Hybrid Simulation Theory for a Classical Nonlinear Dynamical System

Paul L. Drazin
Sanjay Govindjee
Department of Civil and Environmental Engineering
University of California, Berkeley

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The opinions, findings, and conclusions or recommendations expressed in this publication are those of the author(s) and do not necessarily reflect the views of the study sponsor(s) or the Pacific Earthquake Engineering Research Center.
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HYBRID SIMULATION

Hybrid simulation is an experimental and computational technique that allows one to study the time evolution of a system by physically testing a subset of it while the remainder is represented by a numerical model that is attached to the physical portion via sensors and actuators. The technique allows the study of large or complicated mechanical systems while only requiring a subset of the complete system to be present in the laboratory. This results in vast cost savings as well as the ability to study systems that simply cannot be tested due to scale. However, the errors that arise from splitting the system in two requires careful attention if a valid simulation is to be guaranteed. To date, efforts to understand the theoretical limitations of hybrid simulation have been restricted to linear dynamical systems. The research reported herein considers the behavior of hybrid simulation when applied to nonlinear dynamical systems. The model problem focuses on the damped, harmonically-driven nonlinear pendulum. This system offers complex nonlinear characteristics, in particular periodic and chaotic motions. We are able to demonstrate that the application of hybrid simulation to nonlinear systems requires careful understanding of what one expects from such an experiment. In particular, when system response is chaotic we advocate using multiple metrics to characterize the difference between two chaotic systems via Lyapunov exponents and Lyapunov dimensions, as well as correlation exponents. When system response is periodic we advocate using $L^2$ norms. Further, we demonstrate that hybrid simulation can falsely predict chaotic or periodic response when the true system has the opposite characteristic. In certain cases, control system parameters can mitigate this issue.
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# CONTENTS

ABSTRACT .................................................................................................................................. iii  
ACKNOWLEDGMENTS ............................................................................................................. v  
TABLE OF CONTENTS ........................................................................................................... vii  
LIST OF FIGURES ..................................................................................................................... ix  

1. INTRODUCTION .................................................................................................................... 1  
2. GENERAL THEORY OF HYBRID SIMULATION .................................................................... 3  
   2.1 The Reference System .................................................................................................. 3  
   2.2 The Hybrid System .................................................................................................... 4  
3. DAMPED, DRIVEN NONLINEAR PENDULUM .................................................................... 7  
   3.1 The Reference System ................................................................................................ 7  
   3.2 The Hybrid System .................................................................................................... 8  
   3.3 Non-Dimensionalization ............................................................................................ 10  
4. ANALYSIS ............................................................................................................................ 13  
   4.1 Periodic Reference and Hybrid Systems ..................................................................... 14  
   4.2 Chaotic Reference and Hybrid Systems ..................................................................... 15  
      4.2.1 Chaos Error Metrics ............................................................................................ 20  
   4.3 One System Periodic and the Other Chaotic ............................................................. 23  
   4.4 Study of $K_i$ ............................................................................................................... 23  
5. CONCLUSIONS ..................................................................................................................... 27  
REFERENCES ............................................................................................................................. 29  
APPENDIX $\theta_p$ AND $d\theta_p/d\tau$ PLOTS .................................................................................. 31
LIST OF FIGURES

Figure 1.1 A simple diagram of a hybrid system set-up......................................................2

Figure 2.1 (a) A general system with domain $\mathcal{D}$ and state vector $u(x,t)$; and (b) a general system with imposed separation into two substructures for comparison to the hybrid system. $\mathcal{P} \cup I \cup C = \mathcal{D}$ and $\delta \mathcal{P} \cap \delta C = I$ . ..................3

Figure 2.2 The hybrid system separated into the physical, $\mathcal{P}$, and computational, $C$, sub-structures...................................................................................................4

Figure 3.1 The damped, driven nonlinear pendulum with a rigid body rotating about $O$ with applied moment $M(t)$ .................................................................7

Figure 3.2 The hybrid pendulum with the rigid body split into two pieces rotating about $O$ with applied moment $M(t)$ .................................................................8

Figure 4.1 The Lyapunov exponents for the reference, $\lambda_1$, and hybrid systems, $\hat{\lambda}_1$ when $\Omega=1$.............................................................................................................14

Figure 4.2 The $L^2$ error for $\Omega=1$ for three values of $\mu$ with only periodic responses................................................................................................................15

Figure 4.3 The state space trajectories for the reference and hybrid systems with $\mu = 1.114$ . .............................................................................................................16

Figure 4.4 The angular velocity time series of the reference and hybrid systems for $\mu = 1.2$ .............................................................................................................17

Figure 4.5 A zoomed in plot of the angular velocity time series of the reference and hybrid systems for $\mu = 1.2$ .............................................................................................................17

Figure 4.6 The Poincaré Sections of the reference and hybrid systems for $\mu = 1.2$ ......18

Figure 4.7 The angular velocity time series of the reference and hybrid systems for $\mu = 2.2$ .............................................................................................................18

Figure 4.8 A zoomed in plot of the angular velocity time series of the reference and hybrid systems for $\mu = 2.2$ .............................................................................................................19

Figure 4.9 The Poincaré Sections of the reference and hybrid systems $\mu = 2.2$ . ..............19

Figure 4.10 The error between $\lambda_i$ and $\hat{\lambda}_i$ as a function of $\mu$ ..............................................21

Figure 4.11 The error in the Lyapunov dimension as a function of $\mu$ ..............................................22

Figure 4.12 The error in the correlation exponent of the Poincaré Sections as a function of $\mu$ .............................................................................................................22
Figure 4.13 The Lyapunov exponents for the reference and hybrid systems when $K_i = 10$ ..............................................................................................................24

