PEER Researchers' Workshop

Aug 15-16, 2024

Stochastic Simulator-based Uncertainty Quantification for Seismic Responses of Bridges

Jungho KIM*

Postdoc researcher, University of California Berkeley

Marco Broccardo (co-PI)

Associate professor, University of Trento

Ziqi Wang (PI)

Assistant professor, University of California Berkeley

PEER

Berkeley
UNIVERSITY OF CALIFORNIA

Project details (#NCTRZW)

- **Title**: Stochastic simulator-based uncertainty quantification for seismic responses of bridges
- **PI**: Ziqi Wang (UC Berkeley), **Co-PI**: Marco Broccardo (University of Trento)
- **Duration**: Aug 2023 – Jan 2025

Research goals

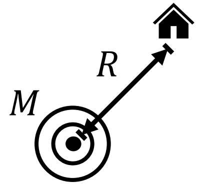
- The goal is to **develop** an efficient **stochastic simulator-based approach** for probabilistic seismic analysis of structural systems
- Technical aims are:
  - Develop **stochastic surrogate models for the stochastic simulator** to efficiently estimate performance measures of seismic response
  - Develop **sensitivity analysis methods** leveraging stochastic surrogate models

Research overview

Input uncertainties

Seismic hazard parameters

Magnitude,
rupture distance,
shear velocity …

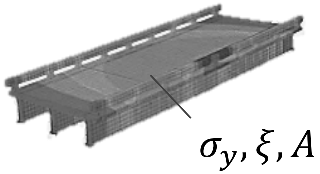


Excitation sequence

White noise process, time-series

Structural parameters

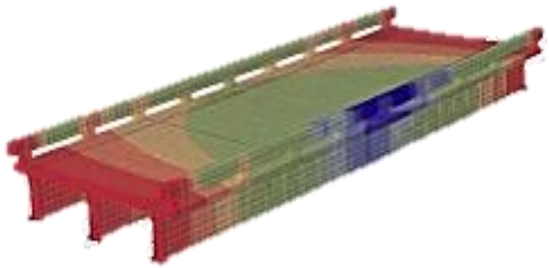
Material parameters,
mass, damping, geometry …



Step 1.
Stochastic simulator

FEM deterministic model

$$\mathcal{M}(X_1, X_2, \dots, X_n)$$

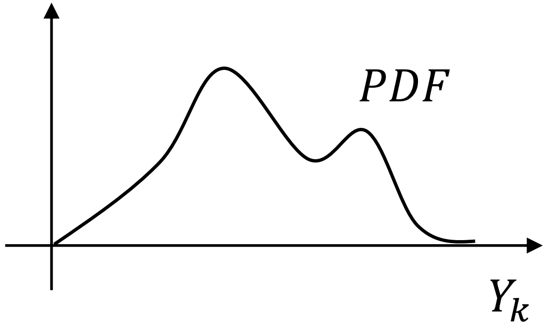


Step 2.
Uncertainty propagation

Seismic response

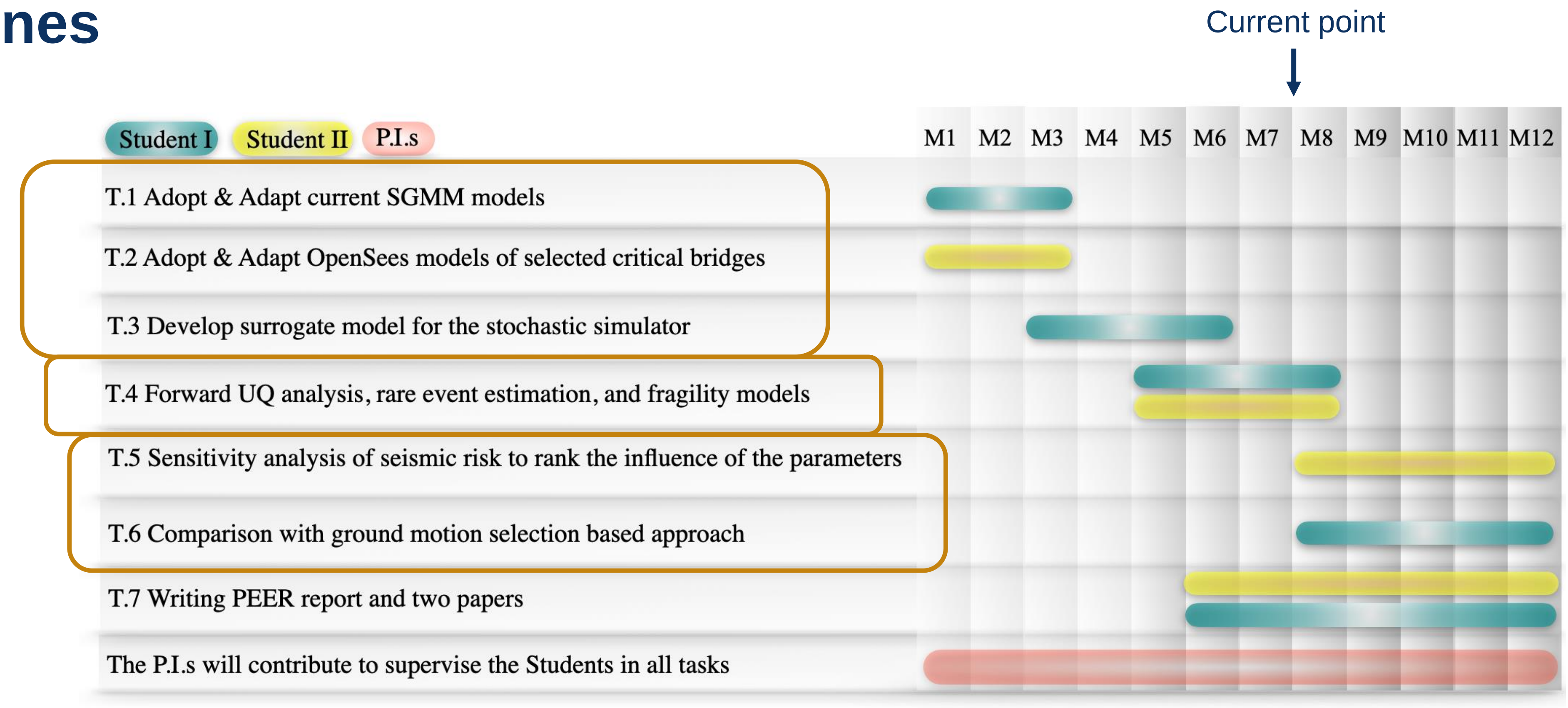
$$\boldsymbol{Y} = \mathcal{M}(\boldsymbol{X})$$

EDP vectors



Step 3.
Sensitivity analysis

Milestones



- Develop surrogate model for stochastic simulator **(done)**
- Uncertainty quantification of seismic responses using stochastic simulator **(done)**
- Sensitivity analysis of seismic response & comparison with ground motion selected-based approach **(on-going)**

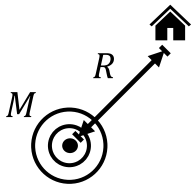
1. Development of stochastic surrogate model

Challenges in surrogate modeling

Input uncertainties

Seismic hazard parameters

Magnitude,
rupture distance,
shear velocity …



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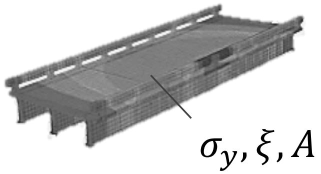

Stochastic surrogate model

FEM deterministic model

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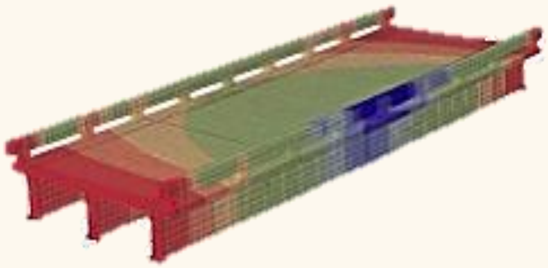
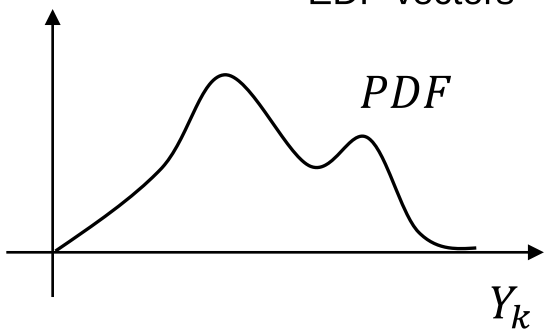


