Frame element with total sectional interaction for nonlinear analysis of reinforced concrete structures

Mauro Poliotti & Jesús-Miguel Bairán

Department of Civil and Environmental Engineering

Universitat Politècnica de Catalunya

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

Department of Civil and Environmental Engineering

Introduction

Structural elements where one dimension is much larger than the others can be assimilated as linear elements. The overall element behavior can be obtained by integration of the sectional response on each cross section along the elements axis.

The need of reproducing different failure modes, as shear or torsional failures, as well as capturing the role of transverse reinforcements, became of special interest the last years with the assessment and reinforcement of existing structures and with the increasing demand of nonlinear analysis by the new codes and provisions.

In this work a sectional model capable of tracing the response of each cross-section under general loading is presented. Each concrete material point of the cross-section is simulated using a 3D plastic-damage model with an evolutive dilatancy parameter. The validation of both models is done by reproducing the experimental behavior of reinforced concrete elements under different loading conditions.

Framework and models

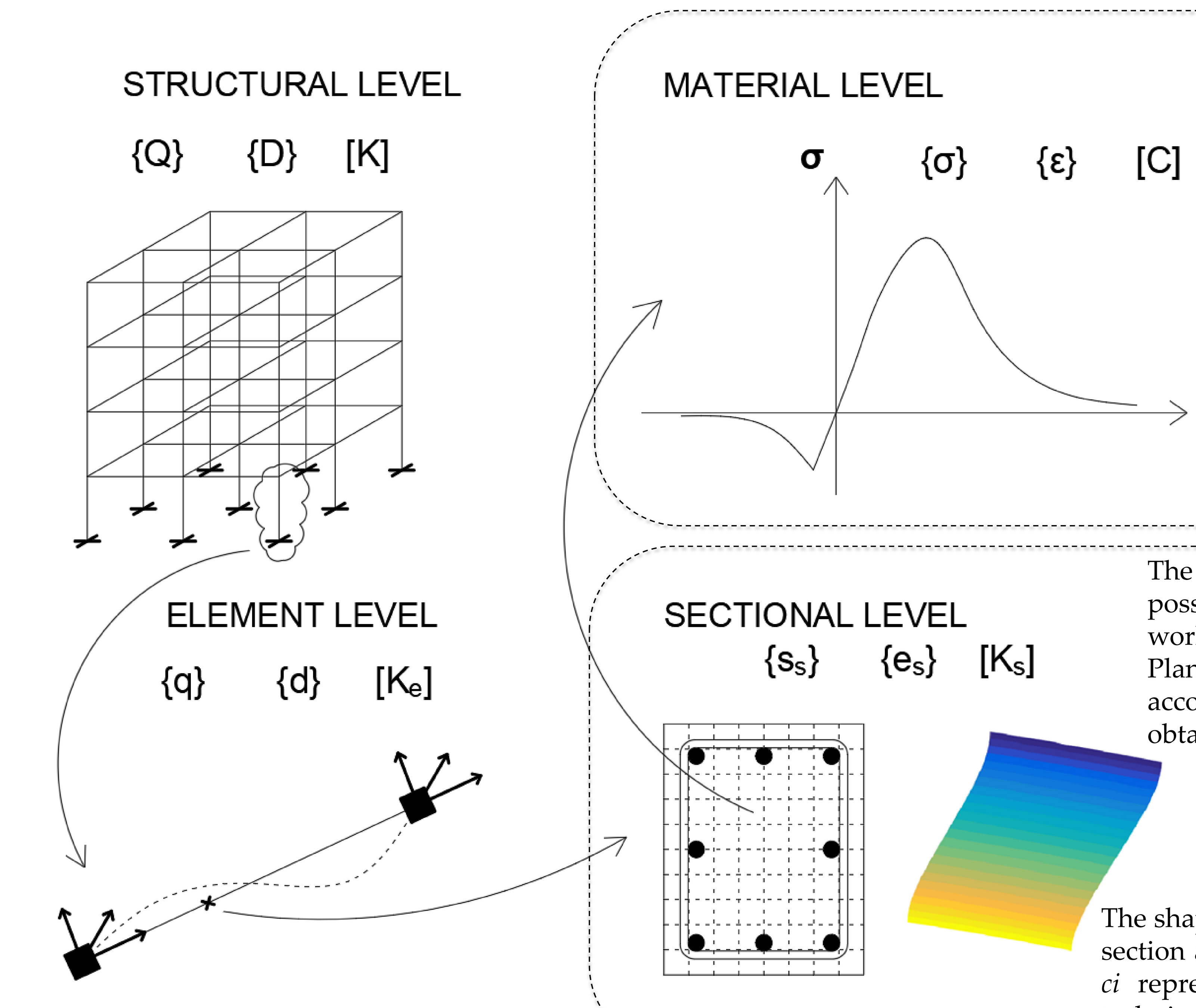

Fig. 1: General Framework

At the material level a 3D plastic-damage model for concrete developed by Poliotti and Bairán [1] is used. There, the constitutive model by Lee and Fenves [2] is enhanced by means of an **evolutive dilatancy** parameter.

$$\bar{\boldsymbol{\sigma}} = \boldsymbol{E}_0 : (\varepsilon - \varepsilon^p) \quad ; \quad \boldsymbol{\sigma} = [1 - D(\boldsymbol{\kappa}, \bar{\boldsymbol{\sigma}})]\, \bar{\boldsymbol{\sigma}} \quad ; \quad \dot{\varepsilon}^p = \dot{\lambda} \frac{\partial \Phi}{\partial \bar{\boldsymbol{\sigma}}} (\bar{\boldsymbol{\sigma}}) \quad ; \quad \dot{\boldsymbol{\kappa}} = \dot{\lambda} \boldsymbol{H}(\bar{\boldsymbol{\sigma}}, \boldsymbol{\kappa}) \tag{1}$$