Figure 4.14 The $E_2^h$ error as a function of $K_i$ for multiple values of $\overline{\mu}$ ........................................24

Figure 4.15 The $E_2$ error as a function of $K_i$ for multiple values of $\overline{\mu}$. ........................................25

Figure 4.16 The Poincaré Sections of the reference and hybrid systems for $\overline{\mu} = 1.2$ and $K_i = 10$ ...........................................................................................................26

Figure A.1 The state space trajectories for the reference and hybrid systems with $\overline{\mu} = 1.114$; compare to Figure 4.3..............................................................................................................31

Figure A.2 The angular velocity time series of the reference and hybrid systems for $\overline{\mu} = 1.2$; compare to Figure 4.4. ..............................................................................................................32

Figure A.3 A zoomed in plot of the angular velocity time series of the reference and hybrid systems for $\overline{\mu} = 1.2$; compare to Figure 4.5..............................................................................................................32

Figure A.4 The Poincaré Sections of the reference and hybrid systems for $\overline{\mu} = 1.2$; compare to Figure 4.6..............................................................................................................33

Figure A.5 The angular velocity time series of the reference and hybrid systems for $\overline{\mu} = 2.2$; compare to Figure 4.7 ..............................................................................................................33

Figure A.6 A zoomed in plot of the angular velocity time series of the reference and hybrid systems for $\overline{\mu} = 2.2$; compare to Figure 4.8. ..............................................................................................................34

Figure A.7 The Poincaré Sections of the reference and hybrid systems for $\overline{\mu} = 2.2$; compare to Figure 4.9..............................................................................................................34

Figure A.8 The Poincaré Sections of the reference and hybrid systems for $\overline{\mu} = 1.2$ and $K_i = 10$; compare to Figure 4.16. ..............................................................................................................35
1. Introduction

Hybrid simulation (or hybrid-testing) is a popular experimental method that is primarily used in civil engineering laboratories [Shing and Mahin 1984; Shing and Mahin 1987]. It originated roughly thirty years ago [Takanashi and Nakashima 1987] and has been used continuously and extensively as a methodology to experimentally assess structural systems under earthquake loadings. Occasionally the methodology has also been used in other disciplines to assess dynamic phenomena; see, e.g., Bursi et al. [2011]. The central problem that hybrid simulation addresses is that it is very difficult and expensive to test full-size civil structures for their structural capacities under seismic loads. The largest testing facility in the world is the E-Defense facility [E-Defense], which can test structures with a 20 m × 15 m plan and 12MN weight. While this represents a large capacity, it precludes the testing of many types of structures, is very expensive due to the need to build full-size prototypes, has limited throughput, and does not easily allow for design exploration.

At its heart, one can think of experimental testing of this variety as the use of an analog computer (algorithm) to simulate the behavior of a structure. Hybrid testing and its many variants (see, e.g., Schellenberg [2008]) tries to leverage this viewpoint in the following manner:

1. The determination of the dynamic response of a structural system is thought of as the integration of the equations of motion for the structure; and
2. The integration of the system of equations is done by a hybrid mix of numerical and analog computing.

In practice, this means that part of the structural system is physically present in the laboratory and the remainder is represented by a computer model. Both parts of the structure are subjected to dynamic excitation, and they interact via a system of sensors and actuators in real- and/or pseudo-time. Figure 1.1 provides a schematic of a typical set-up. Based on the confidence level in the model, a subset of structural response is relegated to a computer model; the physical part typically represents a subset of the structure that lacks a decent computer model; see, e.g., Mosalam and Gunay [2014].

Despite the long history of hybrid-testing, very little is understood about the errors involved when using this methodology to simulate the response of a structure. The bulk of the literature on hybrid testing has focused on improving the accuracy and speed of the numerical computation and the fidelity of the control system, with the implicit assumption that improvements in these aspects will provide a result that is more faithful to an untested physical reality. Recently, however, recent efforts by Bakhaty et al. [2014] and Drazin et al. [2015] have attempted to quantify the theoretical limitations of hybrid testing that are independent of the systematic and random errors that arise from numerical issues and sensor errors. Both research projects used a reference structural system...
that was fully theoretical, split the system into fictitious physical and computational parts, and then explored the fidelity of the hybrid equations with respect to the reference equations. In this way, the true dynamical response of the reference system was known \textit{a priori} in analytic form and could be compared to the hybrid-system response, which was also known in analytic form. The overall methodology thus illuminated directly the central feature of all hybrid simulation methodologies, i.e., the presence of a split system that is patched together with an imperfect interface.

The works of Bakhaty et al. [2014] and Drazin et al. [2015] focused on two linear structural systems: Euler-Bernoulli beams (elastic and viscoelastic) and Kirchhoff-Love (elastic) plates. The research reported herein extends this analysis framework to a nonlinear dynamical system in order to understand the behavior of hybrid-simulation in the presence of kinematic nonlinearities. We considered only the theoretical performance of real-time hybrid simulation as an experimental method and ignored all of the numerical and random errors, as this leads to a best case scenario for a hybrid experiment; see e.g. Shing and Mahin [1987] and Voormeeren et al. [2010]. This approach eliminates the errors associated with time integration methods and signal noise, and focuses only on the errors that are generated by systematic interface mismatch errors—an element that is always present in hybrid simulations. To conduct an in depth analysis of the dynamics of this system, the model problem focused on the damped, driven nonlinear pendulum; see Baker and Blackburn [2005]. This system is one of the most basic nonlinear systems that has a clear physical representation. Despite the simplicity of this system, it is appropriate for this study as it exhibits a rich dynamical response with both periodic and chaotic trajectories. These two behaviors will facilitate studying how a hybrid split affects the overall dynamics of a nonlinear mechanical system. Also considered is a spring-mass-damper actuator system that is controlled by a PI controller. This set-up for the hybrid system gives a more advanced representation of the hybrid system in comparison to the constant error methodology used in Bakhaty et al. [2014] and Drazin et al. [2015].

