bent 5	3.42	3.53	3.35	3.46	2.19	2.23

Error < 3%

UQ for seismic response

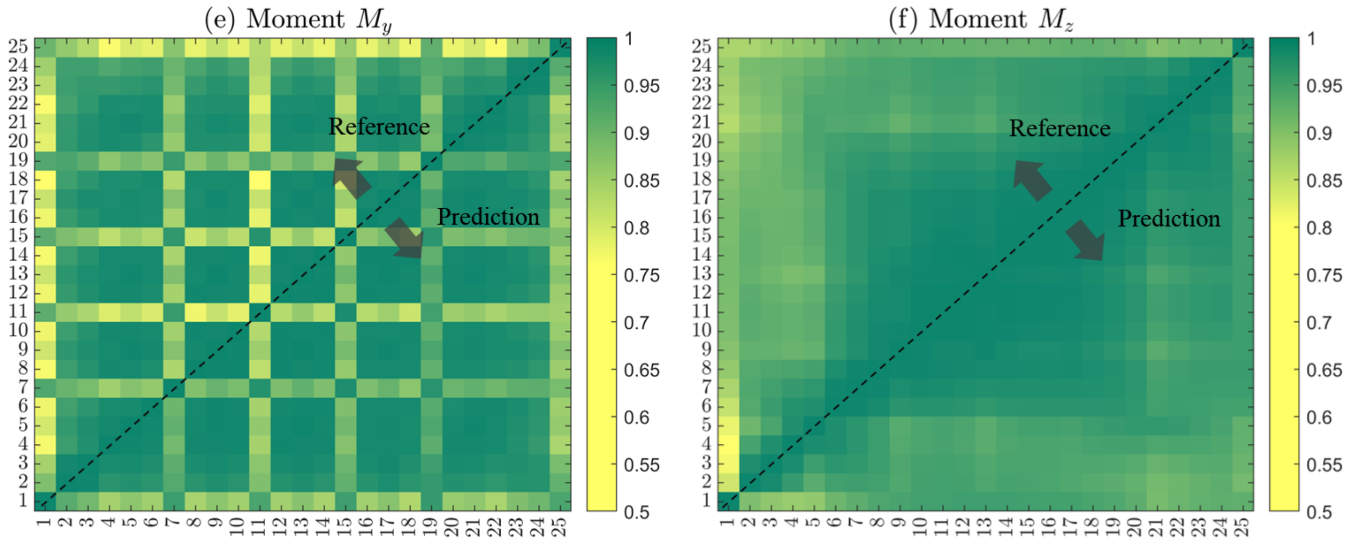
- Inputs: $\mathbf{X} = [\boldsymbol{X}_{GM}, \boldsymbol{X}_S]$
- Outputs: $\boldsymbol{Y} = [\mathrm{GM}_1, \ldots, \mathrm{GM}_{25}]$

- Girder moment prediction by stochastic simulator ($N_{Train} = 600$, $N_{Test} = 1400$)



