A Nonlinear Kinetic Model for Multi-Stage Friction Pendulum Systems

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PEER Researchers' Workshop, August 7-8, 2018

Outline

Motivation

Modeling
- Normal Forces
- Friction Forces
- Contact Forces

Results and discussion
- Uni-directional Motions
- Bi-directional Motions

Summary

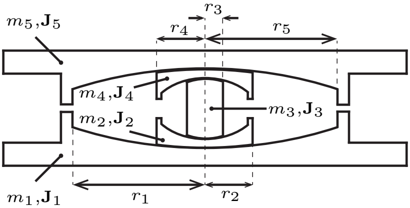
Multi-stage friction pendulum systems: Modeling

- Plasticity analogs
- Planar kinetic models (scalar)

Multi-stage friction pendulum systems: Modeling

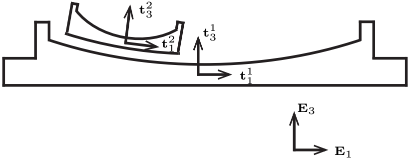
- Plasticity analogs
- Planar kinetic models (scalar)
- Missing fully kinetic models capable of bi-directional motion
  - Permits modeling of the full range of motion
  - Removes modeling guesswork
  - Permit identification of real failure mechanisms

Exact nonlinear kinematics



- Each element of the pendulum is modeled with its mass and full inertia tensor
- Kinematic state $\{\boldsymbol{r}, \boldsymbol{R}\}$ – 1-2-3 (relative) Euler angles
  - Easy enforcement of kinematic sliding constraints

Exact nonlinear kinematics



- Co-rotational frame
  $\boldsymbol{t}_i^1 = \boldsymbol{R}_1 \boldsymbol{E}_i,\ \boldsymbol{t}_i^2 = \boldsymbol{R}_2 \boldsymbol{t}_i^1,\ \ldots,\ \boldsymbol{t}_i^{n+1} = \boldsymbol{R}_{n+1} \boldsymbol{t}_i^n$
- Euler angle singularity occurs at $\theta = \pm\pi/2$, well away from real motions
- Facilitates expressions of bearing center of masses

Contact point computation

- Effective contact point is not the center point of the bearing surfaces, equilibrium needs to be satisfied
- A new set of 1-2 Euler angles is needed to define normal forces

$$\tilde{\mathbf{t}}_i^1 = \tilde{\mathbf{R}}_1 \mathbf{t}_i^1, \quad \tilde{\mathbf{t}}_i^2 = \tilde{\mathbf{R}}_2 \mathbf{t}_i^2, \quad \tilde{\mathbf{t}}_i^3 = \tilde{\mathbf{R}}_3 \mathbf{t}_i^3, \quad \tilde{\mathbf{t}}_i^4 = \tilde{\mathbf{R}}_4 \mathbf{t}_i^4$$

$$\tilde{\mathbf{R}}_1 = \mathbf{R}(\tilde{\psi}_1, \tilde{\theta}_1; \mathbf{t}_i^1), \quad \tilde{\mathbf{R}}_2 = \mathbf{R}(\tilde{\psi}_2, \tilde{\theta}_2; \mathbf{t}_i^2)$$
$$\tilde{\mathbf{R}}_3 = \mathbf{R}(\tilde{\psi}_3, \tilde{\theta}_3; \mathbf{t}_i^3), \quad \tilde{\mathbf{R}}_4 = \mathbf{R}(\tilde{\psi}_4, \tilde{\theta}_4; \mathbf{t}_i^4)$$

Frictional forces

- The frictional forces are easily resolved using the contact point 1-2 Euler angles
- Lots of models are possible, e.g. Coulomb with Bouc-Wen directions HARVEY & GAVIN (2014)
  - Euler direction components in $\tilde{\mathbf{t}}_1^k$- and $\tilde{\mathbf{t}}_2^k$-directions

$$\dot{Y}_1 = \frac{R_1}{R_0}\left((1 - a_1 Y_1^2)\tilde{u}_1 - b_1 Y_1 Z_1 \tilde{v}_1\right),\ a_1 = \begin{cases} 1, & Y_1 \tilde{u}_1 > 0 \\ 0, & Y_1 \tilde{u}_1 \leq 0 \end{cases}$$

$$\dot{Z}_1 = \frac{R_1}{R_0}\left((1 - b_1 Z_1^2)\tilde{v}_1 - a_1 Y_1 Z_1 \tilde{u}_1\right),\ b_1 = \begin{cases} 1, & Z_1 \tilde{v}_1 > 0 \\ 0, & Z_1 \tilde{v}_1 \leq 0 \end{cases}$$

