Ground-Motion Prediction Equations for Arias Intensity Consistent with the NGA-West2 Ground-Motion Models

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Disclaimer

The opinions, findings, and conclusions or recommendations expressed in this publication are those of the author(s) and do not necessarily reflect the views of the study sponsor(s) or the Pacific Earthquake Engineering Research Center.
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ABSTRACT

Following the approach outlined in the Watson-Lamprey and Abrahamson [2006] conditional model for Arias intensity, we use the NGA-West2 database to derive a new scaling model for Arias intensity given peak ground acceleration (PGA), $T = 1$ sec spectral acceleration ($SA_T$), shear-wave velocity in the top 30 m ($V_{S30}$), and magnitude. By combining this conditional model with each of five NGA-West2 ground-motion models for PGA and $SA_T$, we derived five new ground motion prediction equations (GMPEs) for the median and standard deviation of Arias intensity. These five GMPEs for Arias intensity capture the more complex ground-motion scaling effects found in some of the NGA-West2 GMPEs, such as hanging-wall effects, sediment-depth effects, soil nonlinearity effects, and regionalization effects. This allows for Arias intensity values to be estimated that are consistent with the NGA-West2 GMPEs.
ACKNOWLEDGMENTS

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1 Introduction

1.1 TRADITIONAL MODELS FOR ARIAS INTENSITY

Arias intensity ($I_a$) has been recognized as a useful indicator of damage potential for earth dams in seismic analysis [Travasarou et al. 2003]. There have been several empirical models developed for $I_a$ in the past decade (e.g., Travasarou et al. [2003] (TBA03); Stafford et al. [2009] (SBP09); Foulser-Piggott and Stafford [2012] (FPS12); and Campbell and Bozorgnia [2012] (CB12). These $I_a$ models have used a relatively simple parameterization to fit the Arias intensity with the exception of the CB12 model. This creates a potential inconsistency with the pseudo-spectral acceleration (PSA) values developed using ground-motion prediction equations (GMPEs) with more complex parameterization. For example, if the site is located over the hanging wall, then the PSA values at short periods may be increased by a factor of 2 to 3 compared to footwall sites at the same distance; but if the $I_a$ model does not include hanging-wall effects, then the $I_a$ computed for the site may be too small and may not be consistent with the $I_a$ expected for the given PSA. Similarly, if there are strong nonlinear site effects, then the PSA values based on GMPEs that include nonlinearity will be inconsistent with the $I_a$ values based on $I_a$ models that do not include these effects.

1.2 CONDITIONAL GROUND-MOTION MODELS

Watson-Lamprey and Abrahamson [2006] developed a conditional model for $I_a$ that included the observed peak ground acceleration (PGA) and 1-sec spectral acceleration ($S_{AT1}$) as predictive parameters in addition to the earthquake magnitude, distance, and time-averaged shear-wave velocity over the top 30 m ($V_{S30}$). The $I_a$ model is called conditional because it estimates the $I_a$ given the observed PGA and $S_{AT1}$ values.

This report uses the conditional ground-motion model approach to develop an updated conditional model for $I_a$ that is consistent with the NGA-West2 ground-motion models. The advantage of the conditional ground-motion model approach for predicting the $I_a$ is that the more complicated scaling that are included in the NGA-West2 GMPEs—such as short-distance saturation, hanging-wall effects, soil-depth effects, soil nonlinearity effects, and regionalization effects—are accommodated by estimating the median and standard deviations of the PGA and $S_{AT1}$ using the NGA-West2 GMPEs [Abrahamson et al. 2014 (ASK14); Boore et al. 2014 (BSSA14); Campbell and Bozorgnia 2014 (CB14); Chio and Youngs 2014 (CY14); and Idriss, 2014 (I14)]. This allows for estimation of $I_a$ values that are consistent with the estimated PSA.
values from more recent GMPEs to include more complex scaling, such as the NGA-West2 GMPEs.
2 Dataset

2.1 DATASET SELECTION

This study used the Pacific Earthquake Engineering Research Center (PEER) NGA-West2 database [Ancheta et al. 2014], which includes earthquakes that occurred through 2011 and contains over 21,000 three-component recordings from California and worldwide, with moment magnitudes, $M$, ranging from $M3.0$–$M7.9$. We use the same subset of this dataset as the ASK14 model, which consists of 15,730 recordings (distances between 0 and 400 km from 326 earthquakes with magnitudes between 3 and 7.9). There were five recordings with missing $I_a$ values [Record Serial Number (RSN) 4236, 8164, 8165, 8166, 8169]. The initial regression studies also found that there were ten recordings that were significant outliers in terms of only their $I_a$ values (RSN 9048, 13483, 14046, 14727, 15374, 17041, 17206, 17305, 17800, and 20407). We did not correct these ten records and simply removed them from our dataset.