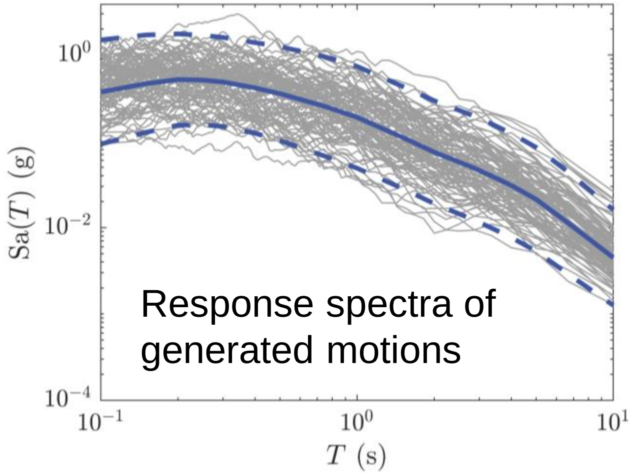
- Marginal response PDFs

- Correlation matrix

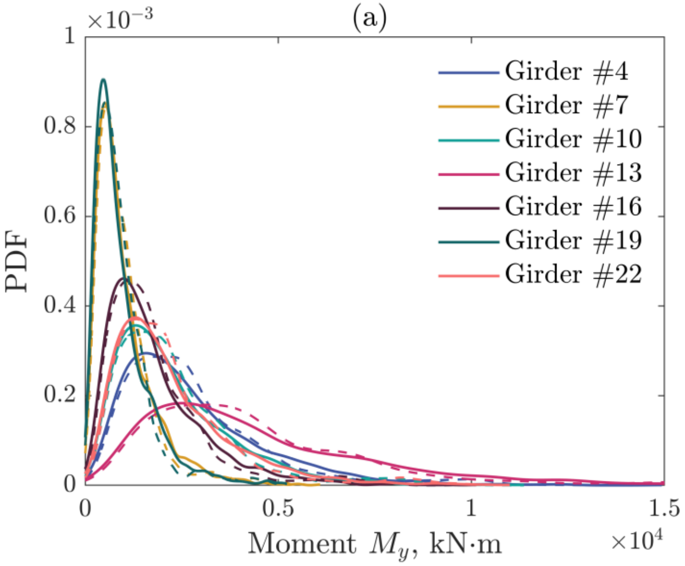


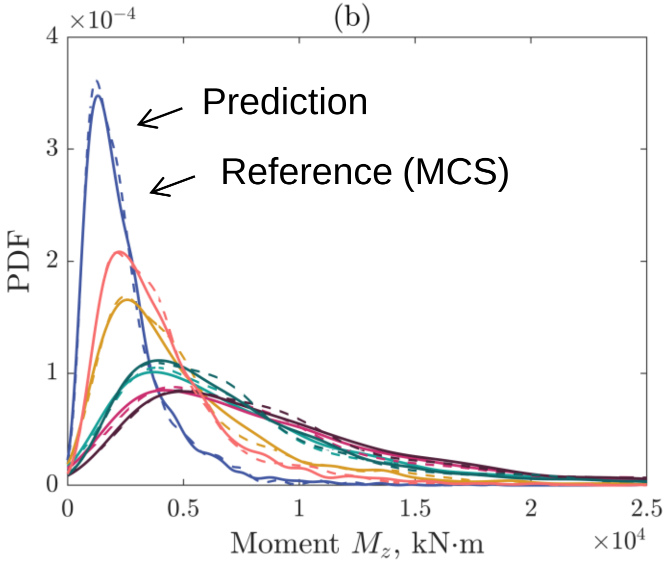
Application to synthetic motions

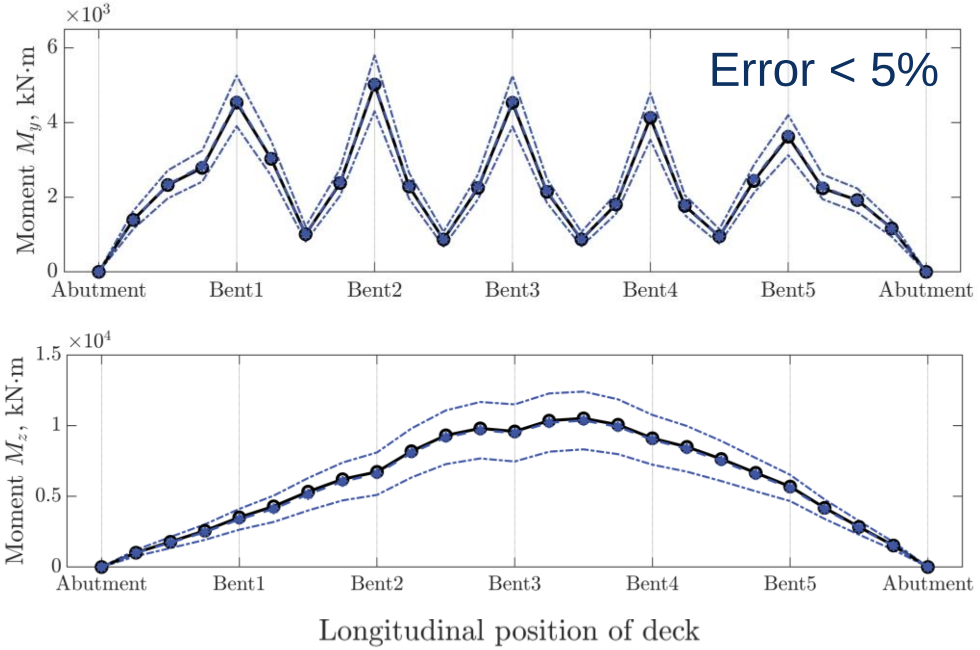
- **2,000 spectrum-compatible synthetic motions** are generated by the ground motion generation algorithm (Yanni et al., 2024)