$$\dot{\lambda} \geq 0 \; ; \; \dot{\lambda} F(\bar{\boldsymbol{\sigma}}, \boldsymbol{\kappa}) = 0 \; ; \; F(\bar{\boldsymbol{\sigma}}, \boldsymbol{\kappa}) \leq 0 \tag{2}$$

$$F(\bar{\boldsymbol{\sigma}}, \boldsymbol{\kappa}) = \frac{1}{1-\alpha} \left[\alpha \bar{I}_1 + \sqrt{3 \bar{J}_2} + \beta(\boldsymbol{\kappa}) \langle \hat{\bar{\sigma}}_{max} \rangle - \gamma \langle -\hat{\bar{\sigma}}_{max} \rangle \right] - c_c(\boldsymbol{\kappa}) \leq 0 \tag{3}$$

$$\Phi(\bar{\boldsymbol{\sigma}}) = \sqrt{(\varepsilon_1 \alpha_p f_{t0})^2 + 2 \bar{J}_2} + \alpha_p \bar{I}_1 \tag{4}$$

$$\boxed{\alpha_p = \alpha_p(\boldsymbol{\kappa}, \bar{\boldsymbol{\sigma}}) = \tan \psi(\boldsymbol{\kappa}, \bar{\boldsymbol{\sigma}})} \tag{5}$$

The adequate control of the dilatant behavior affects the strength and ductility of confined concrete as well as the shear strength and softening.

The **b-spline** sectional model with **total interaction** between the six possible internal forces developed in Poliotti and Bairán [3] is used in this work. The displacement field of a 3D beam is obtained as a sum of the Plane-Section displacement field and a complementary field that accounts for warping and distortion. This complementary field is obtained considering explicitly the 3D inter-fiber equilibrium.

$$\boldsymbol{u} = \boldsymbol{u}_{ps} + \boldsymbol{u}_w = \begin{Bmatrix} u_{ps} \\ v_{ps} \\ w_{ps} \end{Bmatrix} + \begin{Bmatrix} u_w \\ v_w \\ w_w \end{Bmatrix} \qquad \boldsymbol{u}_w = \begin{Bmatrix} u_w \\ v_w \\ w_w \end{Bmatrix} = \begin{Bmatrix} \sum a_i F_i(y,z) \\ \sum b_i F_i(y,z) \\ \sum c_i F_i(y,z) \end{Bmatrix} \tag{6}$$

$$\iiint_\Omega div(\boldsymbol{\sigma})\; \delta \boldsymbol{u}\; d\Omega = 0 \qquad \int_0^L \left(\iint_A div(\boldsymbol{\sigma})\; \delta \boldsymbol{u}\; dA \right) dx = \int_0^L R(x)\, dx = 0 \tag{7}$$

