S1: Earthquake Source Characterization

History and Current Status of Rupture Modeling

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PEER

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Overview

- A quick look ”back into the past” *… obviously, this will be utterly incomplete …*
- Spatial variations of on-fault displacement (aka “slip heterogeneity”)
- Other ingredients for rupture modeling
  - Hypocenter positioning
  - Variability of temporal rupture evolution: rupture velocity & rise time
  - The local slip-rate function: shape & duration
- Further constraints from simulations and observations
- Open questions

Disclaimer: not much detail / review on rupture dynamics

Early Developments

Realizing the earthquake source process is complicated ...

- Earthquake source complexity recognized in the 1960ties and 1970ties
- Omega-square (ω^{-2}) or ω^{-3} model to explain far-field observations (e.g. *Aki, 1967*)
- Theoretical source models developed for point-source like ruptures (e.g. *Brune, 1970*)

Slip, Slip-rate, Slip-Acc.

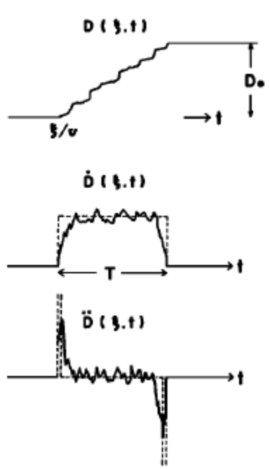


Fig. 1. Schematic diagram of dislocation and its time derivatives at a given point ξ on a fault.

ACF of Slip, Slip-Acc.

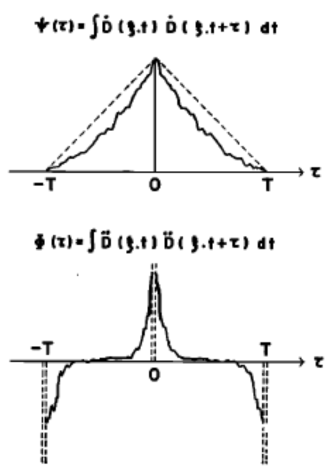


Fig. 2. Schematic diagram of autocorrelation functions of dislocation velocity and dislocation acceleration at a given point ξ on a fault.

Aki, 1967

Far-field spectral decay

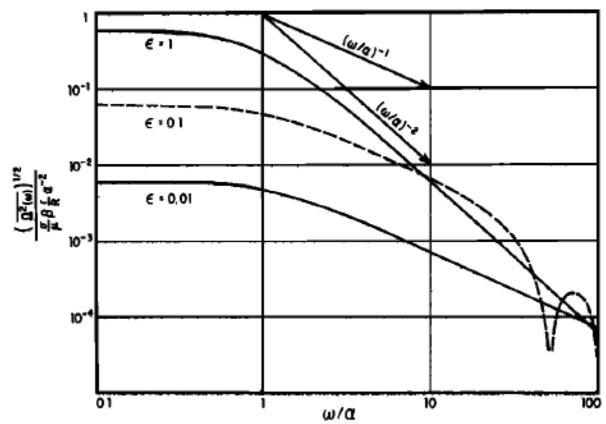


Fig. 5. Average (rms) far-field spectral density curves.

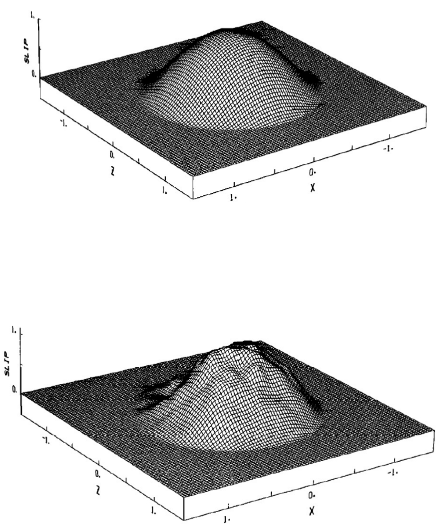
$$\langle \Omega(\omega) \rangle = \langle \Re_{\theta\phi} \rangle \frac{\sigma\beta}{\mu} \frac{r}{R} F(\epsilon) \frac{1}{\omega^2 + \alpha^2}$$

Brune, 1970

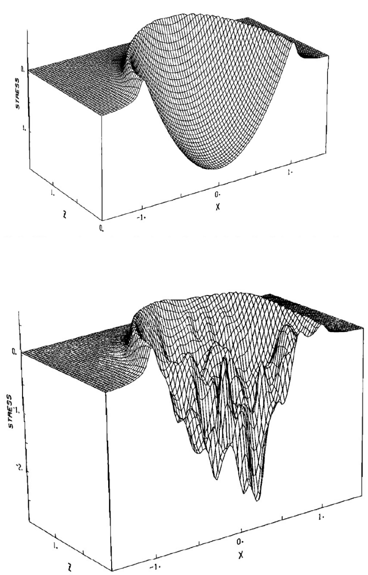
2D Slip- and Stress-functions on fault plane

- Extended-fault slip characterization (e.g. *Andrews, 1980, 1981*)
  - Two-dimensional slip function *D(x,z)* with specific properties (in space & FFT domain)
  - Spectral behavior $D(\mathbf{k})$ constrained to $D(\mathbf{k}) \sim \mathbf{k}^{-2}$ by far-field ω^{-2}-decay

2D Slip Function D(x,z)



Static Stress Function σ(x,y)



Andrews (1980, 1981)

$$\Delta\sigma_{\parallel}(\mathbf{k}) = K_{\parallel}(\mathbf{k}) D(\mathbf{k})$$

$$\Delta\sigma_{\perp}(\mathbf{k}) = K_{\perp}(\mathbf{k}) D(\mathbf{k})$$

$$K_{\parallel}(\mathbf{k}) = -\frac{1}{2}\frac{\mu}{\sqrt{k_{\parallel}^2 + k_{\perp}^2}}\left[\frac{2(\lambda+\mu)}{\lambda+2\mu}k_{\parallel}^2 + k_{\perp}^2\right]$$

$$K_{\perp}(\mathbf{k}) = -\frac{1}{2}\frac{\mu}{\sqrt{k_{\parallel}^2 + k_{\perp}^2}}\left[\frac{2(\lambda+\mu)}{\lambda+2\mu} - 1\right]k_{\parallel}k_{\perp}$$

Notation Ripperger and Mai (2004)

$$|\sigma(k)| \propto k^{-\nu}$$

$$|D(k)| \propto k^{-\nu-1}$$

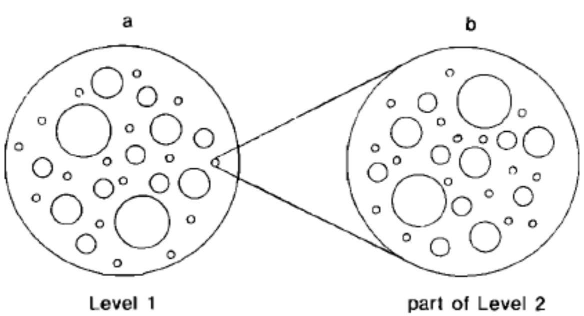
$\nu = 1 \rightarrow |D(k)| \propto k^{-2} \rightarrow$ *"k-square"*