- Seismic UQ for girder moments under synthetic motions:





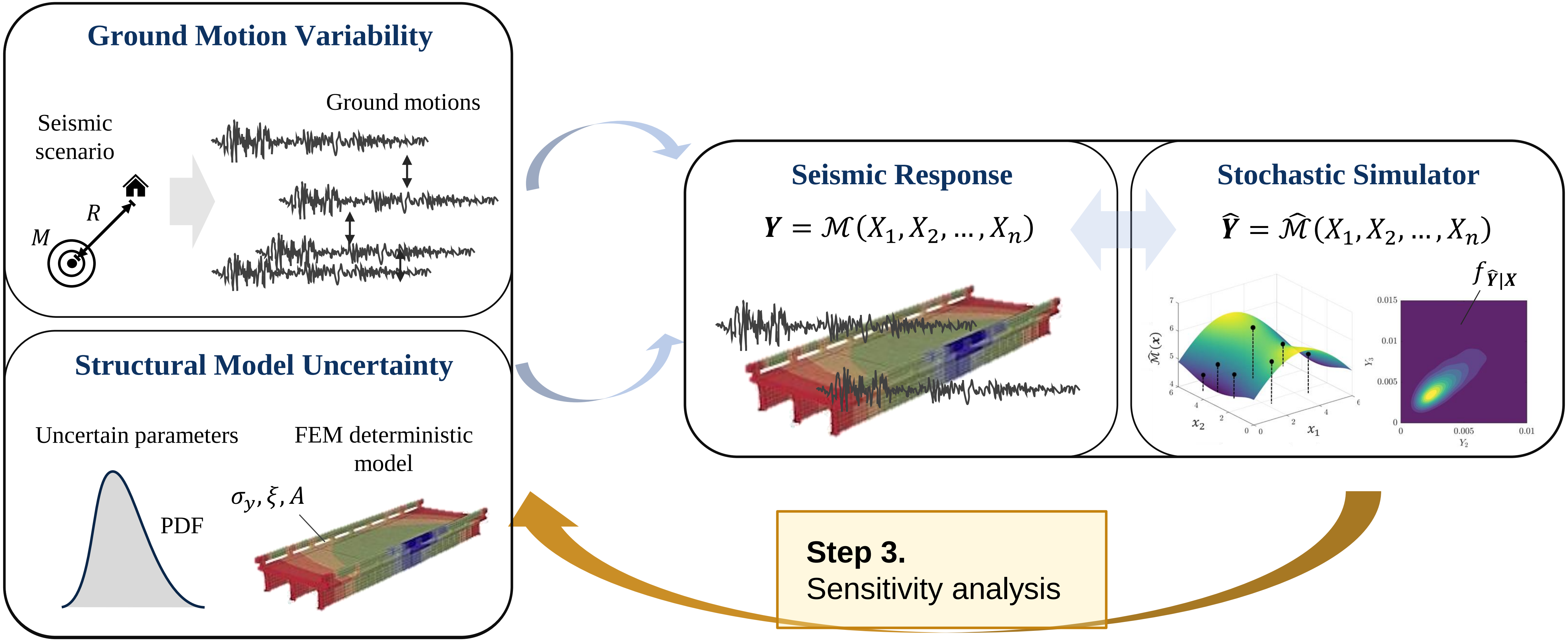


The proposed method is effective to both synthetic and recorded motions

Yanni, H., Fragiadakis, M., & Mitseas, I. P. (2024). Probabilistic generation of hazard-consistent suites of fully non-stationary seismic records. *EESD*, 53(10), 3140-3164.