![Figure 1.1 A simple diagram of a hybrid system set-up.](image-url)
2. General Theory of Hybrid Simulation

2.1 THE REFERENCE SYSTEM

First, we present the reference system to which the hybrid system will be compared. A mechanical system with domain $\mathcal{D}$ is considered; see Figure 2.1a. The mechanical response of the system is characterized by a state vector,

$$ u(x,t) \text{ for } x \in \mathcal{D} $$

where $t$ represents time. In order to compare the reference system response to the hybrid-system response, we imagine that the reference system is split into two substructures: a “physical” substructure ($\mathcal{P}$-side) and a “computational” substructure ($\mathcal{C}$-side); see Figure 2.1b, where $\mathcal{P} \cup I \cup \mathcal{C} = \mathcal{D}$ and $\partial \mathcal{P} \cap \partial \mathcal{C} = I$. The state vector can now be separated into two parts:

$$ u(x,t) = \begin{cases} u_p(x,t) & \text{if } x \in \mathcal{P} \\ u_c(x,t) & \text{if } x \in \mathcal{C} \end{cases} $$

This defines the true response for a given mechanical system. The precise expression for $u(x,t)$ is found by determining the function that satisfies the governing equations of motion on $\mathcal{D}$ and the imposed boundary conditions on $\partial \mathcal{D}$.

(a) (b)

Figure 2.1 (a) A general system with domain $\mathcal{D}$ and state vector $u(x,t)$; and (b) a general system with imposed separation into two substructures for comparison to the hybrid system. $\mathcal{P} \cup I \cup \mathcal{C} = \mathcal{D}$ and $\partial \mathcal{P} \cap \partial \mathcal{C} = I$. 
2.2 THE HYBRID SYSTEM

The response of the hybrid system should be defined in a similar fashion to make the comparison between the two systems straightforward. Using the same boundary defined in Figure 2.1b, the hybrid system is separated into two substructures. To differentiate the reference system from the hybrid system, a superposed hat (\(^\wedge\)) is used to indicate a quantity in the hybrid system. The mechanical response of the hybrid system is represented by the following state vector:

\[
\hat{u}(x, t) = \begin{cases} 
\hat{u}_p(x, t) & \text{if } x \in \mathcal{P} \\
\hat{u}_c(x, t) & \text{if } x \in \mathcal{C}
\end{cases}
\]  

(2.3)

In a hybrid system \(\hat{u}_p\) and \(\hat{u}_c\) are determined from the “solution” of the governing equations of motion for \(\mathcal{P}\) and \(\mathcal{C}\) subjected to the boundary conditions on \(\partial \mathcal{P}\) and \(\partial \mathcal{C}\). The boundary conditions on \(\partial \mathcal{D} \cap \partial \mathcal{P}\) and \(\partial \mathcal{D} \cap \partial \mathcal{C}\) naturally match those of the reference system. However, in the hybrid system one must additionally deal with boundary conditions on the two interface sides of \(I_p\) and \(I_c\), where \(I_p = I \cap \partial \mathcal{P}\) and \(I_c = I \cap \partial \mathcal{C}\). The boundary conditions on \(I_p\) and \(I_c\) are provided by the sensor and actuator system.

\[D_c[\hat{u}_c]\big|_{I_c} = D_p[\hat{u}_p]\big|_{I_p},\]  

(2.4)

where \(D_c[\bullet]\) and \(D_p[\bullet]\) are operators that generate the necessary equations at the interface from the state vectors \(\hat{u}\). As demonstrated later, a simple spring-mass damper system with a PI controller will be used to model the interface that will allow specifying precisely the form of \(D_c[\bullet]\) and \(D_p[\bullet]\). This model allows study of the effects of systematic hybrid system splitting errors, specifically boundary mismatch errors. Such errors directly correlate to errors seen in experimental hybrid systems; see e.g. Shing and Mahin [1987] or Ahmadizadeh et al. [2008].
In an actual hybrid simulation, one only has the physical part, $\mathcal{P}$, the sensor and actuator system, and the computational model for part $\mathcal{C}$. This makes it challenging to know if the determined response $\hat{u}$ is correct to a sufficient degree. To circumvent this issue, this research considered an analytical model for part $\mathcal{P}$ and part $\mathcal{C}$ as well as for the sensor and actuator system. This allows robust computation of the error in the response quantity $\hat{u}$ of the hybrid system by comparing it to the response quantity $u$ of the reference system. The error investigated is then limited to the error in the hybrid system associated with the splitting interface.
3. Damped, Driven Nonlinear Pendulum

3.1 THE REFERENCE SYSTEM

The first system that is discussed herein is that of the reference damped, driven nonlinear pendulum; see Figure 3.1. The pendulum consists of a uniform rigid rod of mass \( m \) and length \( \ell \) that rotates about the point \( O \). There is an applied moment \( M(t) \) at \( O \), and there is linear viscous damping at \( O \) with damping constant \( c \). The kinetic energy of the system is given by

\[
T = \frac{m\ell^2}{6}\dot{\theta}^2, \tag{3.1}
\]

and the potential energy is given by

\[
U = mg \left[ \frac{\ell}{2} - \frac{\ell}{2} \cos(\theta) \right]. \tag{3.2}
\]

Figure 3.1 The damped, driven nonlinear pendulum with a rigid body rotating about \( O \) with applied moment \( M(t) \).
Using Lagrange’s prescription for finding the equations of motion (see, e.g., O’Reilly [2008]), one has

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\theta}} \right) - \frac{\partial T}{\partial \theta} + \frac{\partial U}{\partial \theta} = M_{nc}, \]

where

\[ M_{nc} = -c\dot{\theta} + M(t). \]

This gives

\[ \frac{m_f^2}{3} \ddot{\theta} + c\dot{\theta} + mg \frac{f}{2} \sin(\theta) = M(t), \]

which is the equation that determines the true motion of the system.