The shape functions *Fi(y,z)* in Eq.(6) are B-splines functions defined in the cross-section and constructed from a interpolation grid. The weight factors *ai, bi* and *ci* represent the additional degrees of freedom of the problem. This numerical technique reduces the computational with respect to a finite element solution.

Pure Shear

A reinforced concrete section tested by Kani [4] under pure shear loading, is simulated with the proposed model. The analyzed cross-section is situated on the inflection point of a beam in order to reproduce the pure shear load.

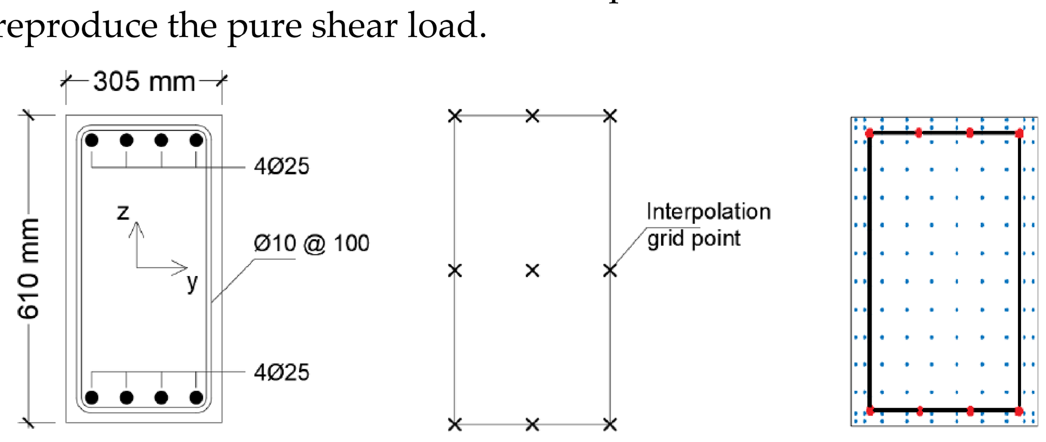

Fig. 2: Specimen: Cross-section, Interpolation grid and Fiber distribution

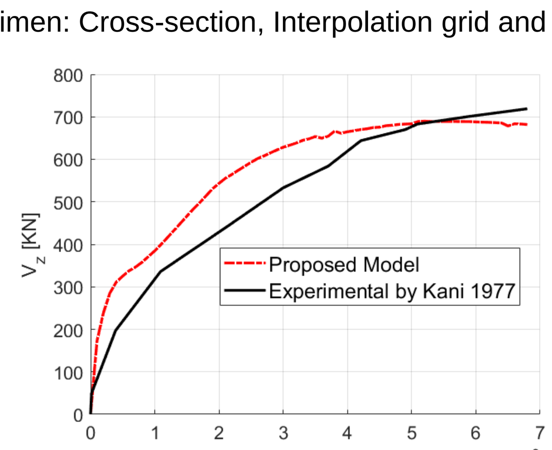

Fig. 3: Shear force-strain curve. Experimental results by [4].