Seismic response

$$\boldsymbol{Y} = \mathcal{M}(\boldsymbol{X})$$

EDP vectors



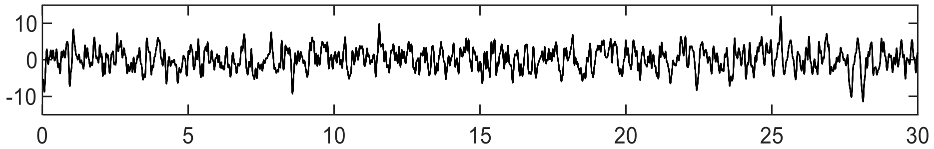
- Surrogate modeling can be **challenging** due to the **complex & high-dimensional input uncertainties**

➡ **Dimensionality reduction** can be useful

Dimensionality reduction-based stochastic surrogate model

- **Main idea**: Perform dimensionality reduction in the input-output space

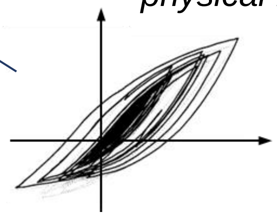
High-dimensional input $\boldsymbol{X}$



Physical model

$$\boldsymbol{Y} = \mathcal{M}(\boldsymbol{X})$$

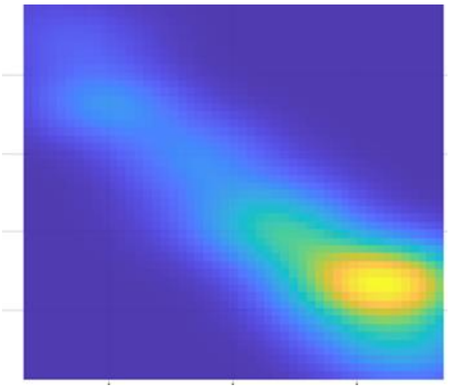
Constrained by physical laws



$$\mathbf{A}\ddot{\mathbf{Y}}(\boldsymbol{X}, t) + \mathbf{R}\mathbf{Y}(\boldsymbol{X}, t) = q$$

Low-dimensional representation via working with $\{(\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}^{n+m} : \boldsymbol{y} - \mathcal{M}(\boldsymbol{x}) = 0\}$

$$\mathcal{H} : \boldsymbol{z} \equiv (\boldsymbol{x}, \boldsymbol{y}) \mapsto \boldsymbol{\psi}_z :$$



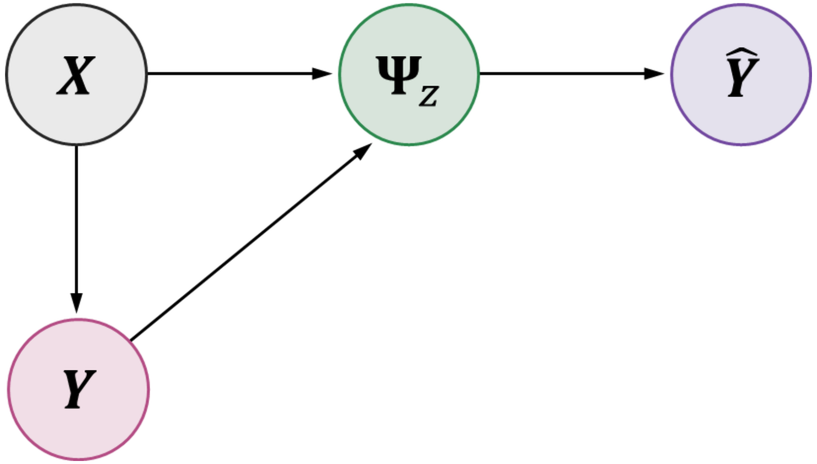
➡ **“Extract” stochastic surrogate model** from results of **dimensionality reduction**

Kim, J., Yi, S. R., & Wang, Z. (2024). Dimensionality reduction can be used as a surrogate model for high-dimensional forward uncertainty quantification. *arXiv preprint*:2402.04582.

Procedures of the proposed stochastic surrogate model

- Dimensionality reduction in the input-output space–construct $\mathcal{H}: \boldsymbol{z} \equiv (\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}^{n+m} \mapsto \boldsymbol{\psi}_z \in \mathbb{R}^d$
- Construct a conditional distribution $f_{\widehat{Y}|\boldsymbol{\Psi}_z}(\widehat{\boldsymbol{y}}|\boldsymbol{\psi}_z)$ to predict $\boldsymbol{y}$ given $\boldsymbol{\psi}_z$
- "Extract" a surrogate model $f_{\widehat{Y}|X}(\widehat{\boldsymbol{y}}|\boldsymbol{x})$ from $\mathcal{H}$ and $f_{\widehat{Y}|\boldsymbol{\Psi}_z}$

training stage



"True" surrogate model:

$$f_{\widehat{Y}|X}(\widehat{\boldsymbol{y}}|\boldsymbol{x}) = \int\int f_{\widehat{Y}|\boldsymbol{\Psi}_z}(\widehat{\boldsymbol{y}}|\boldsymbol{\psi}_z) f_{\boldsymbol{\Psi}_z|XY}(\boldsymbol{\psi}_z|\boldsymbol{x},\boldsymbol{y}) f_{Y|X}(\boldsymbol{y}|\boldsymbol{x})\, d\boldsymbol{\psi}_z d\boldsymbol{y}$$

Condi. distribution *Dimensionality reduction* *Original model*

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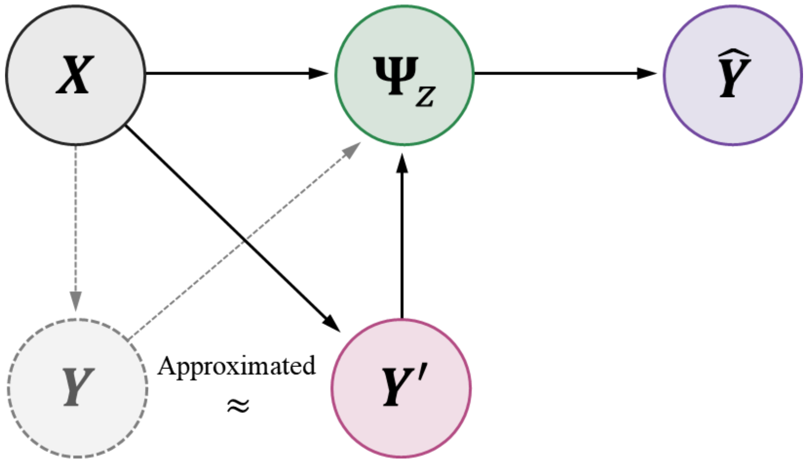
Procedures of the proposed stochastic surrogate model

- Dimensionality reduction in the input-output space–construct $\mathcal{H}$: $\boldsymbol{z} \equiv (\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}^{n+m} \mapsto \boldsymbol{\psi}_z \in \mathbb{R}^d$
- Construct a conditional distribution $f_{\widehat{Y}|\boldsymbol{\Psi}_\mathrm{z}}(\widehat{\boldsymbol{y}}|\boldsymbol{\psi}_\mathrm{z})$ to predict $\boldsymbol{y}$ given $\boldsymbol{\psi}_\mathrm{z}$
- "Extract" a surrogate model $f_{\widehat{Y}|X}(\widehat{\boldsymbol{y}}|\boldsymbol{x})$ from $\mathcal{H}$ and $f_{\widehat{Y}|\boldsymbol{\Psi}_\mathrm{z}}$

prediction stage



$\rightarrow \widehat{\boldsymbol{y}}' \sim f_{\widehat{Y}|X}(\widehat{\boldsymbol{y}}|\boldsymbol{x})$