The resulting magnitude and distance distribution of the 15,715 recordings in the selected dataset is shown in Figure 2.1. Of this dataset, 506 recordings have a minimum usable frequency greater than 1 Hz, indicating that their $SA_{71}$ values are unusable. Removing these 506 recordings leads to our final dataset of 15,209 recordings.
Figure 2.1  Magnitude and distance distribution of the selected $I_a$ dataset.
3 Conditional Arias Intensity Model

3.1 FUNCTIONAL FORM

The Arias intensity is defined as

\[ I_a = \frac{\pi}{2g} \int a^2(t) \, dt \]  

(3.1)

where \( a(t) \) is the acceleration in m/sec\(^2\), \( g \) is the acceleration of gravity, and \( I_a \) is the Arias intensity in m/sec [Arias 1970]. Following Watson-Lamprey and Abrahamson [2006], we develop a conditional model for Arias intensity that includes the observed PGA and \( SA_{T1} \) as predictor variables. Watson-Lamprey and Abrahamson [2006] used the following functional form for their conditional \( I_a \) model:

\[
\ln \left( I_a \, (m/sec) \right) = c_1 + c_2 \ln(V_{S30}) + c_3 M + c_4 \ln(\text{PGA}) + c_5 \ln(SA_{T1}) + c_6 \ln(R_{\text{rup}}) + c_7 \left[ \ln(R_{\text{rup}}) \right]^2
\]  

(3.2)

An initial exploratory analysis using the NGA-West2 database showed that the coefficients for the distance terms (\( c_6 \) and \( c_7 \)) were not significantly different from zero. (\( c_6 \) and \( c_7 \) are highly correlated so both terms cannot be determined. The \( t \)-ratio for \( c_6 \) is -0.16.) Therefore, the distance terms were removed and the form listed in Equation (3.3) was used for developing the conditional \( I_a \) model.

\[
\ln \left( I_a \, (m/sec) \right) = c_1 + c_2 \ln(V_{S30}) + c_3 M + c_4 \ln(\text{PGA}) + c_5 \ln(SA_{T1})
\]  

(3.3)

where \( V_{S30} \) is in m/sec, and PGA and \( SA_{T1} \) are in g.

It is common to use a random-effects regression model for developing GMPEs to account for the correlation in the data through the event terms (e.g., Abrahamson and Youngs [1992]); however, because we developed a conditional model that includes the recorded PGA and \( SA_{T1} \) as input parameters, some of the correlation within an event is already captured through the observed PGA and \( SA_{T1} \) terms. We used the random-effects regression in the statistics program JMP (SAS Institute) to estimate the coefficients for the conditional \( I_a \) model. The resulting coefficients are listed in Table 3.1 along with the standard errors of the estimates. Table 3.1 shows that the \( I_a \) scales more strongly with PGA than \( SA_{T1} \) because the \( I_a \) is a high-frequency
parameter; however, the $SA_{T1}$ term is also statistically significant, shown by the large $t$-ratio of 44.

This conditional model for $I_\alpha$ has a total standard deviation of 0.38 natural log units (intra-event standard deviation $\phi = 0.35$, inter-event standard deviation $\tau = 0.15$). This standard deviation is much smaller than the standard deviations for traditional $I_\alpha$ models (not conditioned on observed PGA and $SA_{T1}$) that are about 1.0 natural log units, indicating that the $I_\alpha$ can be computed with much smaller aleatory variability if PGA and $SA_{T1}$ are known, due to strong correlation of the $I_\alpha$ with the PGA and $SA_{T1}$.