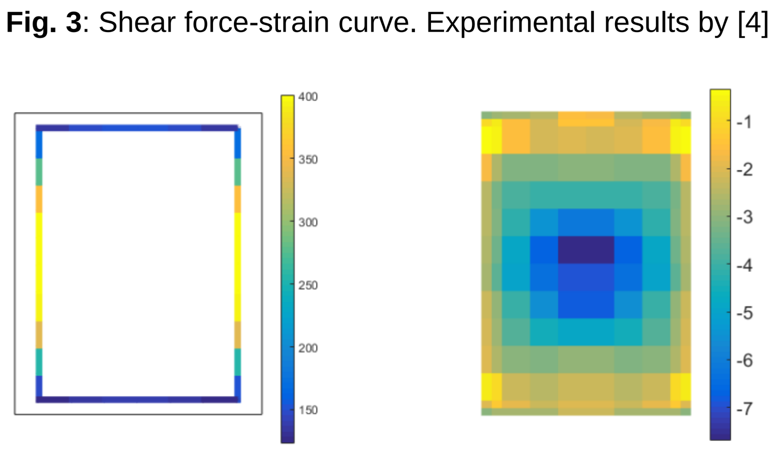

Fig. 4: Section state at maximum load

Coupled Bending and Torsion

A reinforced concrete section under the combined action of torsion and bending moments tested by Onsongo [5] is simulated. The analysis is performed controlling the torsional curvature and constraining the internal force vector to represent the bending to torsion ratio of the selected specimen which was R = T/M = 0.261.

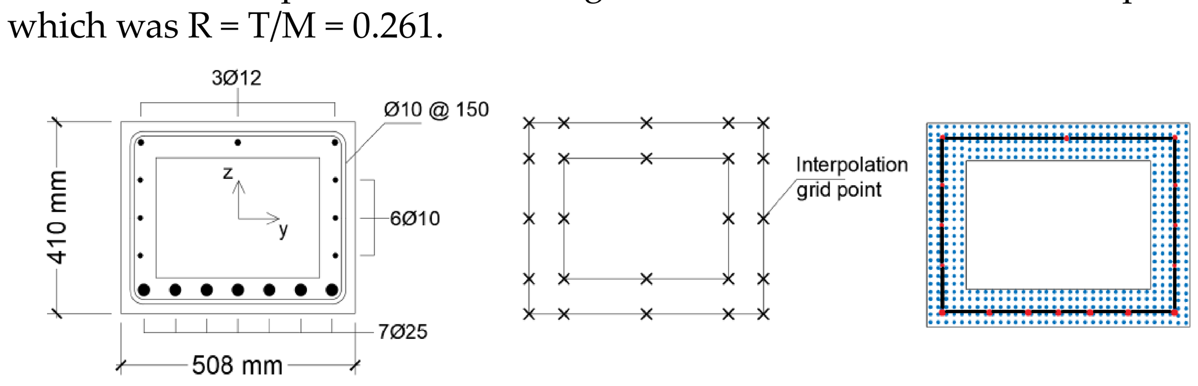

Fig. 5: Specimen: Cross-section, Interpolation grid and Fiber distribution

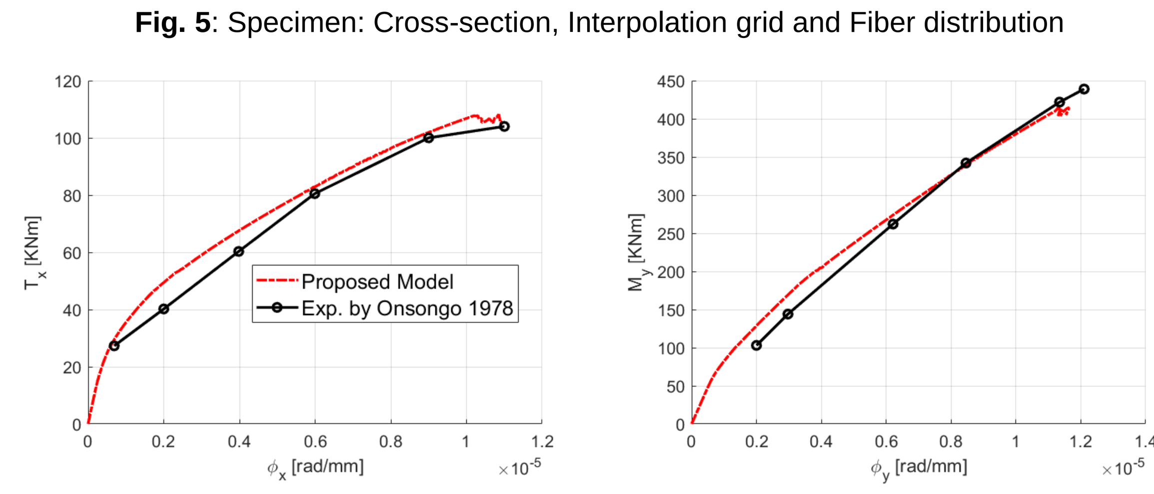

Fig. 6: Torsion-curvature and Moment-curvature curves. Experimental results by [5]