"Stationary" surrogate model:

$$f^{(\infty)}_{\widehat{Y}|X}(\widehat{\boldsymbol{y}}|\boldsymbol{x}) = \int\int f_{\widehat{Y}|\boldsymbol{\Psi}_\mathrm{z}}(\widehat{\boldsymbol{y}}|\boldsymbol{\psi}_\mathrm{z}) f_{\boldsymbol{\Psi}_\mathrm{z}|XY}(\boldsymbol{\psi}_\mathrm{z}|\boldsymbol{x},\boldsymbol{y}') f^{(\infty)}_{\widehat{Y}|X}(\boldsymbol{y}'|\boldsymbol{x})\, d\boldsymbol{\psi}_\mathrm{z} d\boldsymbol{y}'$$

"approximate" surrogate

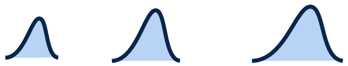
Transition kernel:

$$T\left(\widehat{\boldsymbol{y}}^{(t)}, \widehat{\boldsymbol{y}}^{(t+1)}|\boldsymbol{x}\right) = f_{\widehat{Y}|\boldsymbol{\Psi}_\mathrm{z}}\left(\widehat{\boldsymbol{y}}^{(t+1)}\middle|\boldsymbol{\psi}_\mathrm{z}\right) f_{\boldsymbol{\Psi}_\mathrm{z}|XY}\left(\boldsymbol{\psi}_\mathrm{z}\middle|\boldsymbol{x}, \widehat{\boldsymbol{y}}^{(t)}\right)$$

→ **Outputs: Stochastic surrogate model,** $\widehat{\boldsymbol{y}}^{(t)} \sim f_{\widehat{Y}|X}(\widehat{\boldsymbol{y}}|\boldsymbol{x})$

Kim, J., Yi, S. R., & Wang, Z. (2024). Dimensionality reduction can be used as a surrogate model for high-dimensional forward uncertainty quantification. *arXiv preprint*:2402.04582.

Summary of the proposed stochastic surrogate model



$$\boldsymbol{X} = [\boldsymbol{X}_1, \boldsymbol{X}_2 \dots, \boldsymbol{X}_n] \quad \xrightarrow{\mathcal{M}(\boldsymbol{X})} \quad \boldsymbol{Y} = [Y_1, Y_2, \dots, Y_m]$$

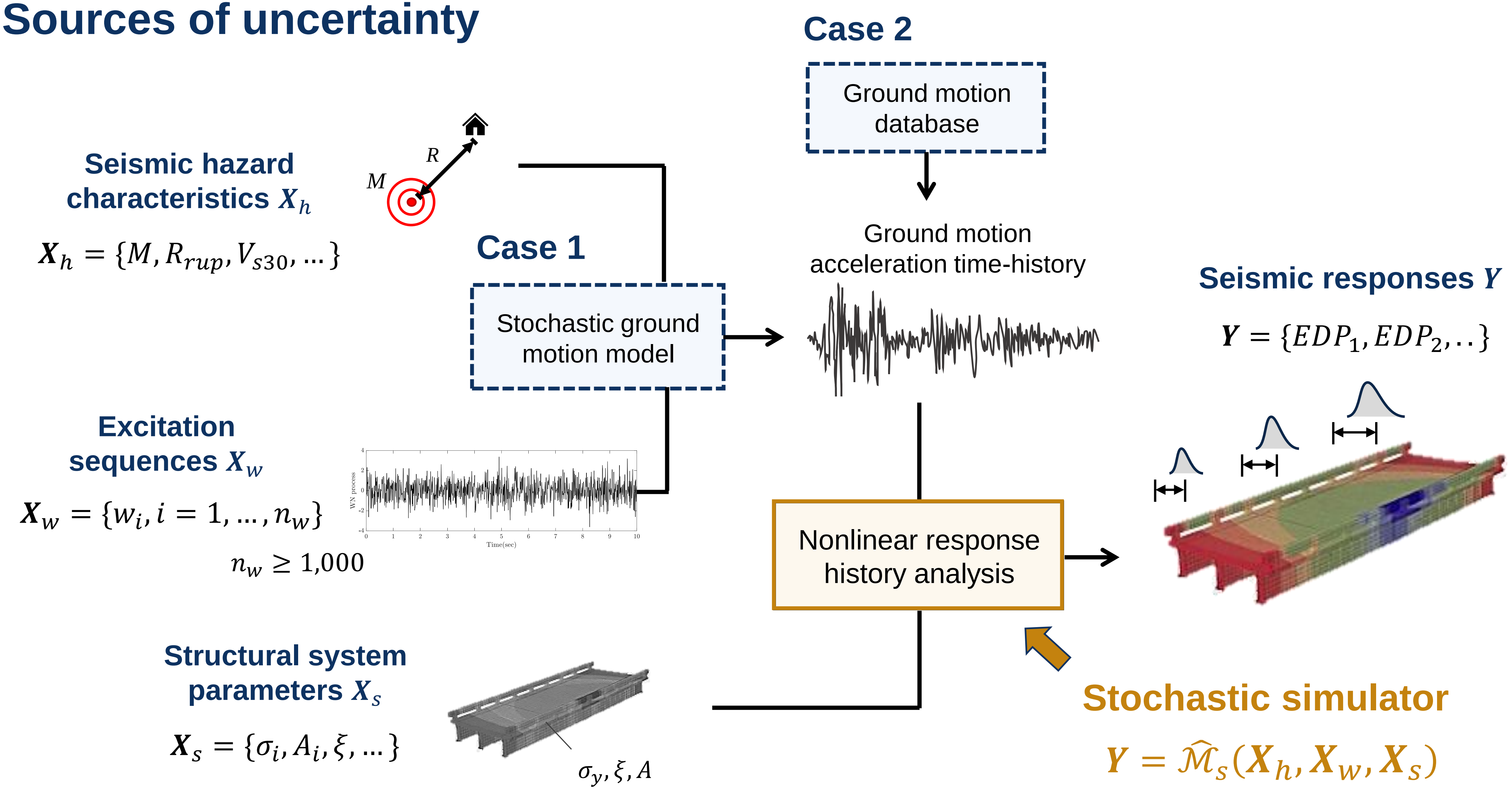
where $n \gg 100$

- We **“extract” a surrogate model** from the results of dimensionality reduction
  - Surrogate model for high-dimensional system
- **Stochastic simulator**: Output predictions are probabilistic distributions
- **Multi-output predictor**: We can quantify interdependencies between multiple outputs

Kim, J., Yi, S. R., & Wang, Z. (2024). Dimensionality reduction can be used as a surrogate model for high-dimensional forward uncertainty quantification. *arXiv preprint*:2402.04582.

2. Uncertainty quantification of seismic response

Sources of uncertainty



Application (Case 1: stochastic ground motion model)

- UQ for seismic response

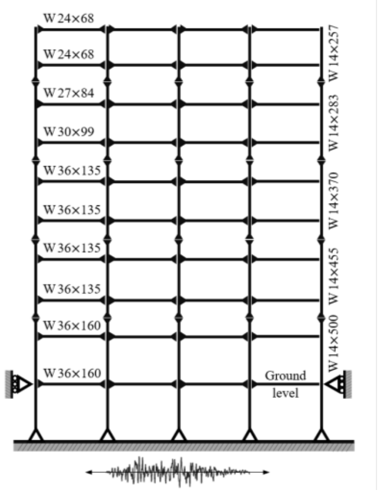
$$\boldsymbol{X} = [\boldsymbol{X}_h, \boldsymbol{X}_w, \boldsymbol{X}_s] \in \mathbb{R}^{\geq 2{,}000}$$

$$\widehat{\mathcal{M}}_s(\boldsymbol{X})$$

Uncertainty propagation using stochastic simulator

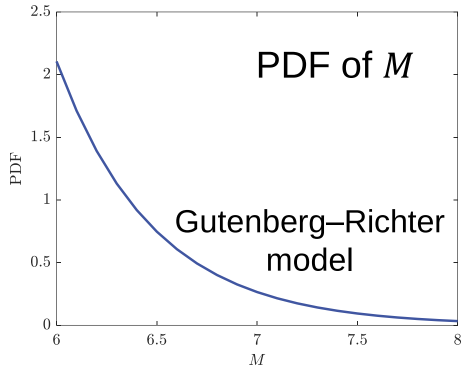
$$\boldsymbol{Y} = [\mathrm{IDR}_1,\ldots,\mathrm{IDR}_9,\mathrm{SD}_1,\ldots,\mathrm{SD}_9] \in \mathbb{R}^{18}$$