Early Developments

Composite Sources

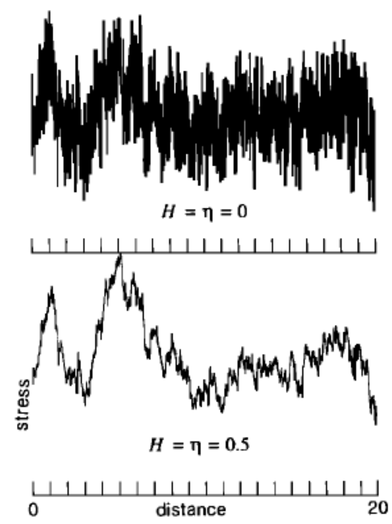
- Apply and extend ideas of *Andrews (1980, 1981)* to earthquake rupture modeling
- Linking spectral decay of far-field displacement to fractal dimension & *b*-values combining many elementary sources (subevents) (*Frankel, 1991*)

Hierarchial Patch Distribution



$$\frac{dN}{d(\ln R_{\mathrm{sub}})} = p\left(\frac{R_{\mathrm{sub}}}{R_{\mathrm{main}}}\right)^{-D}$$

Self-similar Random Stress



If D = 2 and stress-drop is constant (η = 0) → ω^{-2}-decay

$$\Omega(f) \propto \frac{M_0}{1 + (f/f_0)^{\gamma}}$$

$$M_0 \propto R^{3+\eta}$$

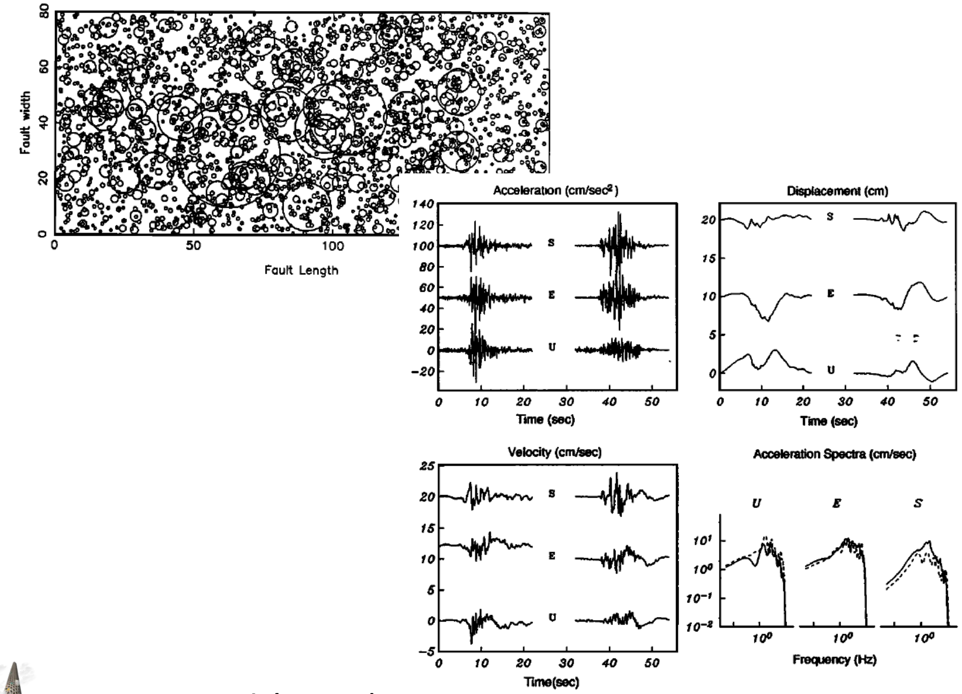
$$\gamma = 3 + \eta - D/2.$$

Frankel (1991)

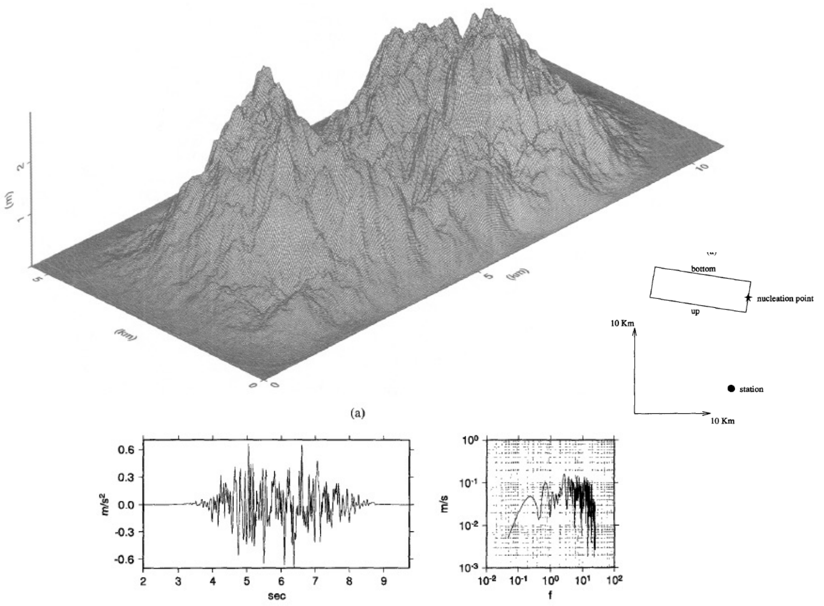
Early Developments

Composite Sources

- Apply and extend ideas of *Andrews (1980, 1981)* to earthquake rupture modeling
- Linking spectral decay of far-field displacement to fractal dimension & *b*-values combining <u>many elementary sources</u> (subevents) (*Frankel, 1991*)
  - Composite source model (*Zeng et al, 1994; Anderson, 2015*)
  - ***k***-square rupture model (*Herrero and Bernard, 1994*)



Zeng et al (1994)



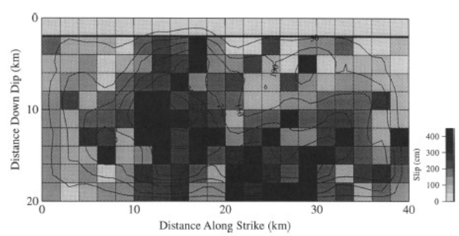
Herrero & Bernard (1994)

Early Developments

From theoretical models to „observation-based“ ones …

- Increasing number of source inversion studies provide “rupture models”
- *Somerville et al (1999)* characterize earthquake slip for 15 such rupture models
  - Analyze 2D slip maps; count “asperities” (large slip regions); statistics, scaling laws
  - Compare with ***k***-square model

Slip-model processing



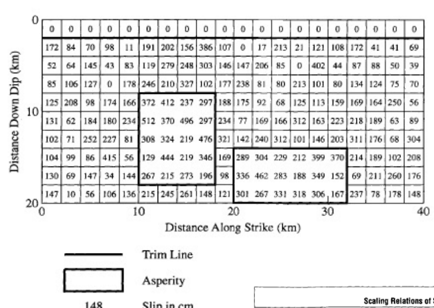
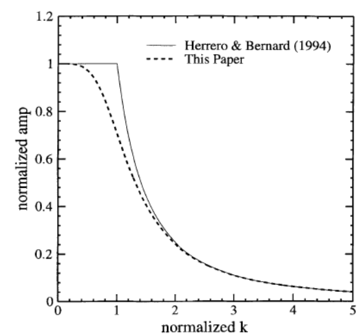