3. Sensitivity analysis of seismic response

Global sensitivity analysis of seismic response

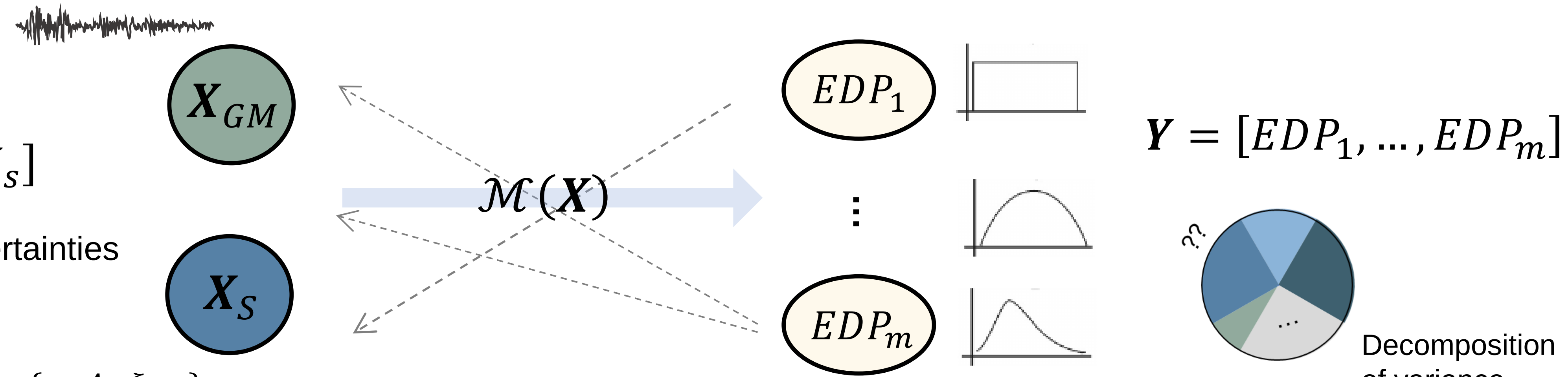


Variance-based sensitivity analysis

- Sensitivity indices for **each EDP** with respect to **each group of input uncertainties**

$$\boldsymbol{X} = [\boldsymbol{X}_{GM}, \boldsymbol{X}_s]$$

Two "**groups**" of uncertainties



$$\boldsymbol{X}_S = \{\sigma_i, A_i, \xi, \dots\}$$

- "**Grouped**" sensitivity index: $\rightarrow \; S_{\mathbf{u}}^{k} = \dfrac{\mathbb{V}\mathrm{ar}_{\boldsymbol{X}_{\mathbf{u}}}\left[\mathbb{E}_{\boldsymbol{X}_{\sim\mathbf{u}}}[EDP_k | \boldsymbol{X}_{\mathbf{u}}]\right]}{\mathbb{V}\mathrm{ar}[EDP_k]}$

- Inevitably *high-dimensional* integral (due to $\boldsymbol{X}_{GM}$)
- *High computational complexity* $(Complexity = d \times N^2)$