- 9-story steel building structure:



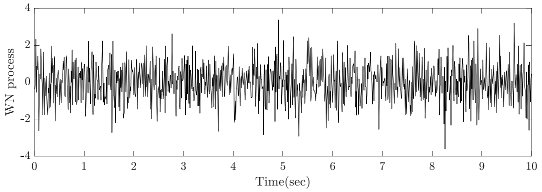
Seismic hazard characteristics

$$\boldsymbol{X}_h = \{M, R_{rup}\} \in \mathbb{R}^2$$



Excitation sequences

$$\boldsymbol{X}_w = \{w_i, i = 1, \ldots, n_w\} \in \mathbb{R}^{\geq 2{,}000}$$



White noise sequence in SGMM

Structural system parameter

$$\boldsymbol{X}_s = \{\zeta, E, f_y^b, \delta_h^b, f_y^c, \delta_h^c\} \in \mathbb{R}^6$$

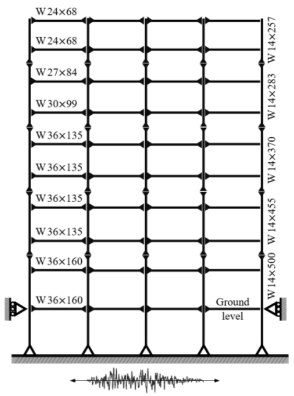
Variable	Mean	c.o.v	Distribution
Damping ratio, ζ (%)	3	0.2	Lognormal
Elastic modulus, E (Mpa)	200,000	0.05	Lognormal
Yield strength for beam, f_y^b (Mpa)	248	0.1	Lognormal
Yield strength for column, f_y^c (Mpa)	345	0.1	Lognormal
Strain hardening ratio for beam, δ_h^b	0.01	0.2	Lognormal
Strain hardening ratio for column, δ_h^c	0.01	0.2	Lognormal

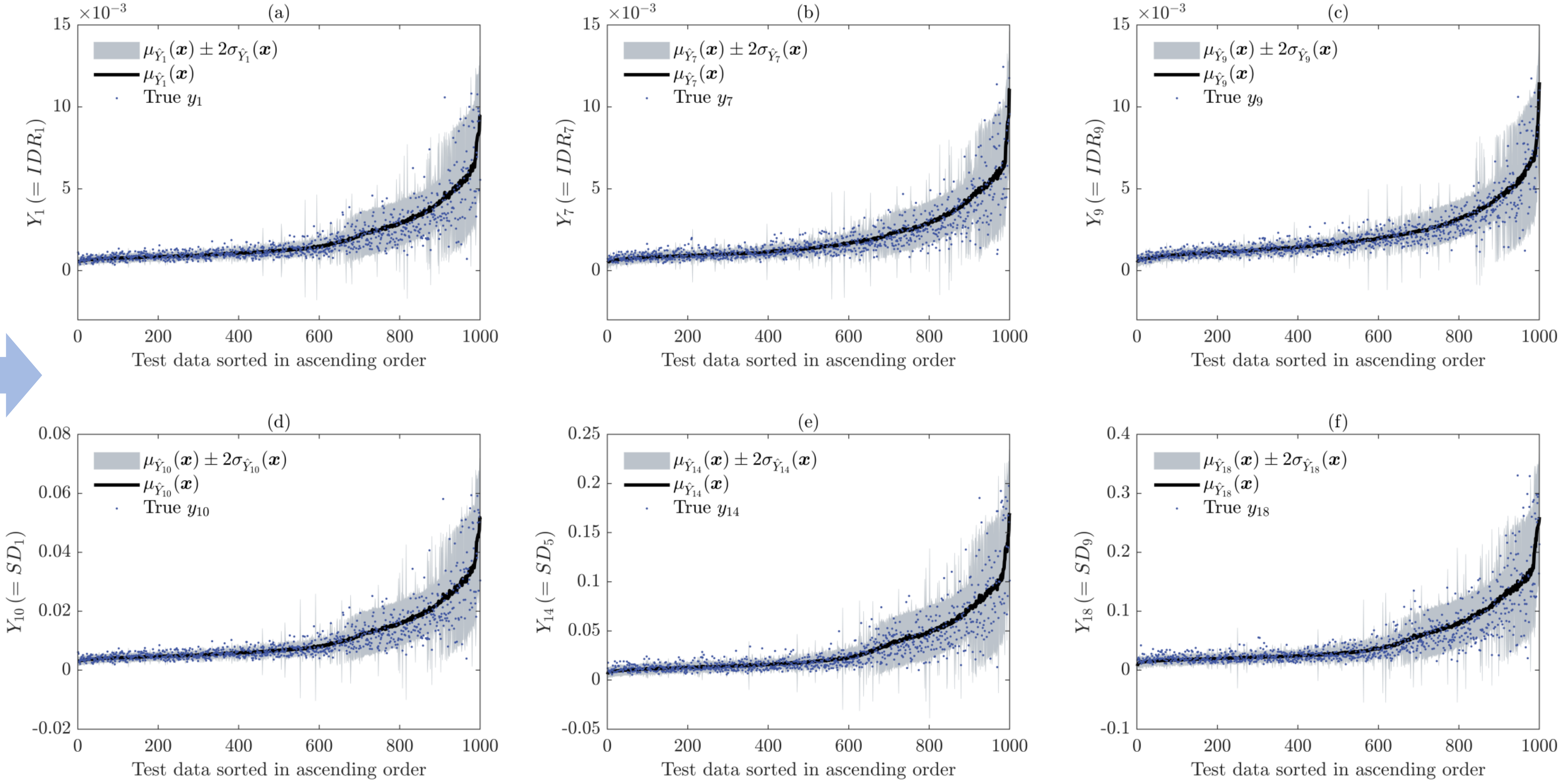
Kim, J., and Z. Wang. Uncertainty quantification for seismic response using dimensionality reduction-based stochastic simulator. (Under Review)

Surrogate modeling of seismic response

- Inputs: $\mathbf{X} = [\boldsymbol{X}_h, \boldsymbol{X}_w, \boldsymbol{X}_s]$
- Outputs: $\boldsymbol{Y} = [\mathrm{IDR}_1,\ldots,\mathrm{IDR}_9,\mathrm{SD}_1,\ldots,\mathrm{SD}_9]$

- Prediction by stochastic simulator ($N_T = 600$)