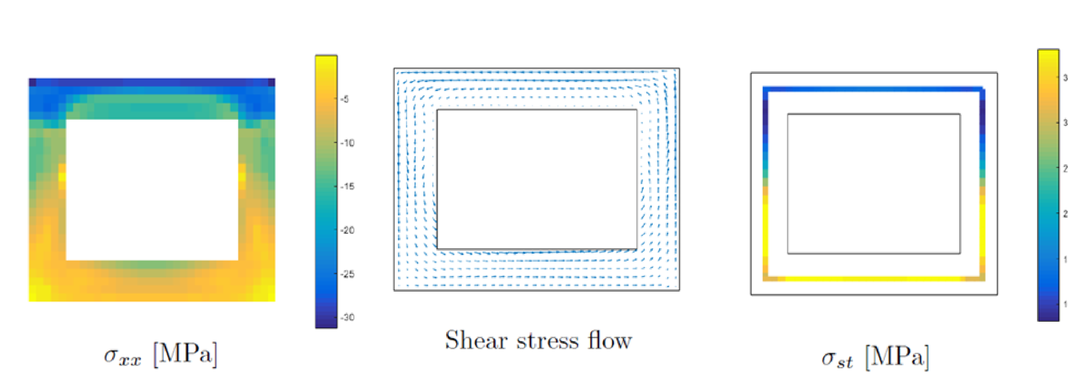

Fig. 7: Section state at ultimate load

Axial load with passive confinement

A circular section of a reinforced concrete column tested by Mander et al. [6] is simulated with the presented model. This specimen is the unit number 4 of a larger experimental campaign which was the basis of the well-known 1D constitutive model for confined concrete of Mander.