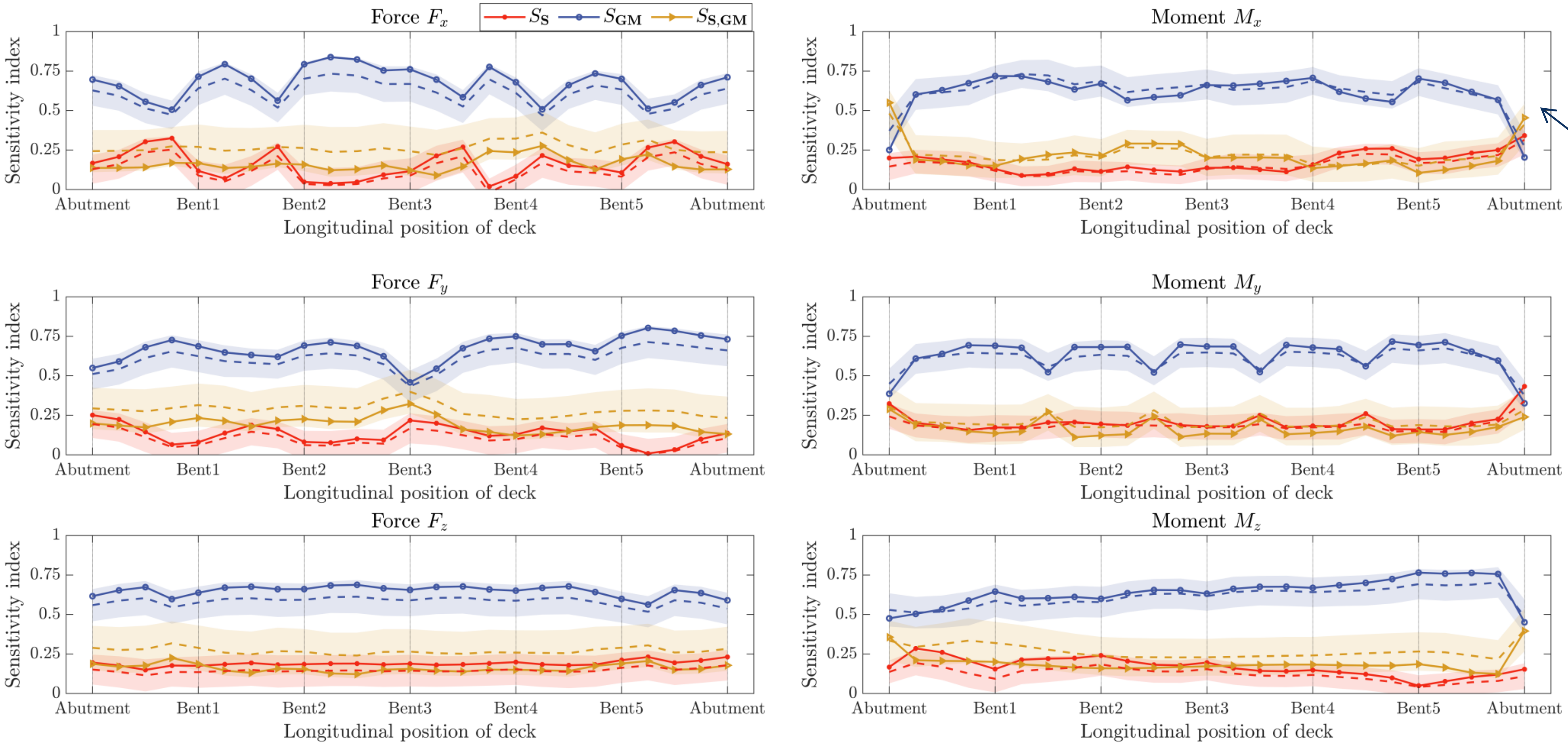
Stochastic simulator
combined with MC sampling

Sensitivity indices of seismic response

- Inputs: $\mathbf{X} = [\boldsymbol{X}_{GM}, \boldsymbol{X}_S]$
- Outputs: $\boldsymbol{Y} = [\mathrm{GM}_1, \dots, \mathrm{GM}_{25}]$

- Sensitivity indices of girder responses w.r.t ground motion and structural parameter uncertainties
- Grouped indices ($S_{\mathbf{GM}}, S_{\mathbf{S}}$) and their interaction effect ($S_{\mathbf{S,GM}} = 1 - S_{\mathbf{GM}} - S_{\mathbf{S}}$)