TABLE 7.
Scaling Relations of Slip Models Assuming Self-Similarity

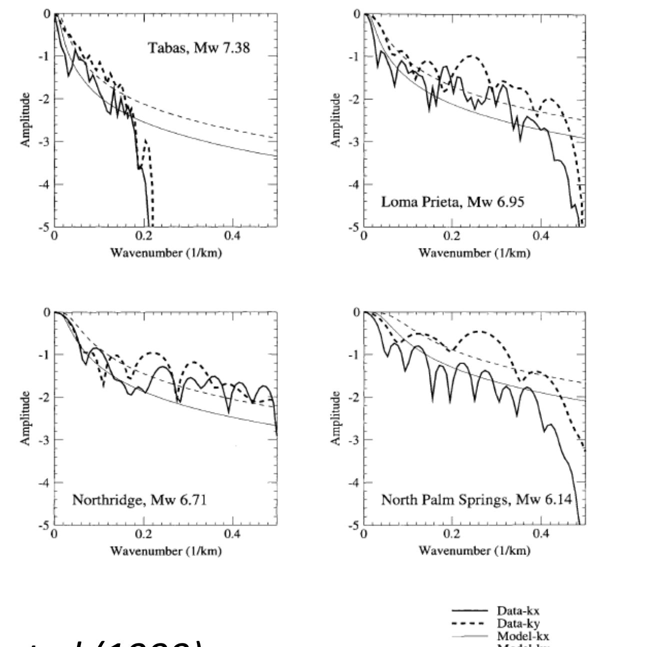
Rupture Area vs. Seismic Moment	$A = 2.23 \times 10^{-15} \times M_o^{2/3}$
Average Slip vs. Seismic Moment	$D = 1.56 \times 10^{-7} \times M_o^{1/3}$
Combined Area of Asperities vs. Seismic Moment	$A_a = 5.00 \times 10^{-16} \times M_o^{2/3}$
Area of Largest Asperity vs. Seismic Moment	$A_l\,(km^2) = 3.64 \times 10^{-16} \times M_o^{2/3}$
Radius of Largest Asperity vs. Seismic Moment	$r_l\,(km) = 1.08 \times 10^{-8} \times M_o^{1/3}$
Average Number of Asperities	2.6
Area of Fault Covered by Asperities	0.22
Average Asperity Slip Contrast	2.01
Hypocentral Distance to Center of Closest Asperity vs. Moment	$R_A = 1.35 \times 10^{-8} \times Mo^{1/3}$
Slip Duration vs. Seismic Moment	$T_R = 2.03 \times 10^{-9} \times Mo^{1/3}$
Corner Spatial Wavenumber Along Strike (km^{-1})	$\log KC_x = 1.72 - 0.5\,M$
Corner Spatial Wavenumber Down Dip (km^{-1})	$\log KC_y = 1.93 - 0.5\,M$

Slip spectrum



$$\mathrm{amp}(kx, ky) = \frac{1}{\sqrt{1+\left(\left(\frac{kx}{KC_x}\right)^2+\left(\frac{ky}{KC_y}\right)^2\right)^{2.0}}}$$

Slip-spectrum fitting



Sommerville et al (1999)

Slip Heterogeneity

From theoretical models to „observation-based" ones …

- Quantify slip heterogeneity from compilations of rupture models
- Slip heterogeneity as spatial random field *(Mai and Beroza, 2002; Lavallee et al, 2006)*
- Auto-correlation function *C(**r**)* in space; power-spectral density *P(**k**)* in Fourier domain

Random-field models

	$C(r)$	$P(k)$
GS	e^{-r^2}	$\frac{a_x a_z}{2} e^{-\frac{1}{4}k^2}$
EX	e^{-r}	$\frac{a_x a_z}{(1+k^2)^{\frac{3}{2}}}$
VK	$\frac{G_H(r)}{G_H(0)}$	$\frac{a_x a_z}{(1+k^2)^{H+1}}$.

$$P(k) \propto \frac{1}{k^{\beta+1}} \propto \frac{1}{\left(k_x^2+k_z^2\right)^{4-D}}$$

$$G_H(r) = r^H K_H(r)$$

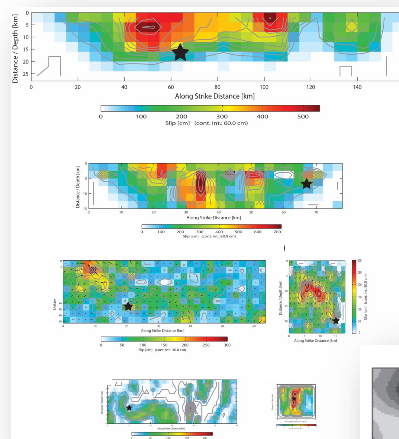
$$r = \sqrt{\frac{x^2}{a_x^2} + \frac{z^2}{a_z^2}}$$

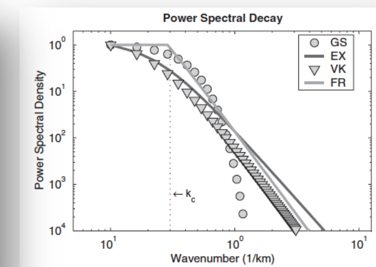
$$k = \sqrt{a_x^2 k_x^2 + a_z^2 k_z^2}$$

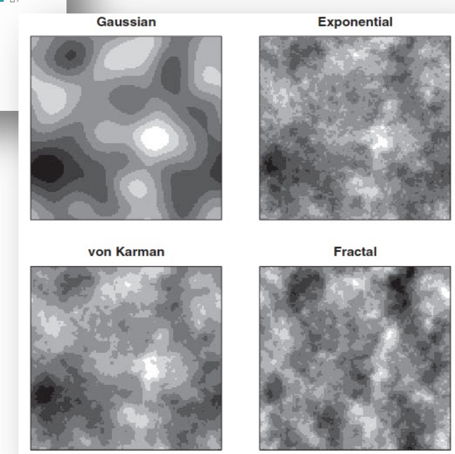
$$D = E + 1 - H$$

- a_x , a_z : correlation lengths
- H: Hurst number ($H = [0; 1]$)
- K_H: modified Bessel function 2nd kind, order H
- k_x, k_z: wavenumber in horizontal and vertical direction
- fractal: "straight-line" in power-spectral decay,
- fractal dimension D (E: Euclidian norm)

Slip in space and spectral domain







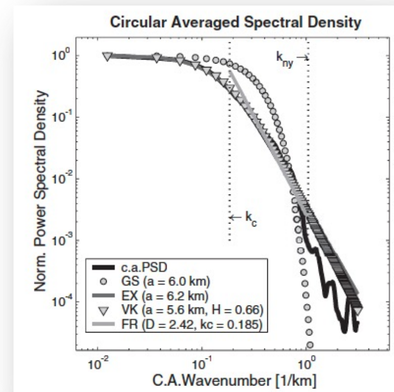
Mai and Beroza (2002)

Properties of slip heterogeneity

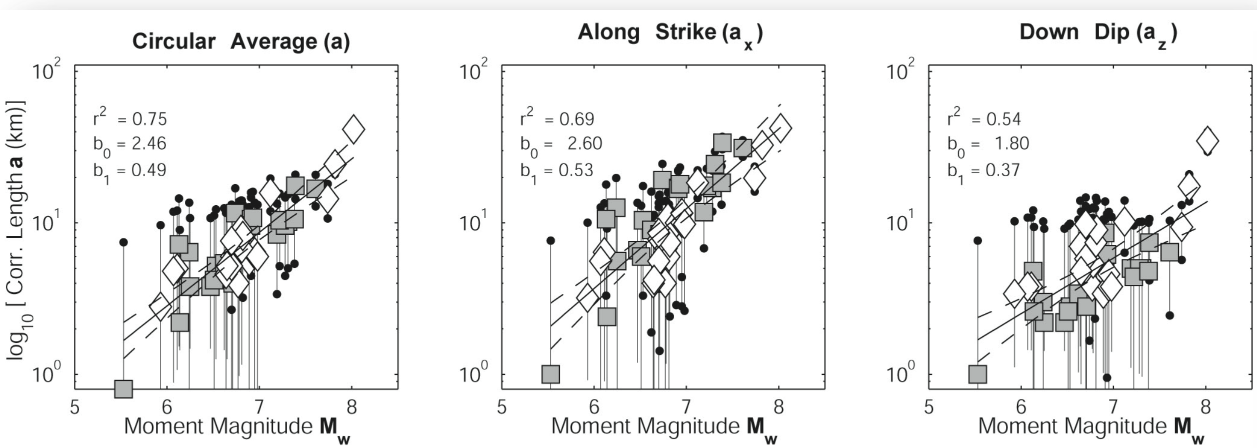
$$P(k) = \frac{4\pi H}{K_0(0)} \cdot \frac{a_x a_z}{\left(1+k^2\right)^{H+1}}$$