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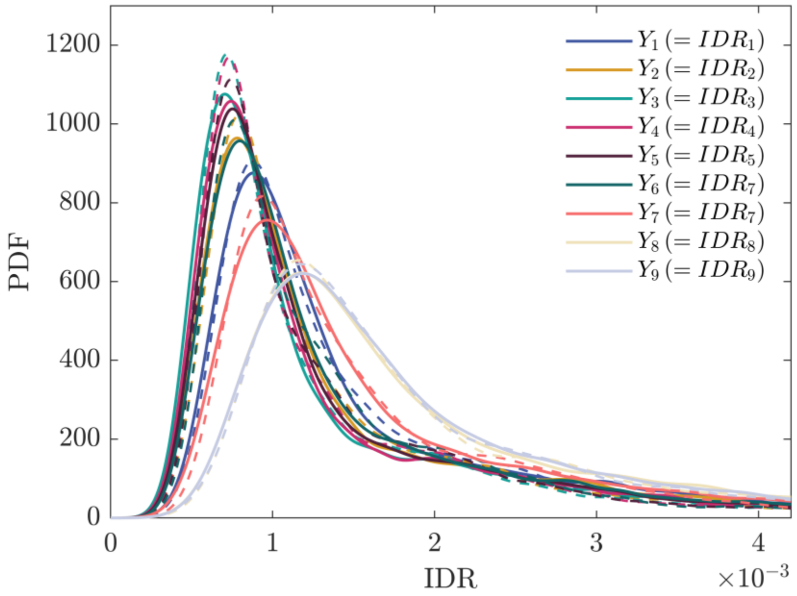
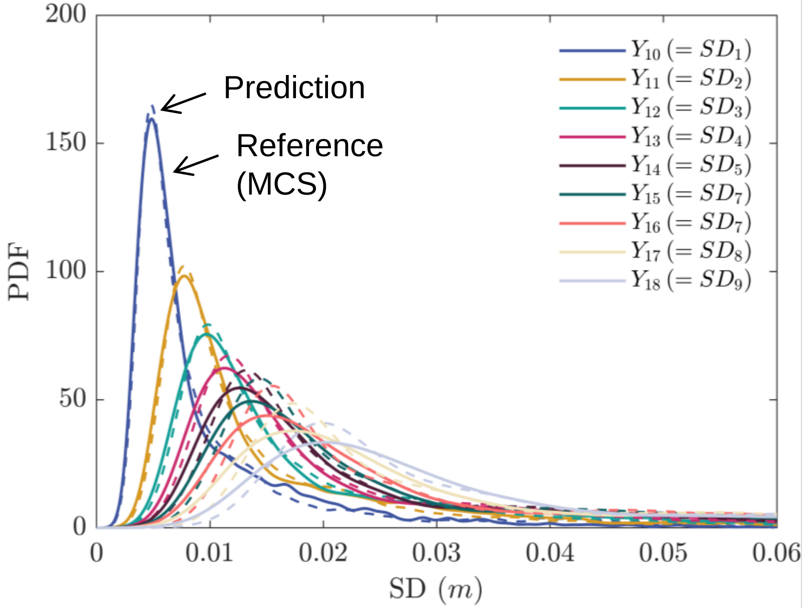
UQ for seismic response

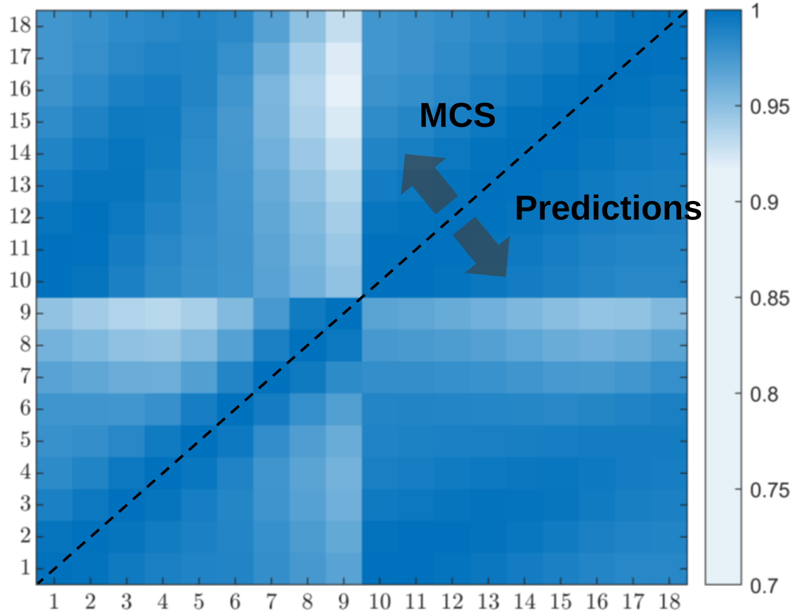
- Inputs: $\mathbf{X} = [\boldsymbol{X}_h, \boldsymbol{X}_w, \boldsymbol{X}_s]$
- Outputs: $\boldsymbol{Y}$ = [IDR_1,...,IDR_9,SD_1,...,SD_9]

- Response PDFs

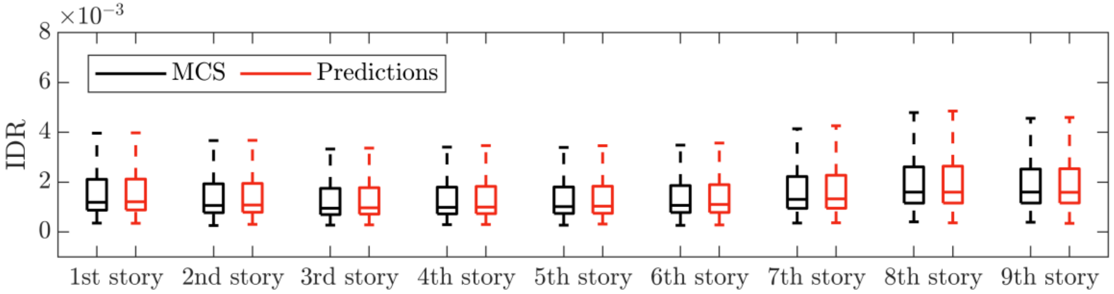


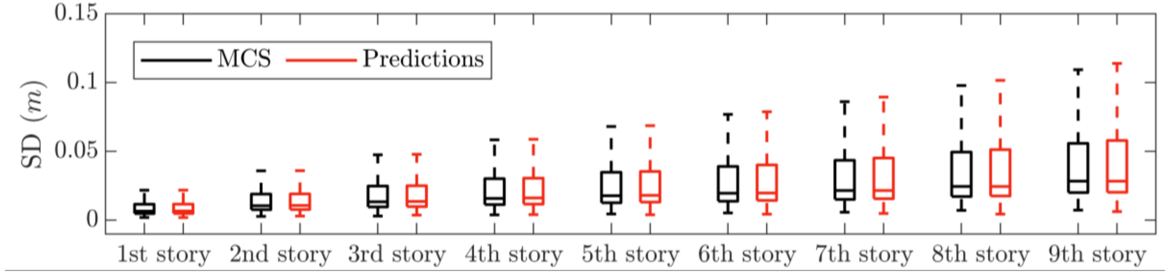


- Correlation matrix



- Median and interquartile ranges





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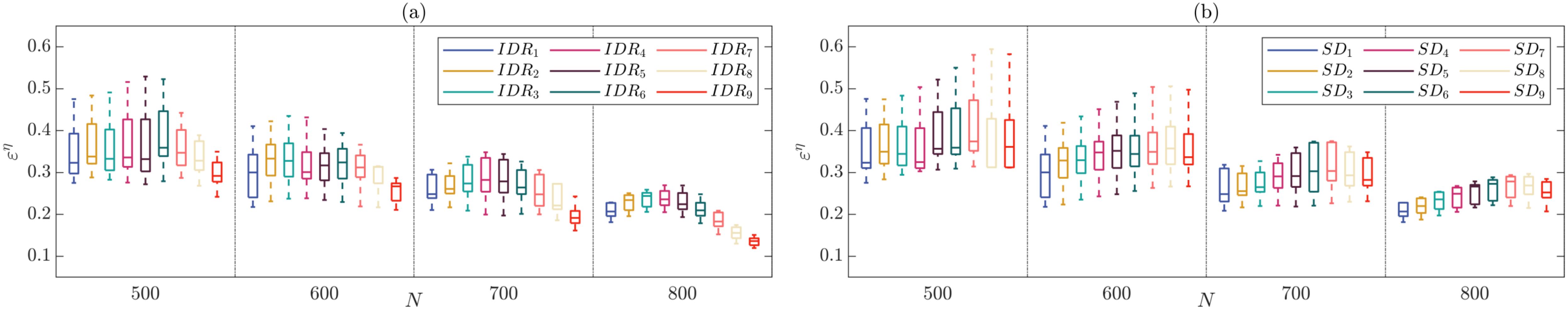
Errors of the surrogate model

- Relative mean square error (RMSE):
- Comparison of RMSE at different numbers of training sets, N_T

Mean prediction

$$\varepsilon_\eta = \frac{\mathbb{E}\left[\left(\mu_{\hat{Y}}(\boldsymbol{X}) - \mathcal{M}(\boldsymbol{X})\right)^2\right]}{\mathbb{V}\mathrm{ar}[\mathcal{M}(\boldsymbol{X})]}$$