3.2 THE HYBRID SYSTEM

Next, we set-up the hybrid pendulum; see Figure 3.2. In this case, the rigid body is split into two distinct bodies that have distinct angles of rotation \( \theta_c \) and \( \theta_p \), with both bodies rotating about \( O \). Also, there are lengths \( \ell_p + \ell_c = \ell \), and masses \( m_p = \ell_p / \ell \) \( m \) and \( m_c = \ell_c / \ell \) \( m \); thus \( m_p + m_c = m \). The kinetic energy is given by

\[ \hat{T} = \frac{m_c\ell_c^2}{6} \dot{\theta}_c^2 + \left( \frac{m_p\ell_p^2}{6} + \frac{m_c\ell_c^2 + m_p\ell_p^2}{2} \right) \dot{\theta}_p^2, \]

and the potential energy is given by

\[ \hat{U} = m_c g \left( \frac{\ell_c}{2} - \frac{\ell_c}{2} \cos(\theta_c) \right) + m_p g \left( \frac{\ell_p}{2} - \left( \frac{\ell_c}{2} + \frac{\ell_p}{2} \right) \cos(\theta_p) \right), \]
where the hat, $\hat{\bullet}$, represents a quantity in the hybrid system. We applied Lagrange’s prescription with respect to $\theta_c$ and $\theta_p$, which is

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_i}\right) - \frac{\partial T}{\partial \theta_i} + \frac{\partial \hat{U}}{\partial \dot{\theta}_i} = \dot{M}_{nci}$$  \hspace{1cm} (3.8)

for $i = c, p$, where

$$\dot{M}_{nci} = -c\dot{\theta}_c + M(t) + M_c, \quad \dot{M}_{mcp} = M_p.$$  \hspace{1cm} (3.9)

Here, $M_c$ is the moment at $I_c$, and $M_p$ is the moment at $I_p$. In this set-up, $M_c$ is an input to the computational model, and $M_p$ is measured by sensors. Expanding Equation (3.8), we obtain

$$\frac{m_c\ell_c^2}{3}\ddot{\theta}_c + c\dot{\theta}_c + m_c g \frac{\ell_c}{2} \sin(\theta_c) = M(t) + M_c,$$  \hspace{1cm} (3.10)

and

$$\left(\frac{m_p\ell_p^2}{3} + m_c\ell_c^2 + m_p\ell_p^2\right)\ddot{\theta}_p + m_p g \left(\frac{\ell_p}{2} + \frac{\ell_p}{2}\right) \sin(\theta_p) = M_p.$$  \hspace{1cm} (3.11)

Note: in the ideal setting with no sensor error, $M_c = -M_p$. We made this assumption to focus on the systematic errors rather than sensor errors. In doing so, Equations (3.10) and (3.11) are then combined into a single equation, given by

$$\frac{m_c\ell_c^2}{3}\ddot{\theta}_c + \left(\frac{m_p\ell_p^2}{3} + m_c\ell_c^2 + m_p\ell_p^2\right)\ddot{\theta}_p + c\dot{\theta}_c +$$

$$m_c g \frac{\ell_c}{2} \sin(\theta_c) + m_p g \left(\frac{\ell_c}{2} + \frac{\ell_p}{2}\right) \sin(\theta_p) = M(t).$$  \hspace{1cm} (3.12)

However, at this point, we only have one equation, Equation (3.12), and two unknowns, $\theta_c$ and $\theta_p$. To obtain a second equation requires a model for the sensor and actuator system that connects the two bodies. Herein, this is modeled as a spring-mass-damper system controlled by a PI controller; see, e.g., Nise [2008]. This model follows the definition from the previous section for internal boundary conditions, or

$$D_c[\dot{\mathbf{u}}_c]|_c = D_p[\dot{\mathbf{u}}_p]|_p.$$  \hspace{1cm} (3.13)

Here, $\dot{\mathbf{u}}_c$ and $\dot{\mathbf{u}}_p$ are given by

$$\dot{\mathbf{u}}_c = [\theta_c], \quad \dot{\mathbf{u}}_p = [\theta_p],$$  \hspace{1cm} (3.14)

and the operators $D_c[\dot{\mathbf{u}}_c]$ and $D_p[\dot{\mathbf{u}}_p]$ have the following definitions:

$$D_c[\dot{\mathbf{u}}_c] = \left[k_c k_i + (k_a k_p + c_a k_i) \frac{d}{dt} + c_a k_p \frac{d^2}{dt^2}\right] \dot{\mathbf{u}}_c,$$  \hspace{1cm} (3.15)
and

\[
D_p [\dot{\ddot{u}_p}] = \left\{ k_a k_i + [k_a (1 + k_p) + c_a k_i] \frac{d}{dt} + [c_a (1 + k_p)] \frac{d^2}{dt^2} + m_a \frac{d^3}{dt^3} \right\} \ddot{u}_p,
\]  