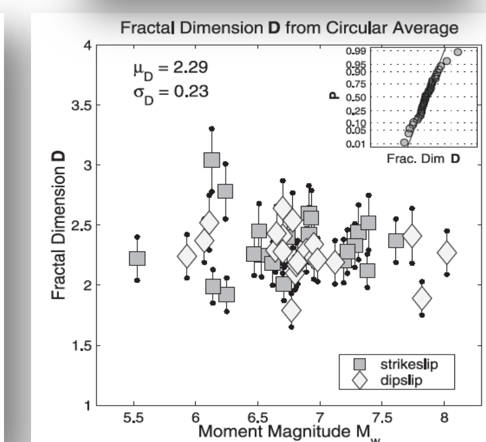
- Patterns emerging from an analysis of many slip models
  - van Karman ACF best replicates the *P(**k**)* of slip distributions
  - Correlation lengths depend on magnitude
  - Hurst exponent *H ~ 0.7*

    → similar to H found for exposed slip surfaces



Scaling of Correlation lengths and Hurst exponent





$$a_x \approx 2.0 + \tfrac{1}{3} L_{eff}; \qquad \log(a_x) \approx -2.5 + \tfrac{1}{2} M_w$$

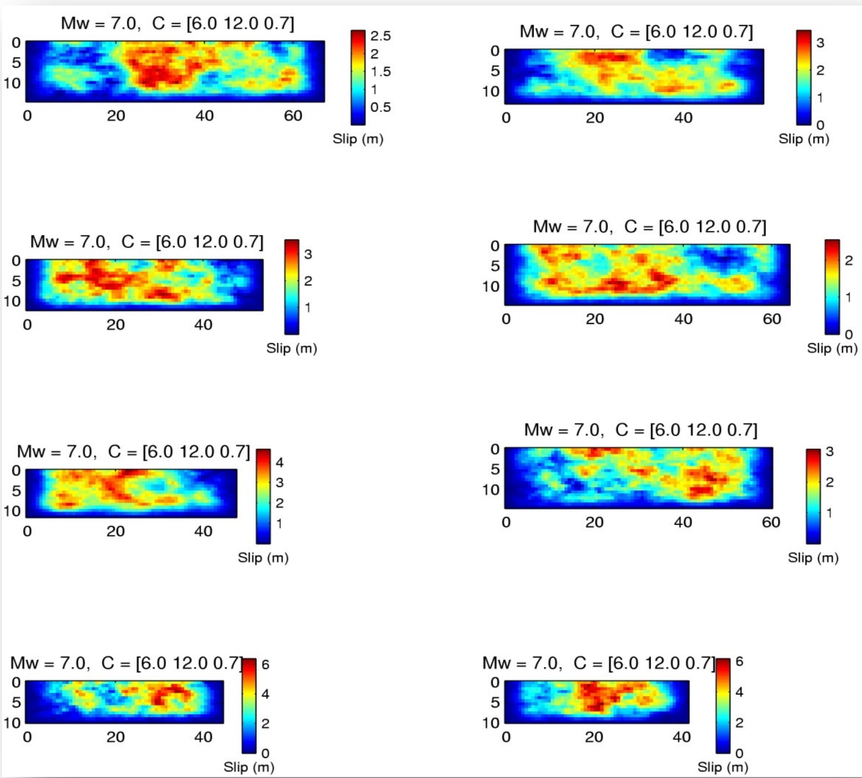
$$a_z \approx 1.0 + \tfrac{1}{3} W_{eff}; \qquad \log(a_z) \approx -1.5 + \tfrac{1}{3} M_w$$

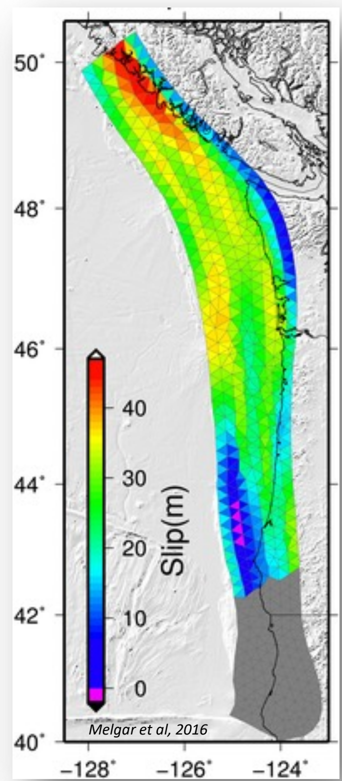
Mai and Beroza (2002)

$$D \sim 2.3 \rightarrow H \sim 0.7$$

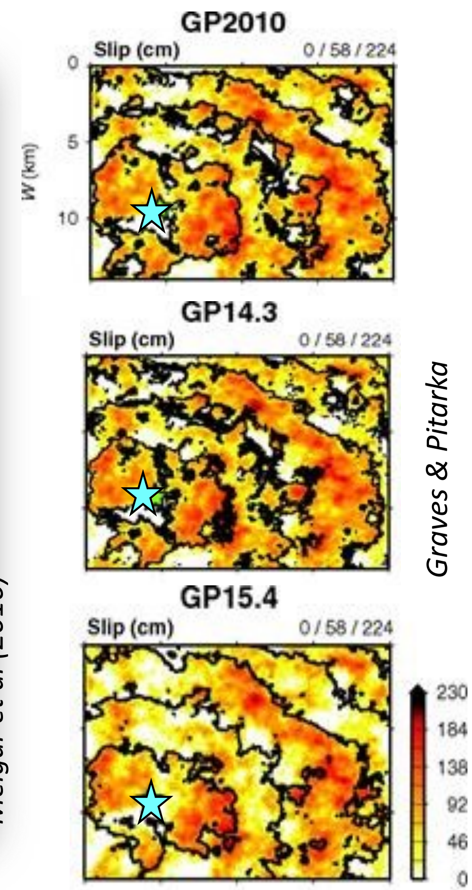
Simulation slip heterogeneity

- For kinematic rupture modeling, tsunami simulations, initial stress for rupture dynamics
  - Assume fault-plane dimensions **or** calculate from source-scaling relations
  - Simulate "random" but realistic heterogeneous slip distribution
  - FFT-methods; geostatistical-methods; Karhunen-Loève expansion (*LeVeque et al, 2016*) …





Melgar et al (2016)