- Rotational friction is also naturally handled in this parameterization

Rim contact

- Motion limit states are also easily expressible in our proposed parameterization
  - e.g. base and first sliding element contact definition

$$s_1 = R_1 \cos^{-1}\left(\mathbf{t}_3^1 \cdot \mathbf{t}_3^2\right)$$

$$g_1 = s_{c1} - s_1$$

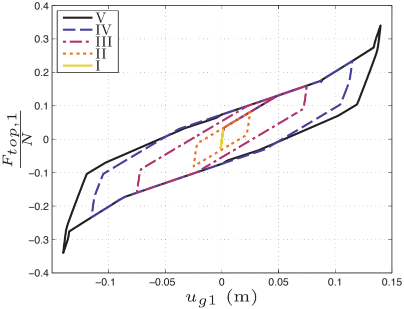
Overall structure

- Parameterization + Linear + Angular momentum balance
  - Model with 24 internal parameters for Euler parameters and normal forces
  - Time integration using DORMAND-PRINCE (1980) (Runge-Kutta) variable order scheme
  - Nonlinear algebraic system

Performance

- One-dimensional behavior FENZ & CONSTANTINOU (2008)
- Bi-directional motion BECKER & MAHIN (2012)
    - Zero fitting parameters, geometry and mass, measured friction coefficients from the experiments

Kinematic state accuracy



Stage I: 2 and 3 surface motion

Stage II: 1 and 3 surface motion

Stage III: 1 and 4 surface motion

Stage IV: 2 and 4 surface motion

Stage V: 2 and 3 surface motion

Kinematic state accuracy

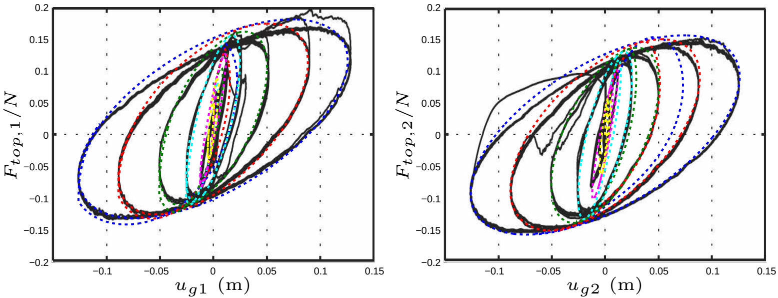
- Displacements and forces at transition points between stages.

	Analytical†	Experimental†	Kinetic Model
u^* (mm)	0.1	2	1.9
u^{**} (mm)	38.4	42	49
u_{dr1} (mm)	92.1	90	87
u_{dr4} (mm)	130.4	130	134
F_{dr1}/N	0.161	0.173	0.175
F_{dr4}/N	0.240	0.272	0.275

† Analytical and Experimental values come from FENZ & CONSTANTINOU (2008)

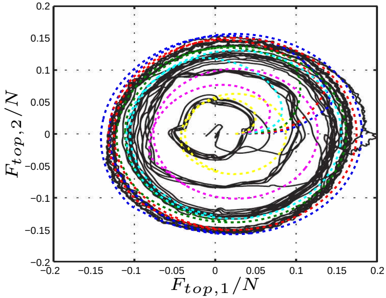
Circular motion: Force vs displacement by component

- $\Omega = 0.1$ rad/s. $\boldsymbol{u}_g = A\,(\cos(\Omega t)\boldsymbol{E}_1 + \sin(\Omega t)\boldsymbol{E}_2)$.



- Becker & Mahin (2012) data solid black curve
  - No fitting involved

Circular motion: Force vs force



- BECKER & MAHIN (2012) data solid black curve

Figure-8 motion: Force vs displacement by component

- $\Omega = 0.1\ \mathrm{rad/s}$. $\boldsymbol{u}_g = A\left(\sin(\Omega t)\boldsymbol{E}_1 + \sin(2\Omega t)\boldsymbol{E}_2\right)$.

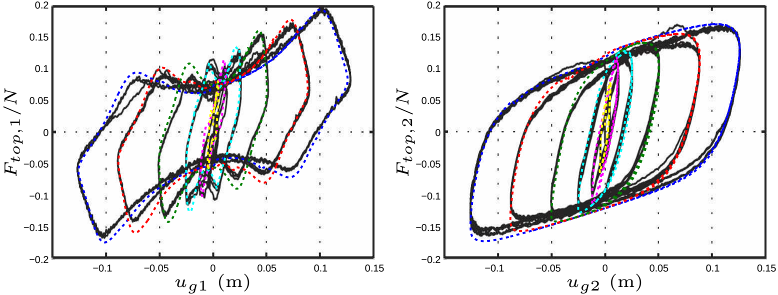


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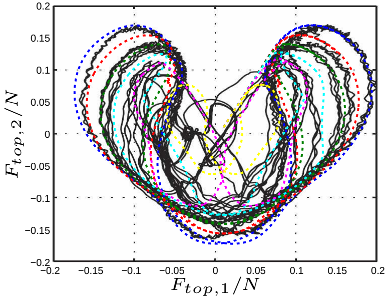
Figure-8 motion: Force vs force



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Summary

- Works for uni- and bi-directional motion
- No linearization assumptions – good agreement with real experiments
- Main assumption – axisymmerty
- Can model many types of bearings, e.g. SARLIS & CONSTANTINOU (2016) $\mu_2 = \mu_3 > \mu_1 = \mu_4$
- Easily connected to FEA codes
- Uplift and loss of compression can be incorporated (where inertia matters)

DRAZIN, P. AND GOVINDJEE S., PEER REPORT-2017/07