### Table 3.1 Parameter estimates, standard errors, and $t$-ratios for scaling model.

<table>
<thead>
<tr>
<th>Term</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>$t$-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>0.47</td>
<td>0.069</td>
<td>7</td>
</tr>
<tr>
<td>$c_2$</td>
<td>-0.28</td>
<td>0.0085</td>
<td>-33</td>
</tr>
<tr>
<td>$c_3$</td>
<td>0.50</td>
<td>0.011</td>
<td>45</td>
</tr>
<tr>
<td>$c_4$</td>
<td>1.52</td>
<td>0.0038</td>
<td>399</td>
</tr>
<tr>
<td>$c_5$</td>
<td>0.21</td>
<td>0.0047</td>
<td>44</td>
</tr>
</tbody>
</table>

### 3.2 EVALUATION OF RESIDUALS

The residuals for the conditional $I_\alpha$ model are shown in Figures 3.1 and 3.2 as a function of the four model input parameters: PGA, $SA_{T1}$, $M$, and $V_{S30}$. Figure 3.1 shows the inter-event residuals as a function of magnitude. Figure 3.2 shows the intra-event residuals as a function of PGA, $SA_{T1}$, and $V_{S30}$. The lack of trends in the residuals shows that the simple scaling in Equation (3.3) adequately captures the dependence of the $I_\alpha$ on these four parameters.

We also checked the residuals for other parameters that are used in GMPEs for PSA but were not used in our conditional $I_\alpha$ model. The lower frame of Figure 3.1 shows the inter-event residuals as a function of depth to top of rupture ($Z_{TOR}$). Figure 3.3 shows the intra-event results as a function of rupture distance ($R_{RUP}$), soil depth to $V_5 = 1$ km/sec ($Z_{1.0}$), and rock PGA (PGA1100) for soil sites with $V_{S30} < 270$ m/sec. There is no trend in the residuals as a function of these parameters, indicating that the $I_\alpha$ scaling with these parameters, including the effects of nonlinear site response on the $I_\alpha$, is captured in the conditional model through the use of the observed PGA and $SA_{T1}$ values. To check the magnitude dependence of the distance scaling, Figure 3.4 shows the intra-event residuals versus distance separated by magnitude. There are no systematic trends in the residuals, indicating that the magnitude dependence of the distance scaling for $I_\alpha$ is also captured through the use of the observed PGA and $SA_{T1}$ values.

Finally, we checked the residuals for sites that are expected to show hanging-wall (HW) effects. The intra-event residuals for sites on the HW side of the rupture ($R_x > 0$) and located over or near the hanging-wall (Joyner-Boore distance, $R_{BB} < 5$ km) from earthquakes with $M > 6$ and dip $< 60^\circ$ show a small under-prediction of the $I_\alpha$. The trend in the residuals is nearly constant with $R_{BB}$, magnitude, and dip. The average amplitude of the offset in the HW residuals (0.11 ln units) is small compared to the size of the HW effects in the GMPEs (e.g., about 0.7 ln units in the ASK14 model for sites over the HW), indicating that the simple conditional model accounts
for most of the HW effect. Because sites located over the HW are an important factor in building design, an additional term is added to the conditional $I_a$ model to improve the model for HW sites. Although the residuals show a constant shift, this needs to be tapered for sites at larger distances, smaller magnitudes, and steeper dips. Therefore, we applied the form of the dip, magnitude, and distance tapers for the HW term from the ASK14 model [Equations (3.5), (3.6), and (3.7)]. The HW term is given by:

$$HW_{\text{resid}} = c_8 F_{HW} T_1(\text{dip}) T_2(M) T_3(R_{jb})$$  \hspace{1cm} (3.4)$$

where $F_{HW}$ is a flag that is 1 for sites located on the hanging-wall side of the rupture and 0 for sites located on the footwall side of the rupture:

$$T_1(\text{dip}) = \begin{cases} 
(90 - \text{dip}) / 45 & \text{for dip} > 30 \\
60 / 45 & \text{for dip} < 30 
\end{cases}$$  \hspace{1cm} (3.5)$$

$$T_2(M) = \begin{cases} 
1 & \text{for } M \geq 6.5 \\
1 + 0.2(M - 6.5) - 0.8(M - 6.5)^2 & \text{for } 5.5 < M < 6.5 \\
0 & \text{for } M \leq 5.5 
\end{cases}$$  \hspace{1cm} (3.6)$$

$$T_3(R_{jb}) = \begin{cases} 
1 & \text{for } R_{jb} = 0 \\
1 - \frac{R_{jb}}{15} & \text{for } R_{jb} < 15 \\
0 & \text{for } R_{jb} \geq 15 
\end{cases}$$  \hspace{1cm} (3.7)$$

The estimate of $c_8$ is 0.09 with a standard error of 0.03.