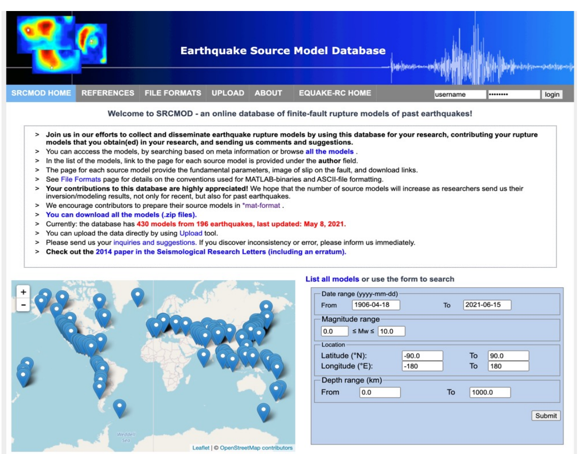


Graves & Pitarka

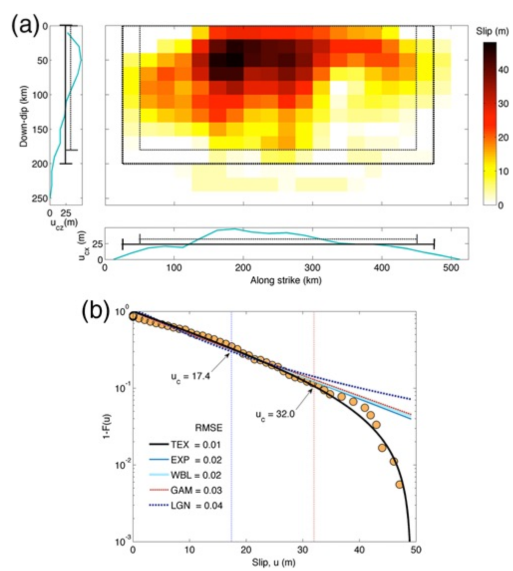
Probability distribution of slip heterogeneity

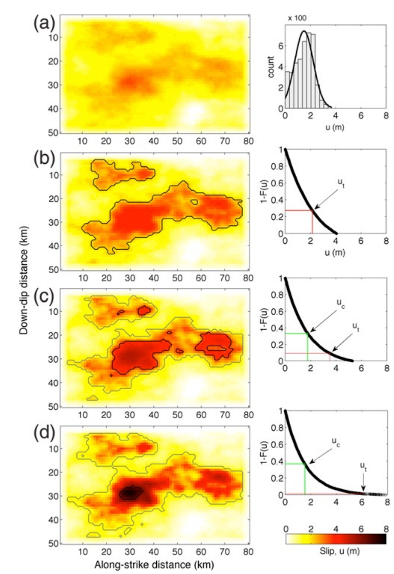
- Several models have been proposed
  - modified log-normal *(Gusev, 2011)*
  - Non-Gaussian Levy law (*e.g., Lavallee et al, 2006)*
  - Statistical properties of slip govern ground motions *(e.g., Song and Dalguer, 2013)*
  - Testing probability distributions using SRCMOD database (*Thingbaijam and Mai, 2016*)

Evidence for truncated exponential distribution



http://equake-rc.info/srcmod





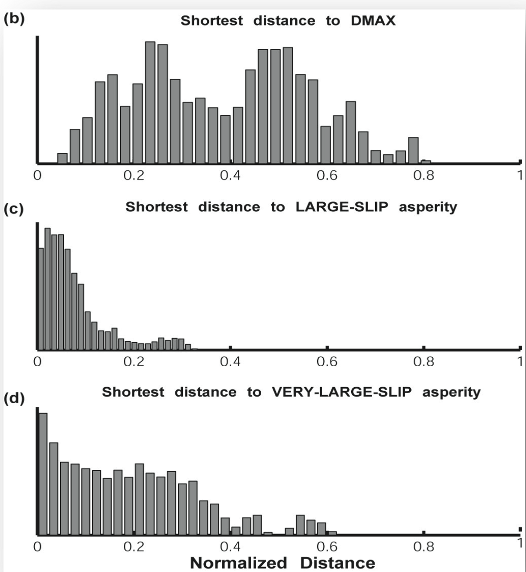
Thingbaijam and Mai, 2016

Rupture-Modeling Ingredients

Where does rupture start?

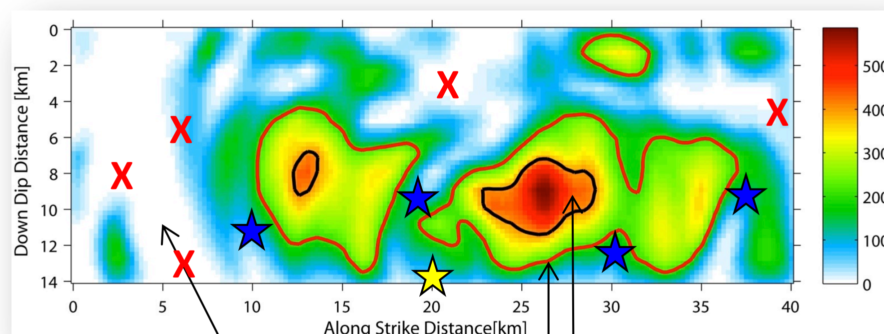
- **Hypocenter location** – not random, but related to slip (stress) on the fault
- from hypocenter locations in finite-source rupture models
  - ruptures starts on, or close to, a large-slip region (“asperity”)
  - consistent with energy-budget consideration of rupture dynamics
  - ruptures may nucleate at any level of slip

distance hypocenter to “asperity”



Hypocenter distance w.r.t. zone of large (very-large) slip

“asperity” definition



$$D < \frac{1}{3} D_{max}$$

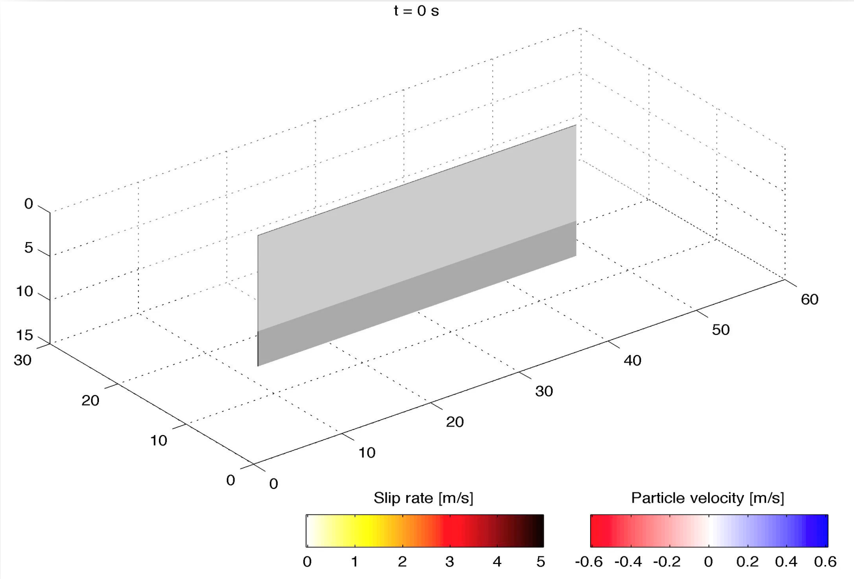
$$\frac{1}{3} D_{max} \leq D < \frac{2}{3} D_{max}$$

$$D \geq \frac{2}{3} D_{max}$$

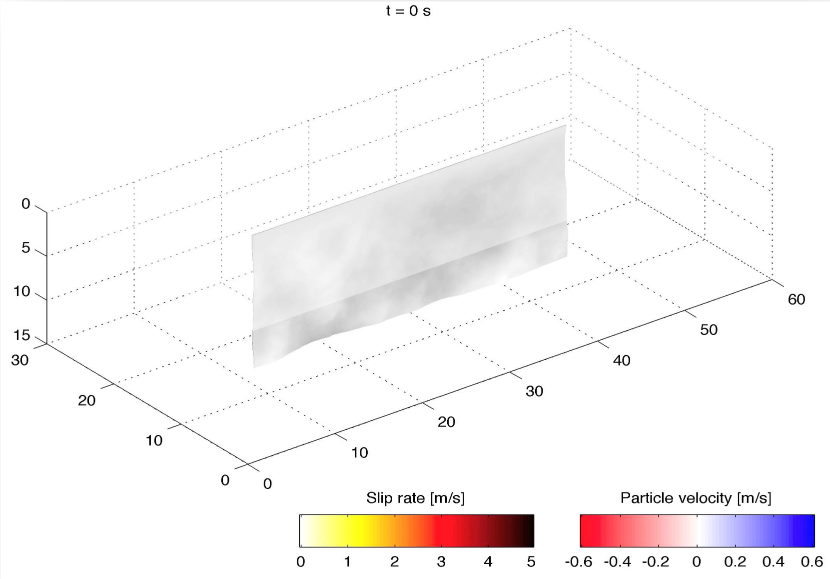
A brief example from rupture dynamics

- **Simple strike-slip fault, M ~7;**
- **Case A – planar fault; Case B – fractally rough fault surface**
  - Enforced nucleation at pre-selected hypocenter
  - Vastly different degrees of complexity in rupture and radiation