Prediction
Reference



95% CI on sensitivity indices

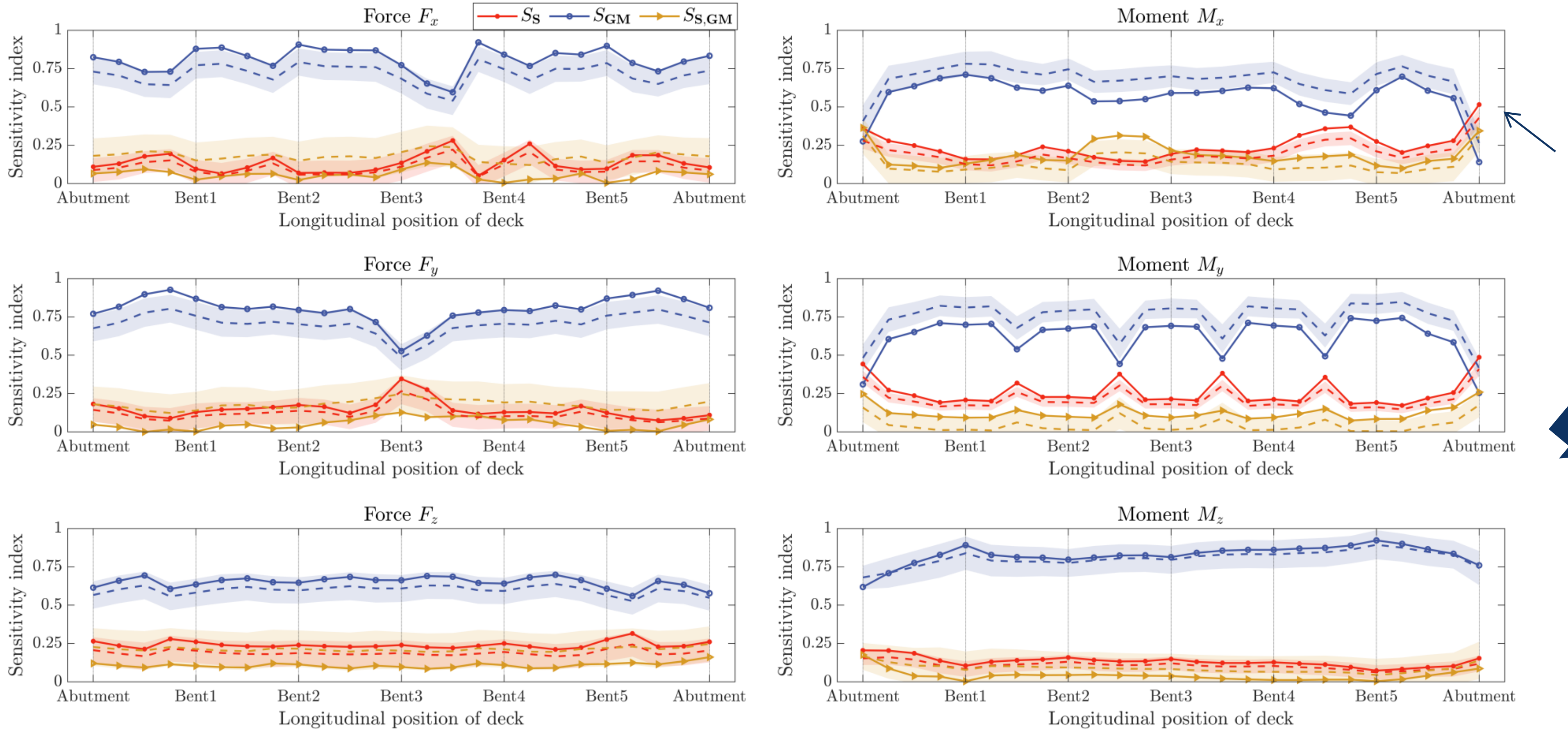
- Error < 10%

- $S_{\mathbf{GM}} \geq 60\%$
- $S_{\mathbf{S}} \leq 20\%$

Application to synthetic motions

- Inputs: $\mathbf{X} = [\boldsymbol{X}_{GM}, \boldsymbol{X}_S]$
- Outputs: $\boldsymbol{Y} = [\mathrm{GM}_{\mathbf{1}}, \dots, \mathrm{GM}_{25}]$

- Applications to synthetic ground motions



Relative contributions (trends) are same with recorded motion case

- Error < 13%

- $S_{\mathbf{GM}} \geq 60\%$
- $S_{\mathbf{S}} \leq 20\%$

➡ Effectively quantify **relative contributions** of **grouped (high-dimensional) input uncertainties**