(3.16)

where the parameters \( m_a, c_a, \) and \( k_a \) are the mass, damping constant, and stiffness, respectively, of the spring-mass-damper system used to model the actuator. Parameters \( k_p \) and \( k_i \) are the proportional and integral gains of the PI controller. Applying these definitions ultimately leads to

\[
c_a k_p \dot{\theta}_c + (k_a k_p + c_a k_i) \dot{\theta}_c + k_a k_i \dot{\theta}_c =
\]

\[
m_a \ddot{\theta}_p + [c_a (1 + k_p)] \dot{\theta}_p + [k_a (1 + k_p) + c_a k_i] \dot{\theta}_p + k_a k_i \theta_p.
\]

(3.17)

Thus, the equations of motion for the hybrid system are given by Equations (3.12) and (3.17). Note that the PI controller is used herein, but the entire exercise is easily repeatable with alternate control methodology; see, e.g., Elkhoraibi and Mosalam [2007] and Mosalam and Günay [2014].

### 3.3 NON-DIMENSIONALIZATION

For further analysis, it is beneficial to non-dimensionalize Equations (3.5), (3.12), and (3.17). In order to do this, we define the following non-dimensional quantities:

\[
\tau = t \sqrt{\frac{g}{l}}
\]

(3.18a)

\[
L_c = \frac{\ell_c}{\ell}, \quad L_p = \frac{\ell_p}{\ell},
\]

(3.18b)

\[
M_c = \frac{m_c}{m} = L_c, \quad M_p = \frac{m_p}{m} = L_p,
\]

(3.18c)

\[
\gamma = \frac{c}{m \ell \sqrt{g \ell}}
\]

(3.18d)

\[
\mu(\tau) = \frac{M}{mg \ell} \left( t = \tau \sqrt{\frac{\ell}{g}} \right)
\]

(3.18e)

\[
M_a = \frac{m_a}{m}, \quad \gamma_a = \frac{c_a}{m \sqrt{g}}, \quad K_a = \frac{k_a \ell}{mg}
\]

(3.18f)

\[
K_p = k_p \quad K_i = k_i \sqrt{\frac{\ell}{g}}
\]

(3.18g)

Using Equation (3.18) allows us to rewrite Equations (3.5), (3.12), and (3.17) as,

\[
\frac{d^2 \theta}{d\tau^2} + 3\gamma \frac{d\theta}{d\tau} + \frac{3}{2} \sin(\theta) = 3 \mu(\tau),
\]

(3.19)
\[
\frac{L_c^3}{3} \frac{d^2\theta_c}{d\tau^2} + \left( \frac{L_p^3}{3} + L_c L_p \right) \frac{d^2\theta_p}{d\tau^2} + \gamma \frac{d\theta_c}{d\tau} + \frac{L_c^2}{3} \sin(\theta_c) + \left( L_c L_p + \frac{L_p^2}{2} \right) \sin(\theta_p) = \mu(\tau) \tag{3.20}
\]

and

\[
\gamma a K_p \frac{d^2\theta_p}{d\tau^2} + (K_a K_p + \gamma a K_1) \frac{d\theta_p}{d\tau} + K_a K_c \theta_c = \\
M a \frac{d^3\theta_p}{d\tau^3} + \left[ \gamma a (1 + K_p) \right] \frac{d^2\theta_p}{d\tau^2} + \left[ K_a (1 + K_p) + \gamma a K_1 \right] \frac{d\theta_p}{d\tau} + K_a K_c \theta_c. \tag{3.21}
\]

This gives us the non-dimensionalized equations of motion for the reference and hybrid systems.
4. Analysis

For the analysis, the applied moment is given by

$$\mu(t) = \bar{\mu} \cos(\Omega \tau)$$

(4.1)

where $\bar{\mu}$ is the non-dimensional magnitude of the applied moment, and $\Omega$ is the non-dimensional frequency of the applied moment. To start, the constants in the system are set as follows: $L_c = 0.6$, $L_p = 0.4$, $M_a = 0.5$, $\gamma = 0.1$, $\gamma_a = 25$, $K_a = 12.5$, $K_l = 3$, and $K_p = 10$.

Since the reference forced pendulum is a two-state non-autonomous system, the system will exhibit either periodic motion or chaotic motion depending on the values of the parameters; see Parker and Chua [1989]. The hybrid forced pendulum is a five-state non-autonomous system and will also exhibit either periodic or chaotic motion. If the motion is periodic, the period of the steady-state motion will be an integer multiple of the forcing period, $nT$, where $n = 1, 2, 3...$ and $T = 2\pi/\Omega$ if $n > 1$; this corresponds to an excited sub-harmonic of period $nT$; see Guckenheimer and Holmes [1983]. In order to determine the character of the motion of the systems, we used Lyapunov exponents; see Nayfeh and Balachandran [1995]. If the largest Lyapunov exponent is positive, then the system will exhibit chaotic motion. If the largest Lyapunov exponent is 0, then the system will experience periodic motion; see Baker and Gollub [1996]. Also, as long as the sum of all of the Lyapunov exponents is negative, then we know that the system is stable in the sense of Lyapunov. The Lyapunov exponents are found using the QR method for small continuous nonlinear systems as outlined by Dieci et al. [2010] and the FORTRAN code provided by “Software: LESLIS/LESLIL and LESNLIS/LESNLL”. We modified the LESNLIS routine to calculate the Lyapunov exponents for our systems.