Planar Fault



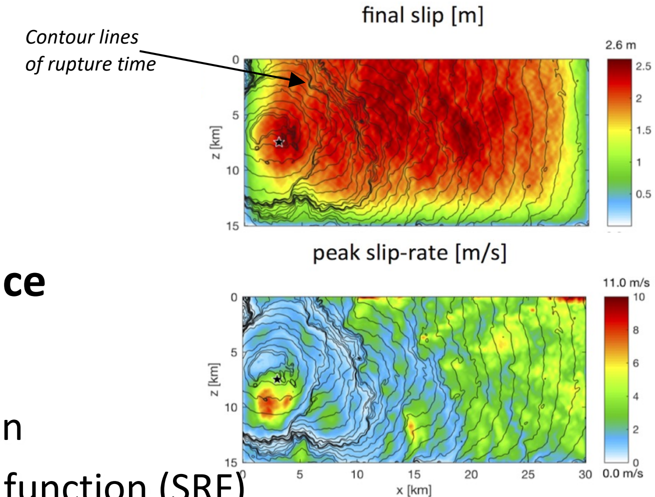
Fractally Rough Fault



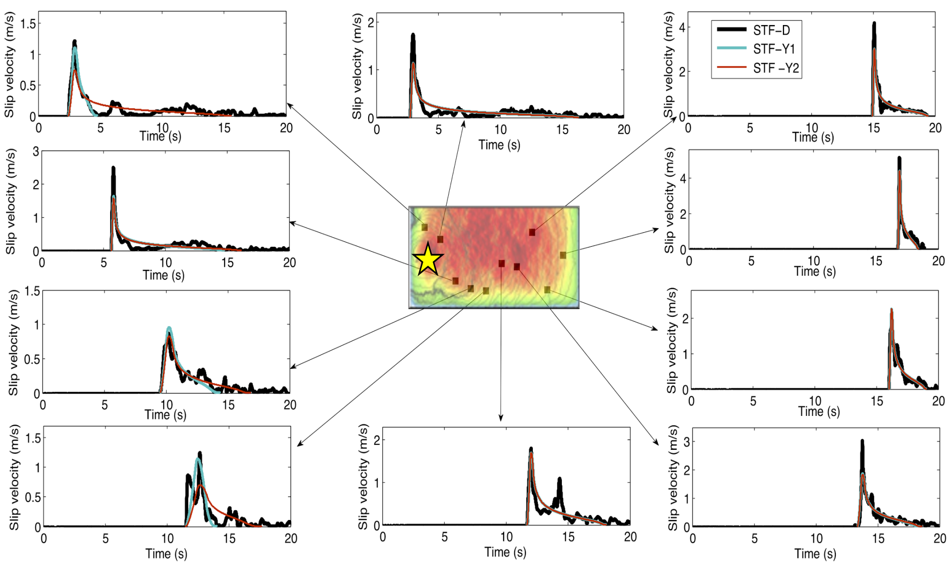
Rupture-Modeling Ingredients

A brief example from rupture dynamics

- **Simple strike-slip fault, M ~7;**
- **Case A – planar fault; Case B - fractally rough fault surface**
  - Enforced nucleation at pre-selected hypocenter
  - Vastly different degrees of complexity in rupture and radiation
  - Encapsulated in temporal rupture evolution → local slip-rate function (SRF)

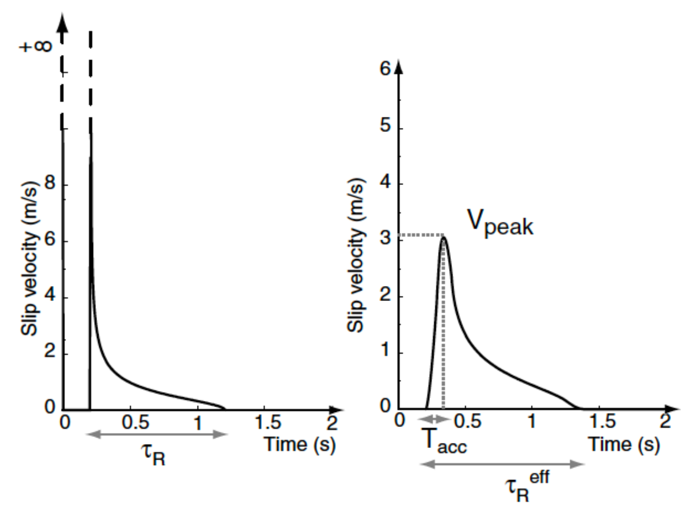


Stark variation in on-fault SRF's



Mai et al, 2018

Yoffe Function as SRF parameterization

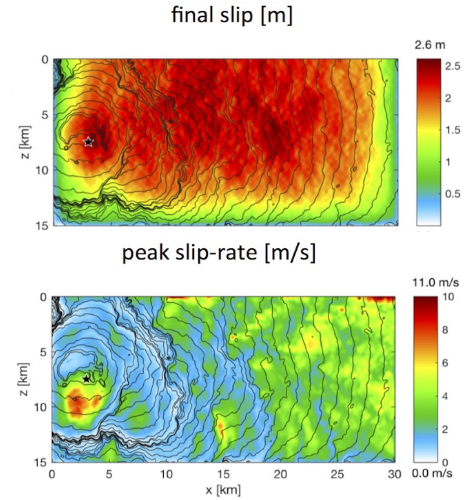


$$V_{peak} = 1.04 \frac{D_{max}}{(T_{acc})^{0.54}(\tau_R)^{0.47}} \simeq C \frac{D_{max}}{\sqrt{T_{acc}}\sqrt{\tau_R}}$$