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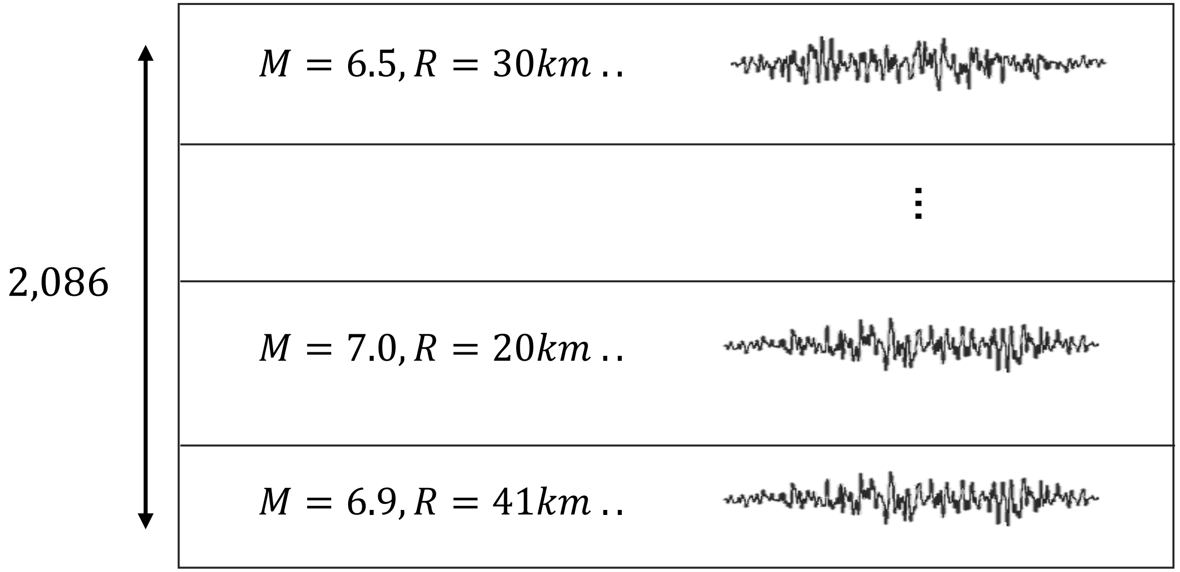
Application (Case 2: ground motion database)

- Use **2086 GM records** from PEER NGA-west database (selected by $6.0 \leq M \leq 8.0, 10 \leq R \leq 50\ km$)
- Applications to the same building structure

▪ Inputs: $\mathbf{X} = [\boldsymbol{X}_h, \boldsymbol{X}_w, \boldsymbol{X}_s]$

▪ Outputs: $\boldsymbol{Y} = [\text{IDR}_1,\ldots,\text{IDR}_9,\text{SD}_1,\ldots,\text{SD}_9]$

Ground motion database



Uncertain structural parameters for 9-story steel building

Variable	Mean	c.o.v	Distribution
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$\boldsymbol{X}_h \in \mathbb{R}^2$ $\boldsymbol{X}_w \in \mathbb{R}^{>3,000}$ $\boldsymbol{X}_s \in \mathbb{R}^6$

→ *Acceleration time-history is considered as realization of* $\boldsymbol{X}_w$

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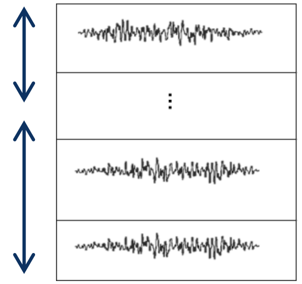
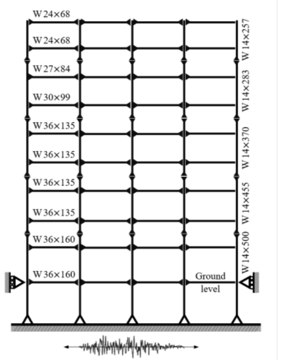
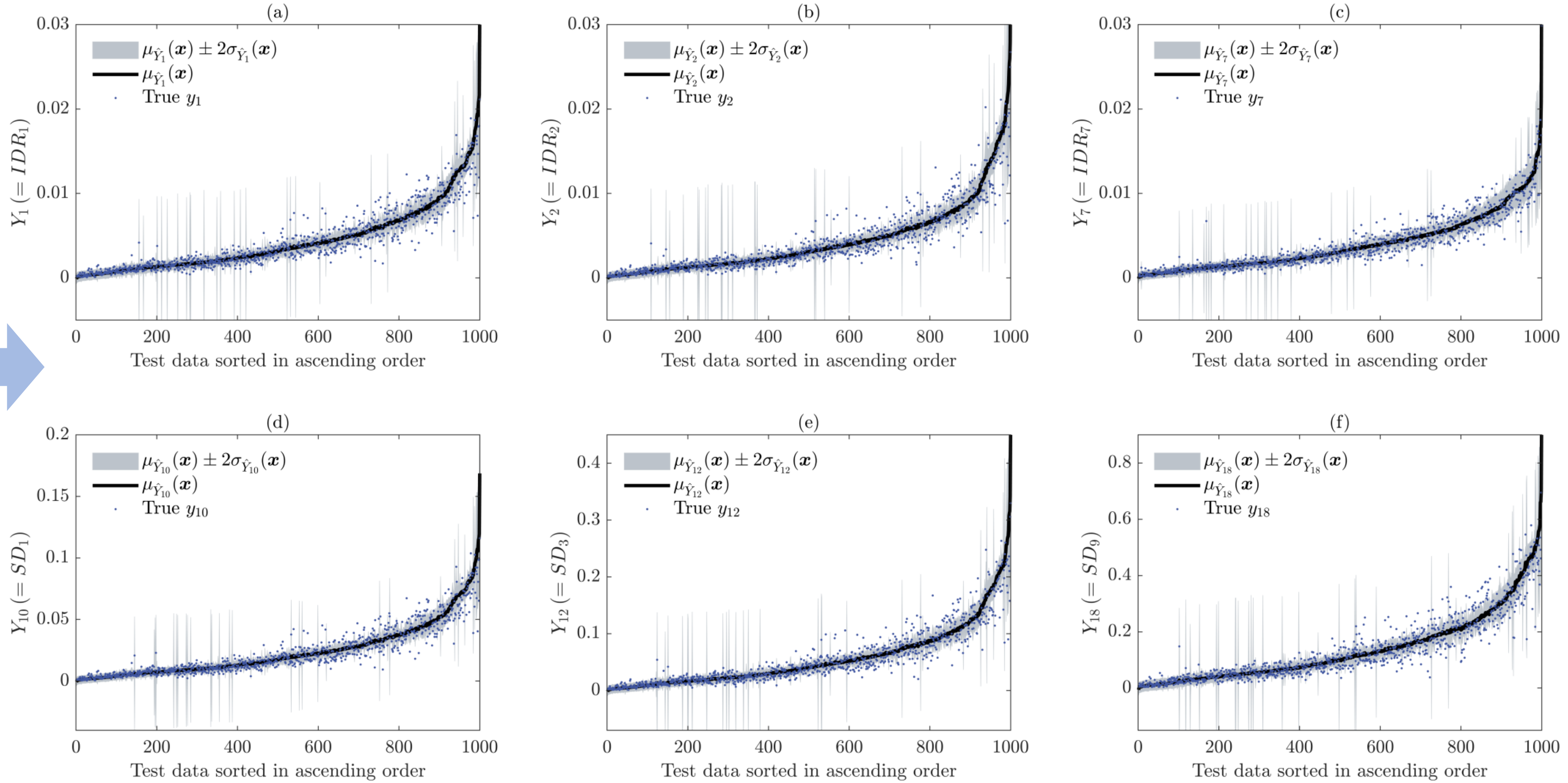
Surrogate modeling of seismic response

Training set ($N = 600$)
Test set ($N = 1{,}000$)



- Prediction by stochastic simulator ($N_T = 600$)