### 3.3 EVALUATION OF ALEATORY VARIABILITY

The regression analysis assumes that the $\ln(I_a)$ residuals have a normal distribution. This assumption is evaluated using normal probability plots. Figure 3.8 shows the normal probability plots of the inter-event and intra-event residuals. The distribution of inter-event residuals is consistent with a normal distribution except for the lower tail, which is not important for engineering applications because we are concerned with large damaging ground motions. The distribution of the intra-event residuals is generally consistent with the normal distribution, with the exception of the upper and lower tails, starting at about the 1% and 99% levels (e.g., about ±2.3 standard deviations). Figure 3.8 shows that the $I_a$ intra-event residuals have fat tails compared to the normal distribution. Similar fat tails have been observed for the intra-event response spectral values using the NGA-West2 dataset [Geopentech 2015 Chapter 11].

The fat tails for the intra-event residuals can be described using a mixture model, given by the weighted average of two normal distributions: one has a mean of 0, standard deviation of 0.40 and weight of 0.56; the other has a mean of 0, standard deviation of 0.27 and weight of 0.44. The normal probability plot using the mixture model is shown in Figure 3.9. The mixture model adequately captures the fat tails in the upper range.
The intra-event residual distribution deviates from a normal distribution at around 2.3 standard deviations, but it is consistent with the mixture model distribution up to 4 standard deviations. If the $I_a$ model is used to compute probabilistic hazard at low probability levels, then the mixture model should be used to compute the conditional probability of exceedance as shown below, where $\Phi(x)$ is the cumulative standard normal distribution.

$$P(I_a > z|PGA, SA_{T1}, M, V_{S30}) = 0.56 \left[ 1 - \Phi \left( \frac{\ln(z) - \ln(I_a)}{\phi_1 + \tau} \right) \right] + 0.44 \left[ 1 - \Phi \left( \frac{\ln(z) - \ln(I_a)}{\phi_2 + \tau} \right) \right]$$

(3.8)

where $\phi_1$ and $\phi_2$ are the intra-event standard deviations from the mixture model ($\phi_1 = 0.40$ and $\phi_2 = 0.27$), and $\tau$ is inter-event standard deviation ($\tau = 0.15$).

The regression also assumes that the $I_a$ residuals are homoscedastic. This assumption is evaluated by visual inspection of the scatter of residuals shown against the predictive parameters in Figures 3.1, 3.2, 3.3, and 3.4. The amplitude of the scatter of residuals does not appear to have any trends with the predicted parameters (magnitude, PGA, $SA_{T1}$, and $V_{S30}$). For PSA GMPEs, the standard deviation is often modeled with a dependence on the earthquake magnitude (e.g., ASK14, BSSA14, CB14, CY14, and I14). The magnitude dependence of the standard deviation for the conditional $I_a$ model is evaluated by computing the intra-event and inter-event standard deviations for different magnitude ranges (Table 3.2). There is no systematic increase or decrease in the inter-event and intra-event standard deviations ($\tau$ and $\phi$) with magnitude.

<table>
<thead>
<tr>
<th>Magnitude Range</th>
<th>Number of Earthquakes</th>
<th>Number of Recordings</th>
<th>$\phi$ (LN units)</th>
<th>$\tau$ (LN units)</th>
<th>$\sigma$ (LN units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5–3.5</td>
<td>25</td>
<td>821</td>
<td>0.38</td>
<td>0.11</td>
<td>0.40</td>
</tr>
<tr>
<td>3.5–4.5</td>
<td>169</td>
<td>7713</td>
<td>0.36</td>
<td>0.14</td>
<td>0.39</td>
</tr>
<tr>
<td>4.5–5.5</td>
<td>54</td>
<td>2235</td>
<td>0.33</td>
<td>0.17</td>
<td>0.37</td>
</tr>
<tr>
<td>5.5–6.5</td>
<td>49</td>
<td>1785</td>
<td>0.29</td>
<td>0.16</td>
<td>0.33</td>
</tr>
<tr>
<td>6.5–7.5</td>
<td>25</td>
<td>2622</td>
<td>0.37</td>
<td>0.14</td>
<td>0.40</td>
</tr>
<tr>
<td>7.5–8.5</td>
<td>4</td>
<td>539</td>
<td>0.35</td>
<td>too few eqk</td>
<td></td>
</tr>
</tbody>
</table>

### 3.4 COMPARISON OF CONDITIONAL $I_a$ MODELS

The conditional $I_a$ model developed in this section is compared to the WLA06 conditional $I_a$ model in Figures 3.10 and 3.11. The coefficients for the magnitude scaling and $V_{S30}$ scaling in the current model are similar to the coefficients in the WLA06. This leads to similar magnitude and $V_{S30}$ scaling between the two models; see Figure 3.10.