To begin, we examine how the magnitude of the applied moment determines the behavior of the responses of both the reference and hybrid systems for a fixed frequency of the applied moment. Setting $\Omega = 1$ for multiple values of $\bar{\mu}$, we can determine when the systems are either periodic or chaotic. Figure 4.1 shows the largest Lyapunov exponent for the reference and hybrid systems as a function of the forcing magnitude; for the most part, the reference and hybrid systems exhibit the same type of behavior. However, there are a few instances where one system is periodic and the other is chaotic. This indicates that there are three separate cases that one needs to consider when performing an error analysis of a nonlinear hybrid simulation system: both responses are periodic, both responses are chaotic, and one response is periodic while the other is chaotic.
4.1 PERIODIC REFERENCE AND HYBRID SYSTEMS

First, we analyze the case when both the reference and hybrid systems are periodic. For this case, we utilized $L^2$ error to gauge how well the hybrid system matches the reference system per Drazin et al. [2015]. The $L^2$ error is given by

$$E_2(\tau) = \sqrt{\int_0^\tau L_c \left( (\theta - \theta_c)^2 + \left(\frac{d\theta}{d\tau} - \frac{d\theta_c}{d\tau}\right)^2 \right) + L_p \left( (\theta - \theta_p)^2 + \left(\frac{d\theta}{d\tau} - \frac{d\theta_p}{d\tau}\right)^2 \right)} + \sqrt{\frac{\theta^2}{\tau} + \left(\frac{d\theta}{d\tau}\right)^2}$$  (4.2)

Note that (1) the $L^2$ error used for the analysis is normalized with respect to the reference system; (2) the difference in angles is always taken to be the smallest angular distance between 0 and $2\pi$. We calculated the $L^2$ error at three different values of $\mu$: $\mu = 0.7, 1.114, \text{ and } 2.6$.

A careful examination of Figure 4.1 shows that all three of these values will produce periodic motion in both systems. The $L^2$ error time series for these three values of $\mu$ are shown in Figure 4.2. This figure shows that when the transients are still present, i.e., small $\tau$, the error varies rapidly. However, as $\tau$ increases, the error approaches a steady-state value. This make sense because both systems are approaching a periodic solution; thus the difference between the two solutions should be approximately constant. However, as shown in Figure 4.2 where $\mu = 1.114$, the
\(L^2\) error approaches a value near 1.3 (or 130\%), indicating that the hybrid system is not tracking the reference system well. Further study reveals that the reference system is traveling in a clockwise direction, while the hybrid system is traveling in a counter-clockwise direction. Thus, the hybrid system is matching the response of the reference system, just in the opposite direction, which caused the large \(L^2\) error. To more fully study the dynamical response, we look at the state space of the two systems, which is shown in Figure 4.3. Note, only \(\theta_c\) and \(d\theta_c/d\tau\) are plotted for clarity in the figures; see the Appendix for similar plots for \(\theta_p\) and \(d\theta_p/d\tau\). This figure shows that although the state-space trajectories are similar in shape, they vary by a rotation in state space. Thus, as long as the exact trajectory is not required, the hybrid response can be useful in understanding the dynamics of the reference system. Note that Figure 4.3 also clearly shows that sub-harmonics are being excited in this case.

\[\text{Figure 4.2 The } L^2 \text{ error for } \Omega = 1 \text{ for three values of } \bar{\mu} \text{ with only periodic responses.}\]

### 4.2 Chaotic Reference and Hybrid Systems

Next, we analyze the case when both systems are chaotic. For the chaotic systems, the \(L^2\) error is no longer a good metric for determining the error in the system. Instead, we compare multiple aspects of the dynamics to fully understand the relationship between the reference and hybrid systems. First, we compare the systems visually before comparing them with error metrics. The time series—specifically, the angular velocity time series—is used to make a visual comparison of the reference and hybrid systems. We then compare the Poincaré Sections of the reference and hybrid systems. Note, for the plotting the Poincaré Sections, the time series was calculated out to \(\tau = 10,000\), with \(\Omega = 1\). This provides just under 1600 points per Poincaré Section, allowing us to compare the nature of the response on a more fundamental level. Two values of \(\bar{\mu}\) are chosen for
the chaotic case: $\mu = 1.2$ and $\mu = 2.2$. Again, Figure 4.1 shows that these values will produce chaotic responses in both systems.

Figures 4.4 and 4.5 show the time series (of the angular velocities) for the systems with $\mu = 1.2$; see the Appendix for $d\theta_p/d\tau$ plots. It is clear that the two system do not track each other very well. However, Figure 4.6 shows the Poincaré Sections for both the reference and hybrid systems with $\mu = 1.2$, and we can easily see the similarity between the two Poincaré Sections. This indicates that even when both systems are chaotic, the fundamental nature of the responses are nearly identical.

Next, we look at the case when $\mu = 2.2$. The angular velocity time series are shown in Figures 4.7 and 4.8, whereby the time series of the reference and hybrid systems match each other fairly well. However, the corresponding Poincaré Sections—see Figure 4.9—show very little correlation. Similar conclusions can be drawn from the plots of $\theta_p$ and $d\theta_p/d\tau$; see the Appendix. In conclusion, even though the time series match well, their Poincaré Sections do not, thus confirming the need to examine multiple aspects of the dynamics.

**Figure 4.3** The state space trajectories for the reference and hybrid systems with $\mu = 1.114$.
Figure 4.4  The angular velocity time series of the reference and hybrid systems for $\bar{\mu} = 1.2$.

Figure 4.5  A zoomed in plot of the angular velocity time series of the reference and hybrid systems for $\bar{\mu} = 1.2$. 
Figure 4.6  The Poincaré Sections of the reference and hybrid systems for $\bar{\mu} = 1.2$.