Tinti et al., 2005

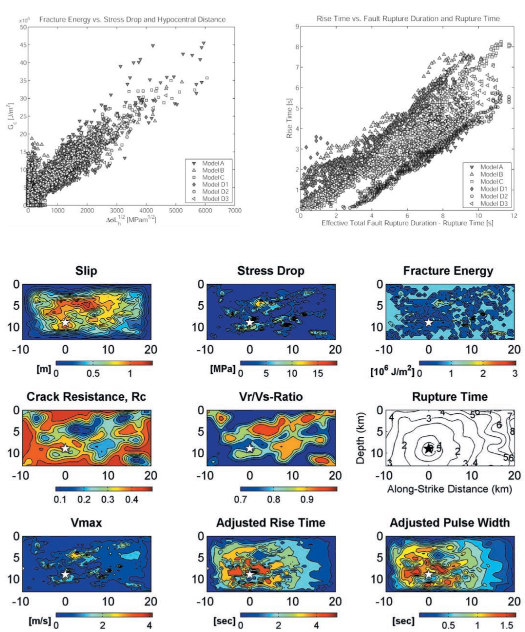
Rupture-Modeling Ingredients

Correlations in temporal rupture parameters …

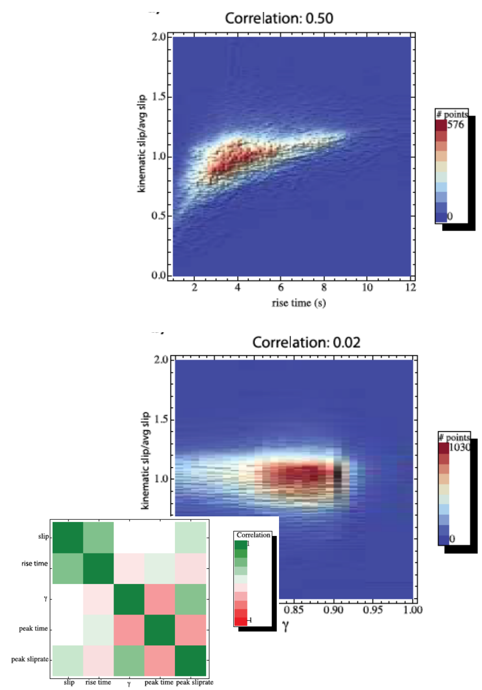


- **From 'databases' of tailor-made dynamic rupture models**
  - Variations in rupture velocity correlated with slip? With stress?
  - Variations in local rise time correlated on slip? Rupture speed?
  - How to constrain variations & limits in peak slip-rate (V_{max})

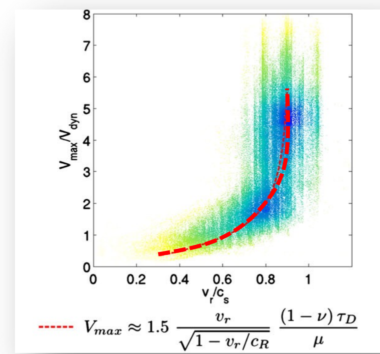
From 'correlation analyses' to kinematic rupture-model generators



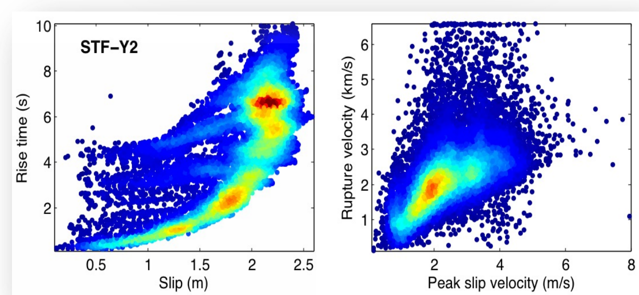
Guatteri et al, 2004



Schmedes et al, 2010



Gabriel et al, 2013



Mai et al, 2018

Correlations in temporal rupture parameters …

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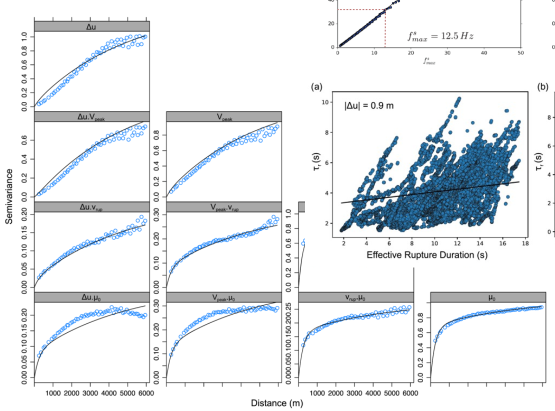
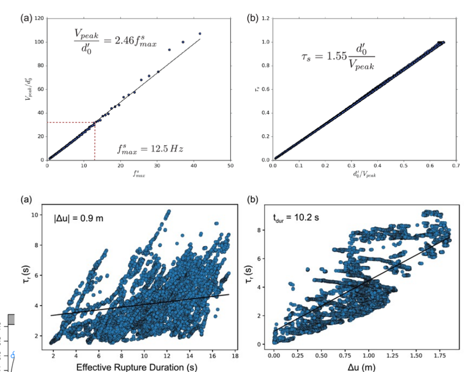
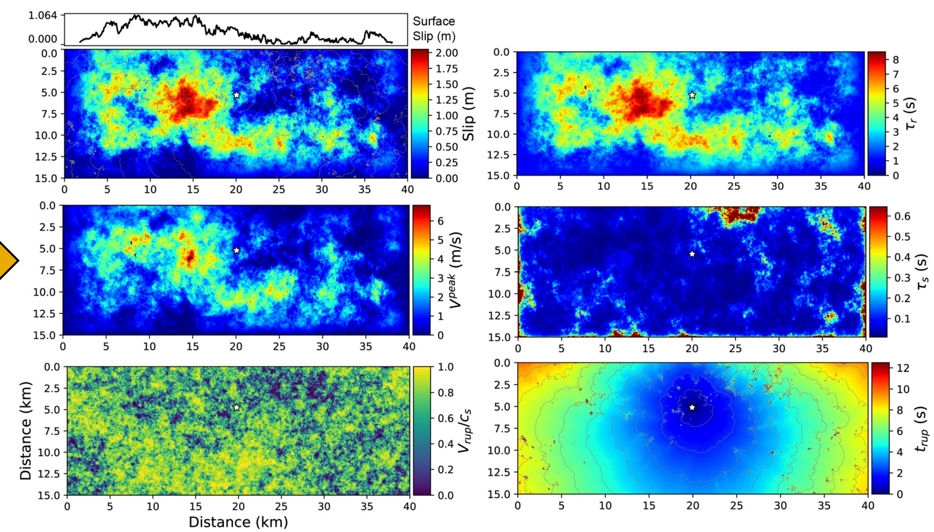
From 'correlation analyses' to kinematic rupture-model generators

- Several approaches, from using scaling laws to advanced geostatistics *(Guatteri et al, 2004; Schmedes et al, 2010; Graves and Pitarka, 2010, 2014, 2016 …; Song et al, 2013;* ***Savran and Olsen, 2020****)*





KRG



Savran & Olsen, 2020

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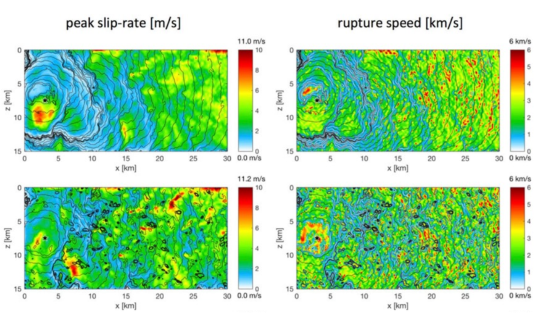
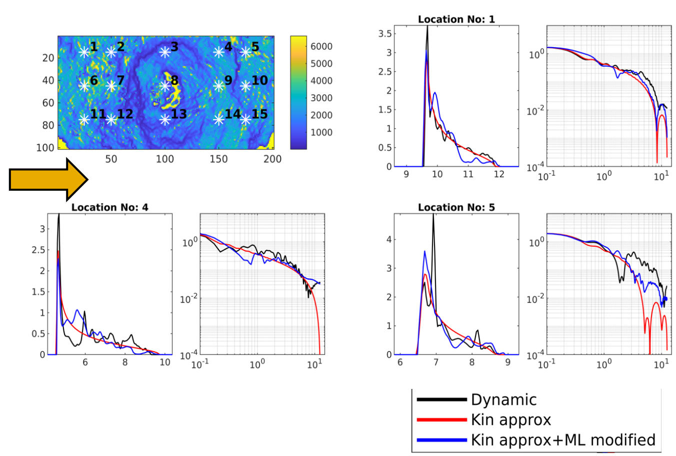
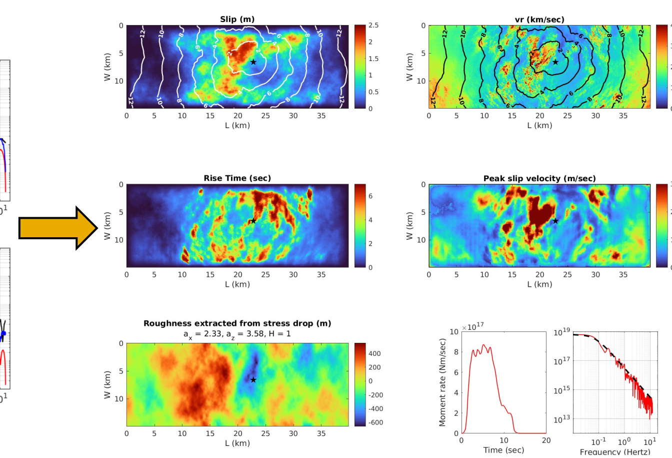
- We currently develop an **ML-based approach to train a KRG from dynamic rupture models**