The coefficient for the $\ln(PGA)$ term is much larger in the current model (1.52) than in the WLA06 model (1.30), suggesting that the current model leads to stronger scaling with PGA than the WLA06 model; however, the $\ln(PGA)$ is correlated with the $\ln[PSA(T=1)]$, and the
coefficient for the ln[PSA(T=1)] term is much smaller in the current model (0.21) than in the WLA06 model (0.33). This difference in the PSA(T=1) scaling offsets much of the difference in the PGA scaling between the two models.

Because the relative values of the PGA and PSA(T=1) depend on the spectral shape, Figure 3.11 shows the scaling with ln(PGA) for magnitudes of 5.0, 6.5, and 8.0. The largest differences in the ln(PGA) scaling is at the M5 range. This reflects the large increase in the number of recordings from moderate magnitude earthquakes in the NGA-West2 dataset used in this study compared to the NGA-West1 dataset used by WLA06. Similarly, the scaling with ln[PSA(T=1)] is shown in the right-hand frame of Figure 3.11 for magnitudes 5.0, 6.5, and 8.0. As with the ln(PGA) scaling, the correlation of the ln(PGA) and the ln[PSA(T=1)] values reduces the differences in the predicted $I_a$ from the two models.

The predicted $I_a$ values using the current conditional $I_a$ model and using the WA06 conditional $I_a$ model are very similar even though the current model uses a much larger database. In contrast, the predicted $I_a$ values using the traditional (non-conditional) approach show a stronger dependence on the dataset used to develop the model (see Chapter 4). Therefore, using the conditional $I_a$ model approach leads to more robust ground-motion models for $I_a$ than using the traditional approach.
Figure 3.1  Inter-event residuals of the conditional $I_s$ model as a function of magnitude (top) and $Z_{TOR}$ (bottom).
Figure 3.2 Intra-event residuals of conditional $I_a$ model as a function of (a) PGA, (b) $SA_{T1}$, and (c) $V_{S30}$. 
Figure 3.3 Intra-event residuals of the conditional $I_a$ model as a function of parameters not included in the conditional model.
Figure 3.4  Intra-event residuals of the conditional $I_a$ model as a function of rupture distance separated by magnitude bin.
Figure 3.5  Intra-event residuals of conditional $I_a$ model for sites with expected hanging-wall effects ($M \geq 6$, dip $\leq 60$, $R_{JB} \leq 5$ km, $R_x > 0$) as a function of $R_{JB}$. The red curve shows the mean residual.
Figure 3.6 Intra-event residuals of conditional $I_a$ model for sites with expected hanging-wall effects ($M \geq 6$, dip $\leq 60$, $R_{jB} \leq 5$ km, $R_x > 0$) as a function of (a) magnitude and (b) dip. The red curve shows the mean residual.
Figure 3.7  Intra-event residuals of conditional $I_h$ model for sites with expected hanging-wall effects ($M \geq 6$, dip $\leq 60$, $R_x > 0$) after including the HW term in Equation (3.4). The red curves shows the mean residual for distance bins and the red dashed curves show the plus and minus one standard error of the mean residual.
Figure 3.8  Normal probability plot for the (a) inter-event residuals and the (b) intra-event residuals for the conditional $I_a$ model.
Figure 3.9 Normal probability plot for the intra-event residuals (in ln units) using the mixture model for the conditional $I_a$ model.
Figure 3.10  Comparison of the scaling for the WLA06 conditional $I_s$ model with the results of this study.
Figure 3.11  Comparison of the scaling for the WLA06 conditional $I_a$ model with the results of this study. Because the PSA ($T=1$) is correlated with PGA, the scaling with PGA or PSA is shown for fixed magnitudes (fixed spectral shape).
4 Arias Intensity Ground Motion Prediction Equations

4.1 MOVING FROM CONDITIONAL MODELS TO TRADITIONAL MODELS

For a given earthquake scenario and site location, we use the median PGA and median $S_{AT1}$ values from the individual NGA-West2 ground motion models combined with the conditional $I_a$ model in Equations (3.3) and (3.4) to develop a median $I_a$ for each NGA-West2 model, as shown in Equation (4.1).

$$
\ln \left[ I_a \text{ (m/sec)} \right] = c_1 + c_2 \ln \left( V_{S30} \right) + c_3 M + c_4 \ln \left( \text{PGA} \right) + c_5 \ln \left( S_{AT1}\text{med} \right) + c_6 F_{IW}^J \begin{cases} 
1 & \text{if } R_{JB} < 5 \text{ km} \\
1 - \frac{R_{JB} - 5}{5} & \text{if } 5 < R_{JB} < 10 \text{ km} \\
0 & \text{if } R_{JB} > 10 \text{ km}
\end{cases}
$$

This median $I_a$ depends on the median PGA and $S_{AT1}$ from the NGA-West2 GMPEs and is no longer conditioned on the observed PGA and $S_{AT1}$.