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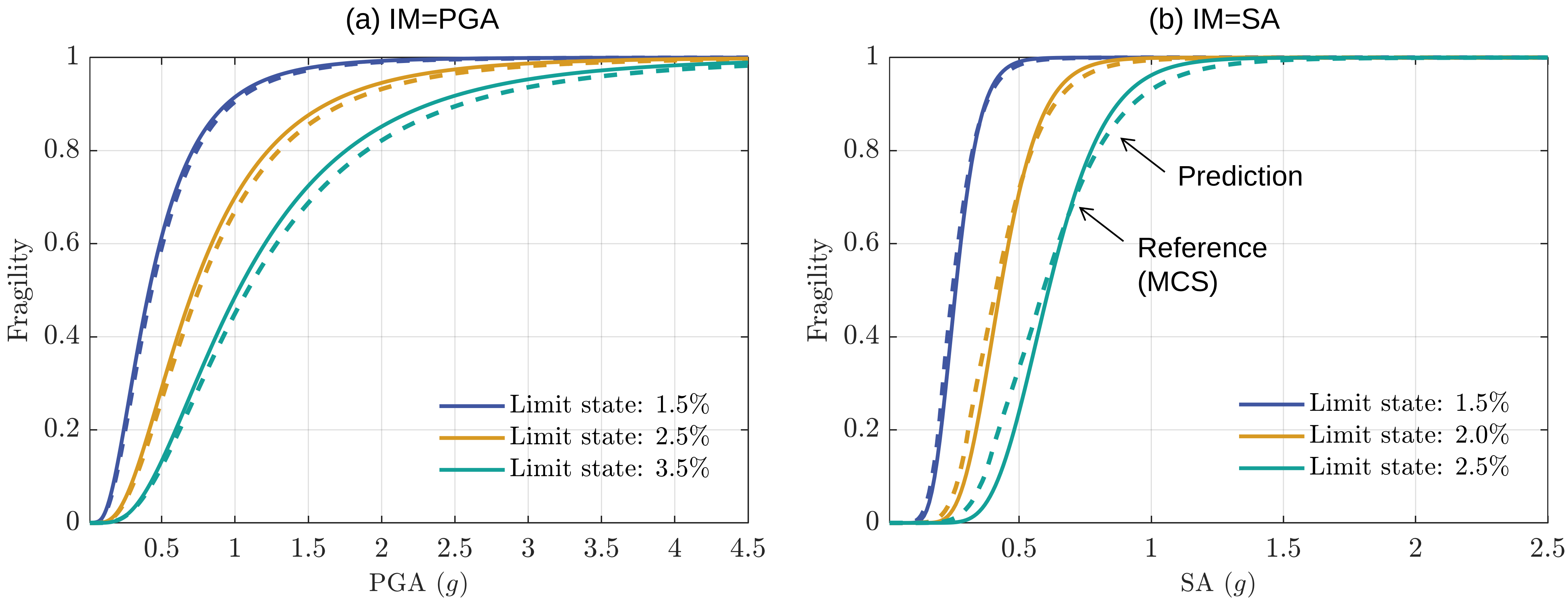
Fragility curve estimation

- Cloud analysis is adopted: $\ln(\text{EDP}) = a + b\ln(\text{IM}) + \varepsilon_R$
- Using the trained surrogate model ($N_T = 600$), fragility curves are estimated from a set of 50 ground motions



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3. Sensitivity analysis of seismic response

Global sensitivity analysis of seismic response

Input uncertainties

Seismic hazard parameters

Magnitude,
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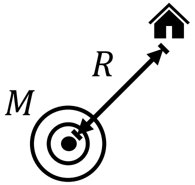

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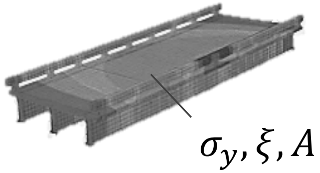
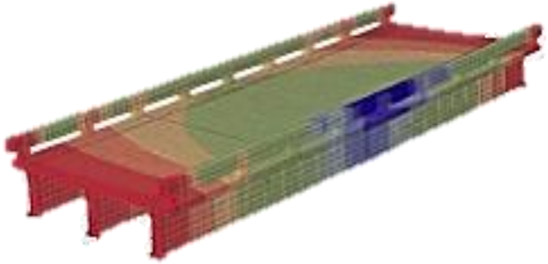


FEM deterministic model

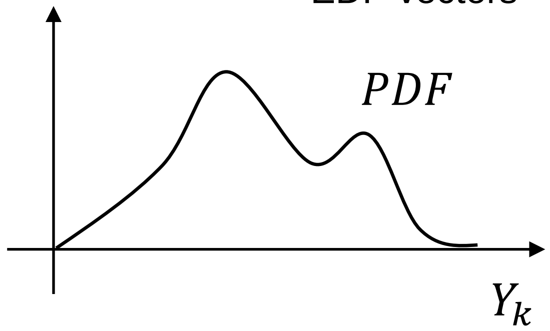
$$\mathcal{M}(X_1, X_2, \ldots, X_n)$$



Seismic response

$$\boldsymbol{Y} = \mathcal{M}(\boldsymbol{X})$$

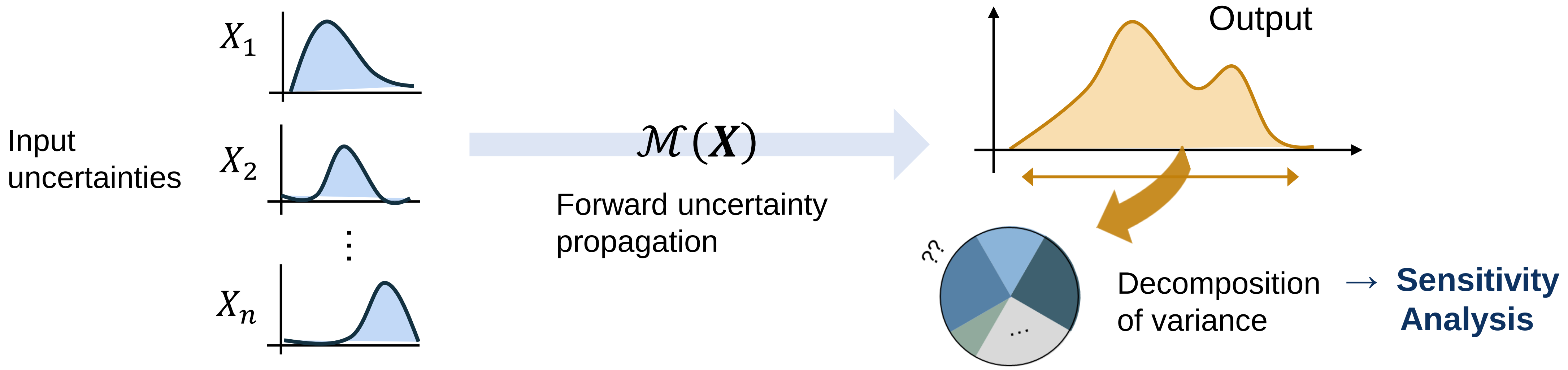
EDP vectors



Step 3.
Sensitivity analysis

Variance-based sensitivity analysis

- Integrated sensitivity over the entire input parameter space: “**Global” sensitivity analysis**



- Two main indices: **First-order (main-effect)** and **total-effect indices**

$$S_i = \frac{\mathbb{V}\mathrm{ar}_{X_i}\left[\mathbb{E}_{\boldsymbol{X}_{\sim i}}[Y|X_i]\right]}{\mathbb{V}\mathrm{ar}[Y]}, \quad S_{T_i} = 1 - \frac{\mathbb{V}\mathrm{ar}_{\boldsymbol{X}_{\sim i}}\left[\mathbb{E}_{X_i}[Y|\boldsymbol{X}_{\sim i}]\right]}{\mathbb{V}\mathrm{ar}[Y]}, \quad 0 \leq S_i \leq 1$$