Summary

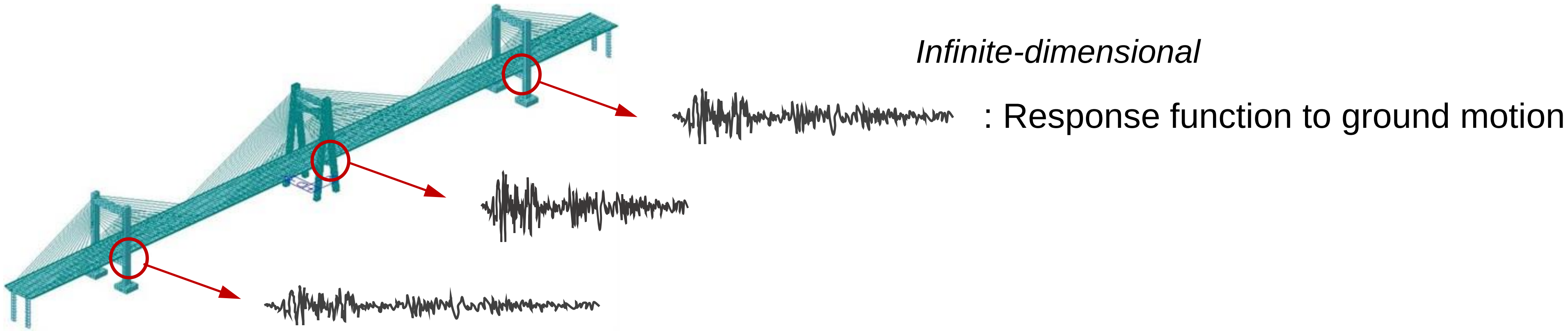
- The research goal is to develop an **efficient stochastic simulator-based approach for seismic UQ analysis of structural systems**
  - Development of a surrogate model for the **stochastic simulator**
  - **Uncertainty quantification** of seismic response using stochastic simulator
  - **Global sensitivity analysis** of seismic response leveraging stochastic surrogate model

- Reference

- Kim, J., Yi, S. R., & Wang, Z. (2024). Dimensionality reduction can be used as a surrogate model for high-dimensional forward uncertainty quantification. *arXiv preprint*:2402.04582.
- Kim, J., & Wang, Z. (2025). Uncertainty quantification for seismic response using dimensionality reduction-based stochastic simulator. *Earthquake Engineering & Structural Dynamics*, 54(2), 471-490.
- Kim, J., Su, M., Wang, Z., & Broccardo, M. (2025). Recorded versus synthetic ground motions: A comparative analysis of structural seismic responses. *arXiv preprint*:2502.19549.

Future Directions: Enhancing Seismic UQ with Neural Operators

- The proposed simulator predicts structural responses at vector scales, such as the EDP vector
  - Prediction of complete **continuous structural response function remains limited**
- Capturing the complete structural response function enables improved long-term maintenance planning, damage detection, and localization



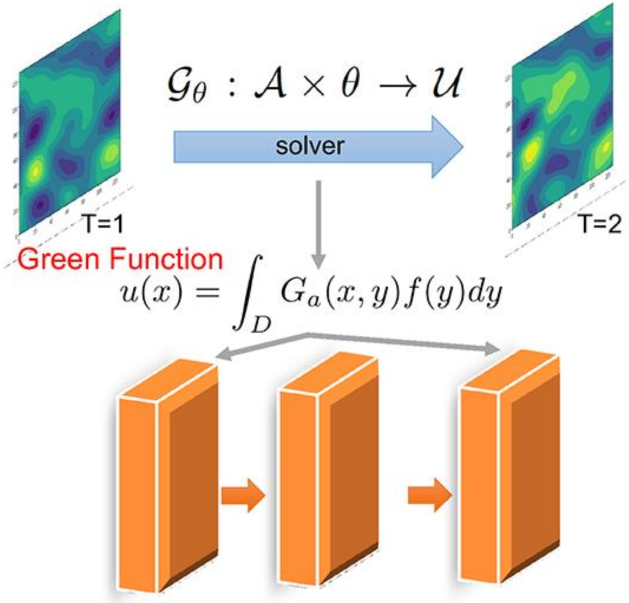
➡ **Neural operator** for describing continuous, infinite-dimensional response functions

Future Directions: Enhancing Seismic UQ with Neural Operators

- **Neural operators** aim to learn the underlying mathematical operator that governs the system rather than pointwise approximations: **Function-based learning**