The variance for the $I_a$ GMPE can be calculated using propagation of errors [Bevington and Robinson 1969] as follows:

$$
\sigma_{\ln(I_a)}^2 = \sigma_{\ln(I_a)\ln(PGA),\ln(S_{AT1})}^2 + \sigma_{\ln(PGA)}^2 \left( \frac{\partial \ln(I_a)}{\partial \ln(PGA)} \right)^2 + \sigma_{\ln(S_{AT1})}^2 \left( \frac{\partial \ln(I_a)}{\partial \ln(S_{AT1})} \right)^2 + 2 \text{COV}[\ln(PGA),\ln(S_{AT1})] \left( \frac{\partial \ln(I_a)}{\partial \ln(PGA)} \right) \left( \frac{\partial \ln(I_a)}{\partial \ln(S_{AT1})} \right)
$$

where

$$
\text{COV}[\ln(PGA),\ln(S_{AT1})] = \rho \left[ \sigma_{\ln(PGA)} \sigma_{\ln(S_{AT1})} \right] \left[ \sigma_{\ln(PGA)} \sigma_{\ln(S_{AT1})} \right]
$$

and $\rho$ is the correlation coefficient, and $\epsilon$ is the normalized residual from the NGA-West2 GMPE. The partial derivatives were calculated from Equation (4.1). The $\sigma_{\ln(PGA)}, \sigma_{\ln(S_{AT1})}$ term is the standard deviation for the conditional $I_a$ in Equation (3.3). We used the correlation
coefficient, \( \rho[\varepsilon_{\ln(PGA)}, \varepsilon_{\ln(SaT_1)}] \), from the Baker and Jayaram [2008] model. Although this correlation model was derived from the NGA-West1 dataset, Carlton and Abrahamson [2014] showed that correlations are stable for different datasets. The values of the partial derivatives, correlation coefficient, and standard deviation of the conditional \( I_a \) model in Equations (4.2) and (4.3) are listed in Table 4.1.

<table>
<thead>
<tr>
<th>( \sigma_{\ln(I_a)\ln(PGA)\ln(SaT_1)} )</th>
<th>( \rho \left[ \varepsilon_{\ln(PGA)}, \varepsilon_{\ln(SaT_1)} \right] )</th>
<th>( \frac{\delta \ln(I_a)}{\partial \ln(PGA)} )</th>
<th>( \frac{\delta \ln(I_a)}{\partial \ln(SaT_1)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.38</td>
<td>0.52</td>
<td>1.53</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Using the values in Table 4.1, the total standard deviation (combined intra-event and inter-event) for the \( I_a \) GMPEs is given by Equation (4.4).

\[
\sigma_{\ln(I_a)} = \sqrt{0.144 + 2.34\sigma^2_{\ln(PGA)} + 0.04\sigma^2_{\ln(SaT_1)} + 0.318\sigma_{\ln(PGA)}\sigma_{\ln(SaT_1)}} \quad (4.4)
\]

where \( \sigma_{\ln(PGA)} \) and \( \sigma_{\ln(SaT_1)} \) are the standard deviations given by the NGA-West2 models. The intra-event and inter-event standard deviations can also be estimated. The correlation for the intra-event and inter-event residuals are very similar [Carlton and Abrahamson 2014]. Assuming the correlations from Baker and Jayaram [2008] are valid for both the intra-event and inter-event residuals, the intra-event and inter-event standard deviations for the \( I_a \) are shown in Equations (4.5) and (4.6).

\[
\phi_{\ln(I_a)} = \sqrt{0.144 + 2.34\phi^2_{\ln(PGA)} + 0.04\phi^2_{\ln(SaT_1)} + 0.318\phi_{\ln(PGA)}\phi_{\ln(SaT_1)}} \quad (4.5)
\]

\[
\tau_{\ln(I_a)} = \sqrt{0.144 + 2.34\tau^2_{\ln(PGA)} + 0.04\tau^2_{\ln(SaT_1)} + 0.318\tau_{\ln(PGA)}\tau_{\ln(SaT_1)}} \quad (4.6)
\]