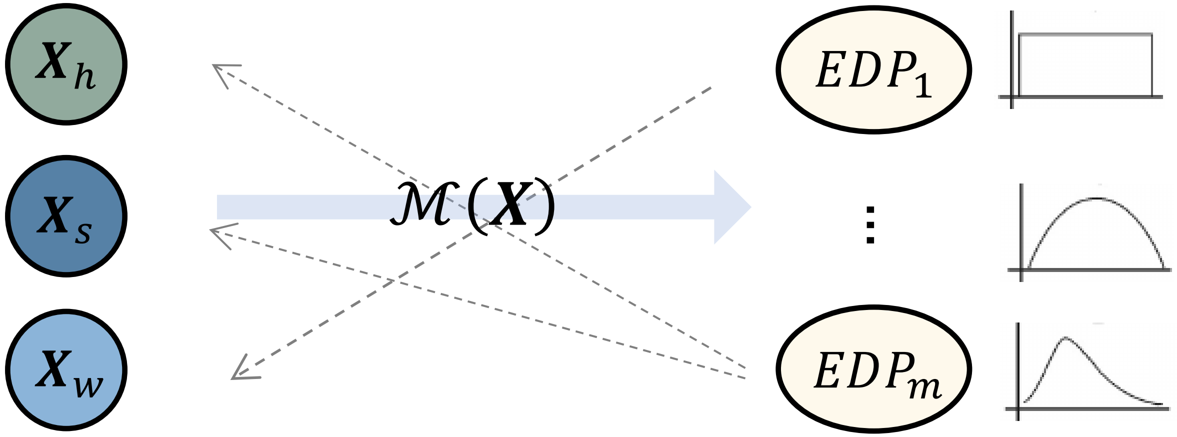
⟶ Quantify the **additive effect of each variable** and **interactions with the other variables**

Variance-based sensitivity analysis

- Sensitivity indices for **each EDP** with respect to **each group of input uncertainties**

$$\boldsymbol{X} = [\boldsymbol{X}_h, \boldsymbol{X}_s, \boldsymbol{X}_w]$$

3 “groups” of random variables



$$\boldsymbol{Y} = [EDP_1, \dots, EDP_m]$$

$$\rightarrow \quad S_{\mathbf{u}}^{k} = \frac{\mathbb{V}\mathrm{ar}_{\boldsymbol{X}_{\mathbf{u}}}\left[\mathbb{E}_{\boldsymbol{X}_{\sim\mathbf{u}}}[EDP_k|\boldsymbol{X}_{\mathbf{u}}]\right]}{\mathbb{V}\mathrm{ar}[EDP_k]}, \quad S_{T_{\mathbf{u}}}^{k} = 1 - \frac{\mathbb{V}\mathrm{ar}_{\boldsymbol{X}_{\sim\mathbf{u}}}\left[\mathbb{E}_{\boldsymbol{X}_{\mathbf{u}}}[EDP_k|\boldsymbol{X}_{\sim\mathbf{u}}]\right]}{\mathbb{V}\mathrm{ar}[EDP_k]}$$

- Inevitably *high-dimensional* integral (due to white noise sequence)
- *High computational complexity* ($Complexity = d \times N^2$)

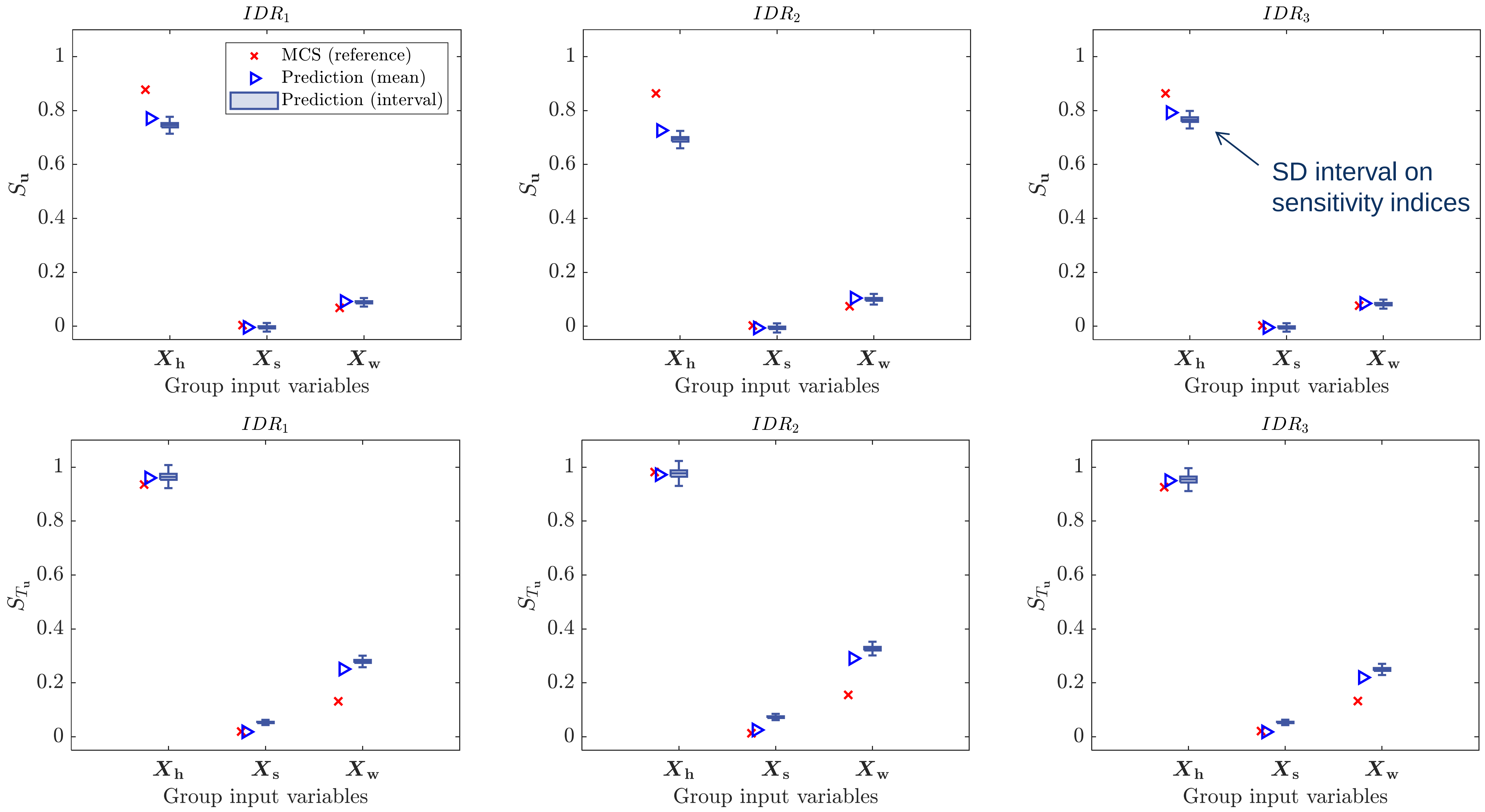
➡ **Stochastic simulator**

Sensitivity indices of seismic response

- Inputs: $\mathbf{X} = [\boldsymbol{X}_h, \boldsymbol{X}_w, \boldsymbol{X}_s]$
- Outputs: $\boldsymbol{Y} = [\mathrm{IDR}_1, \ldots, \mathrm{IDR}_3]$

First-effect:

Total-effect:



Concluding remarks

- The project goal is to develop an **efficient stochastic simulator-based approach for probabilistic seismic analysis of structural systems**
  - Development of a surrogate model for the **stochastic simulator**
  - **Uncertainty quantification** of seismic response using stochastic simulator
  - **Global sensitivity analysis** of seismic response leveraging stochastic surrogate model
- On-going research includes
  - Comparison of GSA-based approach and **ground motion selected-based approach**
  - Application to **bridge structures** (Auburn Ravine Bridge & Penstock Bridge)
- Reference

- Kim, J., Yi, S. R., & Wang, Z. (2024). Dimensionality reduction can be used as a surrogate model for high-dimensional forward uncertainty quantification. *arXiv preprint*:2402.04582.
- Kim, J., and Z. Wang. Uncertainty quantification for seismic response using dimensionality reduction-based stochastic simulator. (Under Review)

Thanks for listening