Figure 4.7  The angular velocity time series of the reference and hybrid systems for $\bar{\mu} = 2.2$. 
Figure 4.8 A zoomed in plot of the angular velocity time series of the reference and hybrid systems for $\bar{\mu} = 2.2$.

Figure 4.9 The Poincaré Sections of the reference and hybrid systems $\bar{\mu} = 2.2$. 
4.2.1 Chaos Error Metrics

In addition to the visual error analysis, we computed three different error metrics used to give a numerical value to the error between two chaotic systems. First, we compared Lyapunov exponents of the two systems. This allowed us to directly compare the level of chaos in each system as the Lyapunov exponent defines how quickly trajectories will diverge from each other due to small variations in the trajectories; see Gilmore and Lefranc [2011]. The second value we compared was the Lyapunov dimension, \( d_L \), which defines the dimension of the strange attractor and is calculated by

\[
d_L = j + \frac{\hat{\lambda}_1 + \hat{\lambda}_2 + \cdots + \hat{\lambda}_j}{|\lambda_{j+1}|}
\]  

(4.3)

where \( j \) is the largest integer for which \( \hat{\lambda}_1 + \hat{\lambda}_2 + \cdots + \hat{\lambda}_j \geq 0 \), see Frederickson et al. [1983]. The Lyapunov dimension can be used to classify the complexity of a strange attractor, since a strange attractor will have a fractional dimension, whereas a non-strange attractor will have an integer dimension. For our systems, \( j = 2 \). Third, we employed the correlation exponent, \( \nu \), as described by Grassberger and Procaccia [1983a]. The correlation exponent is used to measure the local structure of a strange attractor or Poincaré Section; see Grassberger and Procaccia [1983b]. The correlation exponent is based on how close the points on a strange attractor or Poincaré Section are to one another, which is another measure for the complexity of a strange attractor or Poincaré Section. Herein, the correlation exponent was calculated using the points in the Poincaré Section. The errors with respect to these three metrics are calculated as follows:

\[
err_\lambda = \frac{|\hat{\lambda}_i - \lambda_i|}{\lambda_i}, \quad (4.4)
\]

and

\[
err_{d_L} = \frac{|d_L - \hat{d}_L|}{d_L}, \quad (4.5)
\]

\[
err_\nu = \frac{|\nu - \hat{\nu}|}{\nu}. \quad (4.6)
\]

where the hat, \( \hat{\cdot} \), again, represents quantities for the hybrid system. Figures 4.10, 4.11, and 4.12 show these error measures versus applied moment magnitude. Note, points are only calculated for values of \( \bar{\mu} \) for which both the reference and hybrid system are chaotic.
Examination of Figure 4.10 shows a wide variety of errors in the largest Lyapunov exponents; however, about half of all errors are less than 0.2 (or less than 20%). This shows that about half the time the levels of chaos in both systems are equivalent; however, there are times when the two systems vary greatly. Figure 4.11 shows that all of the errors are below 0.4, and a significant portion, more than nine-tenths, are less than 0.2. Thus, there is much less deviation between the Lyapunov dimension of the reference and hybrid systems, indicating that the dimension of their strange attractors stay near one another.

Examination of Figure 4.12 shows that there is a high density of points below 0.2, with about two-thirds of all points below 0.2. Thus, most of the time the Poincaré Sections of the two systems match fairly well; however, there are still instances in which the two systems do not match well. For the cases visually examined above, $err_{\lambda_1} = 0.1203$, $err_{d_1} = 0.1552$, and $err_{r} = 0.0526$ when $\bar{\mu} = 1.2$, and $err_{\lambda_1} = 0.3680$, $err_{d_1} = 2.810 \times 10^{-4}$, and $err_{r} = 0.2792$ for $\bar{\mu} = 2.2$. These values again fit with our conclusion that multiple quantities are needed to properly assess the error between two chaotic responses.
Figure 4.11 The error in the Lyapunov dimension as a function of $\bar{\mu}$.

Figure 4.12 The error in the correlation exponent of the Poincaré Sections as a function of $\bar{\mu}$. 
4.3 ONE SYSTEM PERIODIC AND THE OTHER CHAOTIC

The third case is when one system has a chaotic response and the other system has a periodic response. In this situation it is not possible to compare the two systems as the $L^2$ error breaks down for chaotic systems, and the Poincaré Section for a periodic system will be a single point, whereas the Poincaré Section for a chaotic system will be Cantor-like; see Rao [2004] or Parker and Chua [1989]. For these reasons, it is clear the correlation between the two responses will be nonexistent.

4.4 STUDY OF $K_i$

All of the above analysis was done with specific values of the control parameters. If we use $K_i = 10$ instead, which was arbitrarily chosen, we can see how the Lyapunov exponents of the hybrid system match those of the reference system much better, as seen by comparing Figures 4.1 and 4.13. This potentially indicates that increasing the integral gain, $K_i$, results in better matching between the reference and hybrid systems. To investigate this further, we now look at the effects of changing the integral gain, $K_i$. We studied three specific values of $\bar{\mu}$: $\bar{\mu} = 1.114, 1.2, \text{ and } 3.0$. The first value was chosen because although both the hybrid and reference systems were periodic at $K_i = 3$, the hybrid system was going the opposite direction of the reference system. The second value was chosen because the response is chaotic for both systems at $K_i = 3$. And the third value was chosen because the reference response is periodic, while the hybrid response is chaotic at $K_i = 3$. For analyzing the effect of changing $K_i$, we looked at the hybrid $L^2$ error once the transients had died out and the error had reached steady state:

$$E^h_2(\tau = 1000) = \sqrt{\int_0^\tau (\theta_e - \theta_p)^2 + \left(\frac{d\theta}{d\tau} - \frac{d\theta_p}{d\tau}\right)^2}$$