As a check, the \( I_a \) values are computed using the median PGA and \( SaT_1 \) from the ASK14 GMPE, i.e., it is no longer a conditional model. Using these estimated \( I_a \) values, the inter-event and intra-event residuals of the \( I_a \) are computed using a random-effects regression with only a constant term. The estimated constant term is small: 0.02 natural log units. The inter-event residuals and the intra-event residuals do not show a trend with magnitude or distance. The small constant term and the lack of trends in these residuals indicate that \( I_a \) based on the conditional \( I_a \) model combined with the ASK14 GMPE for the median PGA and \( SaT_1 \) is consistent with the scaling of the observed \( I_a \) values.

### 4.2 MODEL RESULTS

The resulting \( I_a \) models using the five NGA-West2 GMPEs are compared to previously published \( I_a \) models. Figures 4.1(a-d) compare the distance scaling of the \( I_a \) models for vertically dipping strike–slip earthquakes for \( V_{S30} \) of 520 m/sec for magnitude 5, 6, 7, and 8, respectively. Default values are used for the focal depth and depth to top of rupture terms. The default values
for \(Z_{1.0}\) and \(Z_{2.5}\) are 0.22 km and 0.94 km for \(V_{S30} = 520\) m/sec, respectively, based on the scaling in ASK14 for \(Z_{1.0}\) and the scaling in CB14 for \(Z_{2.5}\). For \(V_{S30} = 270\) m/sec, the \(Z_{1.0}\) and \(Z_{2.5}\) values are 0.47 km and 1.98 km, respectively. At large distances, the \(I_a\) models have a steeper slope than the previously published models because they scale with the NGA-West2 GMPE’s, which are better constrained at large distances. The resulting models are also more tightly grouped than the other models are, comparatively. This shows that the resulting models are more robust than models based on the common approach of conducting an independent regression for the \(I_a\) model, which generally does not have the physical constraints built into how the model extrapolates as in the case of the PSA GMPEs.

Figures 4.2 and 4.3 compare the magnitude scaling for the \(I_a\) models for vertically dipping strike–slip earthquakes for a rupture distance of 10 km and two \(V_{S30}\) values: 270 m/sec and 520 m/sec. In the \(M_6\)–\(M_7\) range, the NGA-West2 \(I_a\) models are similar to prior models, but they differ at \(M_5\) and \(M_8\) magnitudes where the previous models have fewer data and are extrapolated. Here, the CB14 model is similar to the CB12 model because of similar complex forms but mainly differ at \(M_8\) magnitudes with lower values in the CB12 model.

Figure 4.4 compares the short-distance scaling for the \(I_a\) models for dipping faults for \(M_7\). This shows a larger difference between the NGA-West2 \(I_a\) model and prior models that did not include hanging-wall effects (all except CB12) for sites located over the hanging wall due to the simple forms used in prior models.

Figure 4.5 compares the standard deviations for the \(I_a\) models developed herein with the standard deviations from previously published models. The standard deviations for the new models are generally consistent with the FPS12 empirical model but are significantly larger than the TBA03, SBP09 and CB12 empirical models. Both the FPS12 and CB12 use the NGA-West1 dataset and use a direct regression on the \(I_a\) data, but the selected subsets are different. As a result, the FPS12 standard deviation is much larger than the CB12 standard deviation.

To understand the cause of the difference in the standard deviations, Figure 4.5 includes a comparison with the standard deviation computed from the CB12 dataset from NGA-West1 and the ASK14 dataset from NGA-West2. The intra-event and inter-event standard deviations for \(\ln(I_a)\) computed using these two subsets are listed in Table 4.2. Applying the results of the conditional \(I_a\) model to the CB08 equation for PGA and \(SA_{T1}\) and corresponding dataset for computing residuals leads to \(\phi = 0.78\) and \(\tau = 0.36\), which are similar to the standard deviation terms from CB12 (\(\phi = 0.77, \tau = 0.31\)). In contrast, applying the conditional \(I_a\) model to the ASK14 model for PGA and \(SA_{T1}\) and corresponding dataset leads to much larger \(\phi\) and \(\tau\) values (\(\phi = 0.89, \tau = 0.61\) for \(M > 5.5\)), which are consistent with the standard deviations for the five new models based on the NGA-West2 equations. They are also consistent with the standard deviation of the FPS12 empirical model. This shows that the increase in the standard deviations for the new \(I_a\) models compared to several of the previous empirical models, shown in Figure 4.5, is not an artifact of the use of a conditional \(I_a\) model. Instead, it is related to the selected dataset.
Table 4.2 Inter-event, intra-event, and total standard deviations for two different datasets and reference GMPEs.