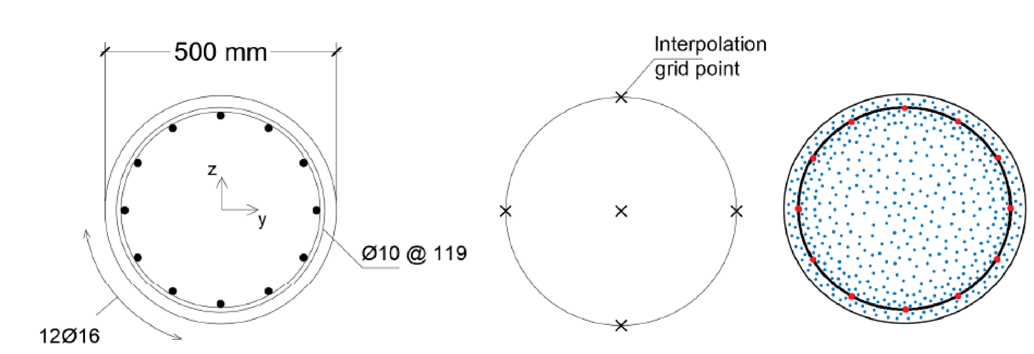

Fig. 8: Specimen: Cross-section, Interpolation grid and Fiber distribution

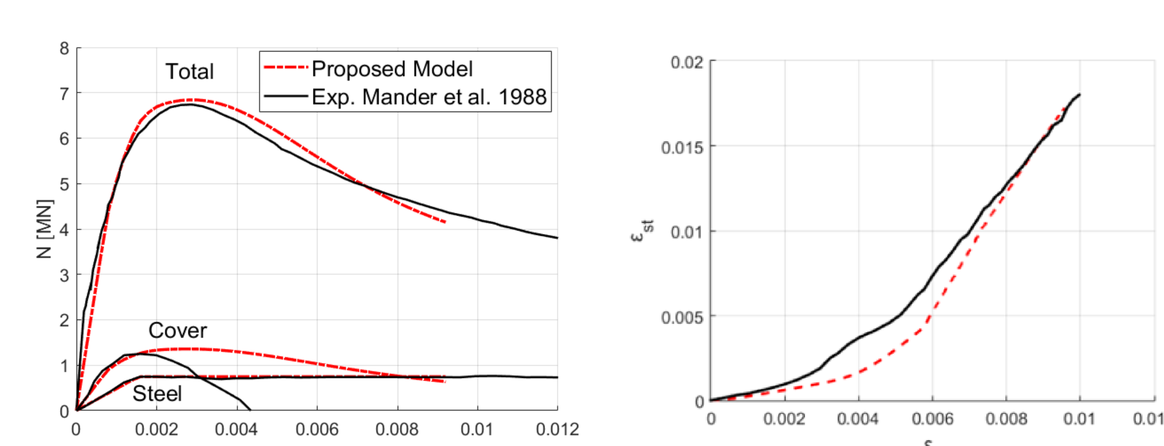

Fig. 9: Axial force-strain and Sprial strains curves. Experimental results by [6]

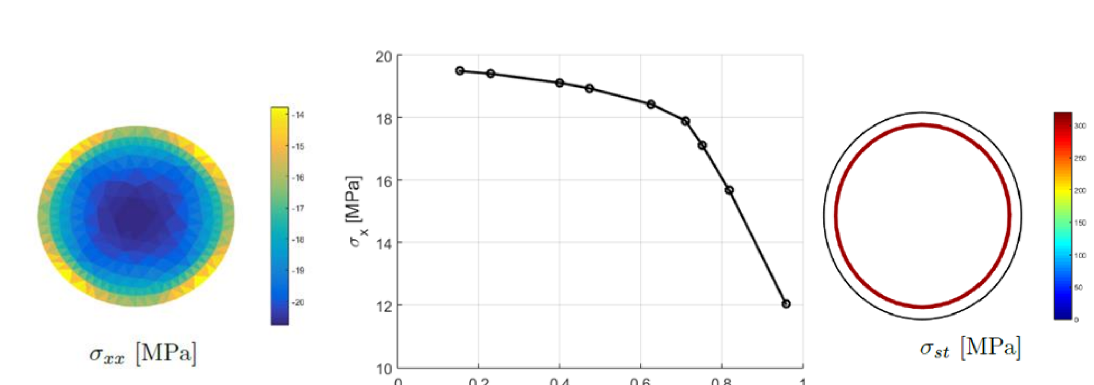

Fig. 10: Section state at ultimate load.

References

[1] M. Poliotti, J.-M. Bairán, A new concrete plastic-damage model with an evolutive dilatancy parameter, Engineering Structures 189 (2019) 541 - 549.

[2] J. Lee, G. L. Fenves, Plastic-damage model for cyclic loading of concrete structures, Journal of Engineering Mechanics 124 (1998) 892-900.

[3] M. Poliotti, J.-M. Bairán, B-spline sectional model for general 3D effects in reinforced concrete elements, Engineering Structures *In Press* (2020).

[4] M. Kani, An experimental investigation of reinforced and prestressed beams in shear, Ph.D. thesis, University of Toronto, Toronto, 1977.

[5] W. Onsongo, The diagonal compression field theory for reinforced concrete beams subject to combined torsion, flexure and axial load, Ph.D. thesis, University of Toronto, Toronto, 1978.

[6] J. B. Mander, M. J. N. Priestley, R. Park, Observed stress-strain behavior of confined concrete, Journal of Structural Engineering 114 (1988) 1827-1849.

Conclusions

- The sectional model presented in this work constitutes and efficient framework capable of reproducing the nonlinear behavior of concrete elements under different loading states and reproducing complex failure modes, such as shear, torsion and coupled forces.

- The adequate control of the dilatant behavior of concrete is shown to be of paramount importance as it controls the volumetric expansion and, consequently, affects the strength and ductility of confined concrete as well as the shear strength and softening.

- The sectional model considers the distortion of the section. Then, it is able to capture the interaction of transverse reinforcements and the concrete mass. Confinement can be simulated in an objective manner for different confining materials, section shape and reinforcement arrangements.