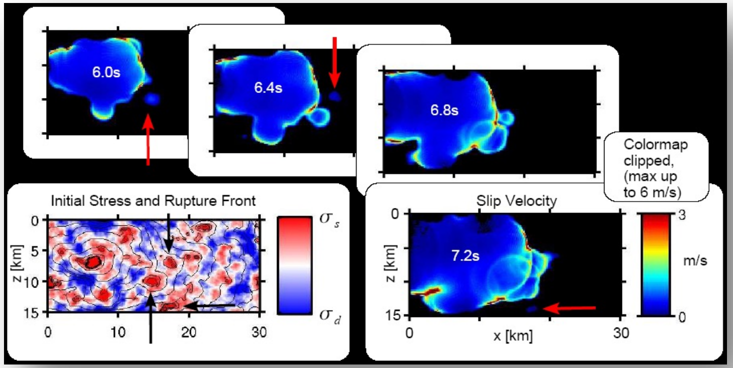




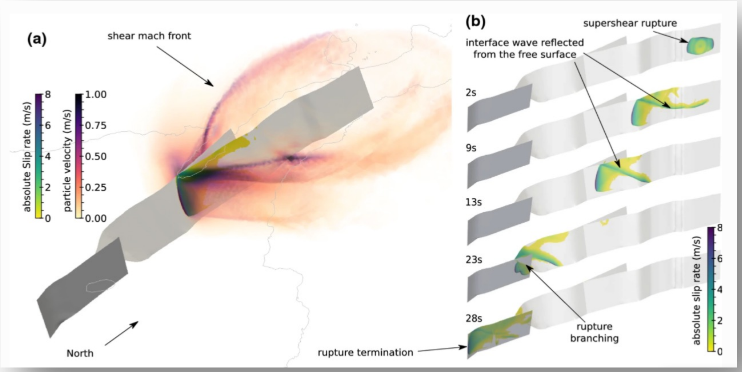
Further constraints: observations & simulations

Earthquakes keep surprising us ….

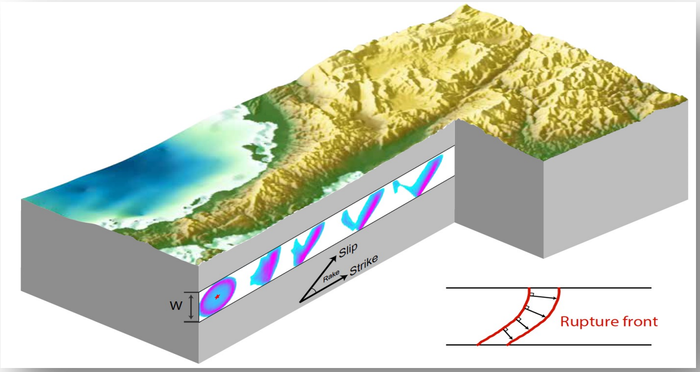
- **Even on quasi-planar faults, small-scale variations (in stress, roughness) lead to intricate rupture properties:**
  - Dynamic triggering, multiple rupture fronts, super-shear rupture-speed episodes



Ripperger et al, 2007



Ulrich et al, 2019



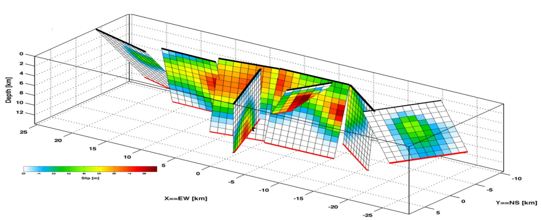
Weng and Ampuero, 2020

Further constraints: observations & simulations

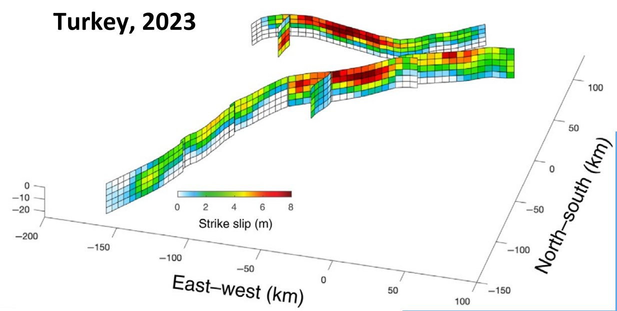
Earthquakes keep surprising us ….

- **Even on quasi-planar faults, small-scale variations (in stress, roughness) lead to intricate rupture properties:**
  - Dynamic triggering, multiple rupture fronts, super-shear rupture-speed episodes
- **Large-scale fault segmentation profoundly affects rupture and radiation process**
  - Depends on nucleation point; currently nowhere accounted for in kinematic rupture generators

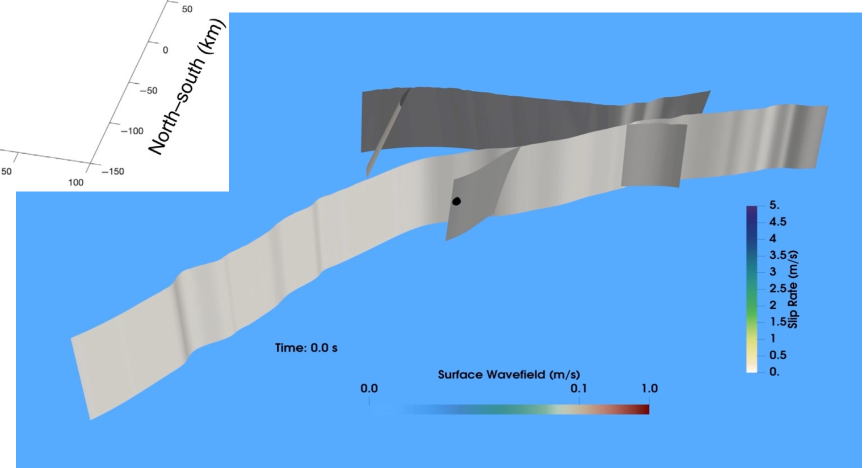
Darfield, NZL (2010)



Turkey, 2023



Mai et al, 2023



Lit et al, in preparation

Open Questions

A few final thoughts … but no conclusions …

- Current rupture-model generators are all based on essentially a single class of models
  ⟹ near-vertical quasi-planar strike-slip earthquakes, M ~ 6.5 – 7.2

- We tend to avoid dealing with super-shear rupture velocity
- Dynamic triggering & multiple rupture fronts are currently not considered in KRG's

- Multi-scale geometric fault complexity (roughness & segmentation) to be included
- Other variations in fault-plane geometry to be added: listricity, variations in along-strike dip, etc ….

Thank You

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