<table>
<thead>
<tr>
<th>Dataset and reference GMPE</th>
<th>Magnitude range</th>
<th>$\phi$ (LN units)</th>
<th>$\tau$ (LN units)</th>
<th>$\sigma$ (LN units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASK14 ($R_{rup} &lt; 80$ km)</td>
<td>$3.0 &lt; M &lt; 4.0$</td>
<td>1.23</td>
<td>0.72</td>
<td>1.43</td>
</tr>
<tr>
<td></td>
<td>$4.0 &lt; M &lt; 5.5$</td>
<td>1.16</td>
<td>0.72</td>
<td>1.37</td>
</tr>
<tr>
<td></td>
<td>$M &gt; 5.5$</td>
<td>0.89</td>
<td>0.61</td>
<td>1.08</td>
</tr>
<tr>
<td>CB08</td>
<td>$M &gt; 4.3$</td>
<td>0.78</td>
<td>0.36</td>
<td>0.86</td>
</tr>
</tbody>
</table>
Figure 4.1 Comparison of the distance scaling of the median $I_a$ models for strike–slip faults for $V_{S30} = 520$ m/sec (TBA03 class C, SBP09 class B). (a) $M = 5$, (b) $M = 6$, (c) $M = 7$, and (d) $M = 8$. 
Figure 4.2  Comparison of the magnitude scaling of the median $I_a$ models for strike-slip faults for a rupture distance of 10 km and $V_{S30} = 520$ m/sec (TBA03 class C, SBP09 class B).

Figure 4.3  Comparison of the magnitude scaling of the median $I_a$ models for strike-slip faults for a rupture distance of 10 km and $V_{S30}=270$ m/sec (TBA03 class D, SBP09 class D).
Figure 4.4  Comparison of the median $I_a$ for dipping faults for $M = 7$ and $V_{530} = 520$ m/sec (TBA03 class D, SBP09 class D).
Figure 4.5  Comparison of the standard deviations for the $I_s$ models based on the NGA West2 GMPEs using Equation (4.4) with the standard deviations from previously published $I_s$ models. Also shown are the standard deviation estimated from the NGA-West2 data within 80 km using the ASK14 $I_s$ model and the standard deviation estimated from the NGA-West1 data used by CB12 with the Campbell and Bozorgnia [2008] GMPE (CB08).
5 Conclusions

It is common to develop estimates of the $I_a$ in addition to the PSA values for defining the design ground motion. If the $I_a$ is estimated using an $I_a$ GMPE based on a simple functional form but the PSA is estimated using GMPEs that considers more complex effects, such as hanging-wall effects and nonlinear site effects, then there may be inconsistencies between the estimated $I_a$ value and the estimated PSA. This inconsistency can be avoided by using conditional $I_a$ models combined with the GMPEs for PGA and $SA_{T1}$. A key advantage of the approach used in this paper is that the extrapolation and more complex scaling in the GMPEs can be captured in the $I_a$ model. Also, because the conditional $I_a$ model has a small standard deviation, it is more robust than traditional $I_a$ models, leading to a suite of $I_a$ models that have a much smaller range of median $I_a$ than the traditional models. The derived models have a larger standard deviation than many of the previous models, which we attribute to dataset differences.

The conditional $I_a$ model [Equation (3.3)] can be used directly (without the NGA-West2 GMPEs) to develop design $I_a$ values that are consistent with the design spectrum by using the PGA and $SA_{T1}$ of the design spectrum as inputs to the conditional $I_a$ model. The full $I_a$ model [Equations (4.1) and (4.4)] combines the conditional $I_a$ with a PSA GMPE-producing $I_a$ models that include complex features of the PSA GMPE, such as hanging-wall effects, nonlinear site effects, directivity effects, magnitude, and/or distance dependent standard deviations. This paper combined the conditional $I_a$ model with the five NGA-West2 GMPEs, but the conditional $I_a$ model, with its small variability and simple form, can be combined with other crustal GMPEs. In addition, as new PSA GMPEs are developed in the future, the conditional $I_a$ model developed herein can be combined with new PSA GMPEs to rapidly develop new $I_a$ GMPEs consistent with new PSA GMPEs because the standard deviation of the conditional $I_a$ model is very small relative to the other aleatory terms in GMPEs.
6 References


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