$\mathcal{G}: \mathcal{A} \rightarrow \mathcal{U}$ $\quad$ $:\mathcal{A}, \mathcal{U}$ are infinite-dimensional function spaces

$\mathcal{G}(a) = u$



Zhang et al. (2024)

Layer $t = 1, \ldots, T$, acting on functions

$$\mathcal{G}_\theta := \mathcal{Q} \circ \underbrace{\sigma(\mathcal{K}_T + W_T + b_T) \circ \cdots \circ \sigma(\mathcal{K}_1 + W_1 + b_1)} \circ \mathcal{P}$$

"Projection" from intermediate to output

"Convolution" of kernel operator

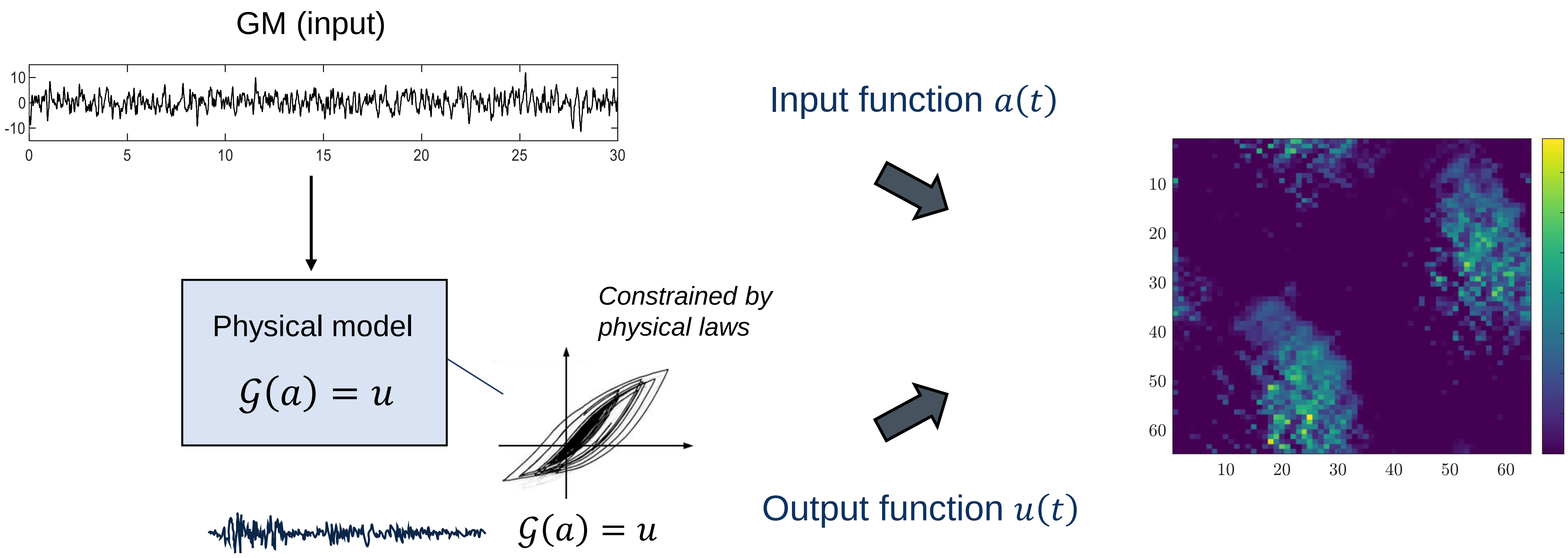
"Lifting" from input to higher dimensional space

- **Iterative kernel integrations $\mathcal{K}_t$**: Encodes the mapping between features across layers

Zhang, T., et al. (2024). Emulator of PR-DNS: Accelerating dynamical fields with neural operators in particle-resolved direct numerical simulation. *J. of Adv. Mod. Earth. Sys.*, 16(2).

Future Directions: Enhancing Seismic UQ with Neural Operators

- Integrating **neural operators** into the stochastic simulator for predicting time-series displacement and velocity profiles, and full-field stress distribution under seismic excitation



➡ Joint PDF for representing complex spatial and temporal patterns in structural response

Thanks for listening