Note the $E^h_2$ is normalized to the top piece of the hybrid pendulum. The hybrid $L^2$ error determines how well the two pieces of the hybrid pendulum are matching each other and is an error measure we can apply independent of the chaotic or periodic nature of either system. As shown in Figure 4.14, as $K_i$ is increased, the hybrid $L^2$ error decreases for all three values of $\bar{\mu}$, which makes sense because $K_i$ affects the steady-state response; thus the two pieces should match better for larger values of $K_i$; see Nise [2008]. However, if we look at the steady-state $L^2$ error in Figure 4.15, the $L^2$ error does not decrease as $K_i$ is increased; in fact, all three values of $\bar{\mu}$ have different responses to increasing $K_i$. 

23
Figure 4.13  The Lyapunov exponents for the reference and hybrid systems when $K_i = 10$.

Figure 4.14  The $E_2^h$ error as a function of $K_i$ for multiple values of $\bar{\mu}$.
For $\overline{\mu} = 1.114$, the error approximately goes between three values as $K_i$ increases. This indicates that even though the hybrid pieces are matching each other better, the hybrid pendulum does not always match the reference pendulum better. In fact, the highest value represents the hybrid pendulum spinning in the opposite direction of the reference pendulum, the middle value represents the hybrid pendulum spinning in the same direction as the reference pendulum but takes a long time to reach the steady-state solution, and the low value represents the hybrid pendulum spinning in the same direction as the reference pendulum and reaching the steady-state solution more quickly.

For $\overline{\mu} = 1.2$, the $L^2$ error is not a good metric for analyzing the error. Instead, we again look at the Poincaré Sections, as shown in Figure 4.16; see the Appendix for $\theta_p$ and $d\theta_p/d\tau$ plots. From a close comparison of Figures 4.6 and 4.16, we can see that with $K_i = 10$, the Poincaré Sections match better than when $K_i = 3$. This indicates that the hybrid response is better for larger values of $K_i$. Evaluating the error metrics from before, we find that $err_{\lambda_1} = 0.5722$, $err_{\mu_1} = 0.0919$, and $err_c = 0.0332$. Comparing these values to those found earlier, we find that the Lyapunov dimension error and correlation exponent error have decreased, while the Lyapunov exponent error has increased. Again, this indicates the need for multiple metrics to gauge the chaotic response; although it appears that increasing $K_i$ resulted in improved hybrid response, there is a metric in which it became worse.

Finally, for $\overline{\mu} = 3.0$, the $L^2$ error sharply dropped around $K_i = 4$. This occurred because the hybrid system changed from chaotic to periodic, while the reference system was periodic.
throughout. After the transition, the hybrid system had the same response type as the reference system. The $L^2$ error remained low because the hybrid system was traveling in the same direction as the reference system, and did not change direction—unlike the case of $\bar{\mu} = 1.114$. This confirms, for the most part, the conclusion regarding $K_i$ reached as determined from Figure 4.13.

Figure 4.16 The Poincaré Sections of the reference and hybrid systems for $\bar{\mu} = 1.2$ and $K_i = 10$. 
5. Conclusions

This paper focused on the fundamental interface mismatch error that occurs during a nonlinear hybrid simulation experiment. To study this intrinsic error, we examined the behavior of a kinematically nonlinear hybrid system with a spring-mass-damper actuator system, controlled by a PI controller. This is a relatively simple model, but it provided considerable control over the study of this system discussed herein. We chose to use a single forcing frequency, which is a parameter that can be applied in future work. Most importantly, the set-up was entirely theoretical, thus providing a true reference against which to compare hybrid results.

Analysis of the reference and hybrid systems found that there are three unique cases that need to be identified when discussing the responses of the reference and hybrid systems: (1) both responses are periodic, (2) both responses are chaotic, and (3) one response is periodic while the other is chaotic.

1. For the periodic-periodic case, we discovered that sometimes the hybrid system tracks the reference system well, resulting in a low $L^2$ error; however, at other times it did not track the reference system well, resulting in a high $L^2$ error. In the case of high $L^2$ error, we noted that the two systems experienced similar motions, despite not tracking well; see Figure 4.3. This leads to a fundamental question regarding hybrid simulation: what does one expect to get from hybrid simulation? Hybrid simulation loses its utility if perfect tracking is the goal given that even with adjustment of the control parameters, perfect tracking is not to be expected or assumed when testing a nonlinear system. However, if one wishes to understand the general response of the dynamical system in that the same parts of the phase space are traversed and at the same frequency, then hybrid simulation can still be useful, and the hybrid system can provide a good representation of the reference system response. Put another way, if one is content that the hybrid system experiences the same states as the true system, independent of temporal ordering, then hybrid simulation retains its utility in the nonlinear setting.

2. This trend carries into the second case where both systems were chaotic. The first example where $\mu = 1.2$ resulted in poor time series matching but a good matching of Poincaré Sections, indicating a clear correlation in the dynamics of the two systems. The second example where $\mu = 2.2$ resulted in good time series matching, but little correlation between the two Poincaré sections. Given these results, it was necessary to compare more than one aspect of the dynamics. Herein, the largest Lyapunov exponents, the Lyapunov dimension, and the correlation exponent were used to analyze the correspondence between the responses. As shown in Figure 4.6, it was clear that
responses were similar. Even though the time series of the reference and hybrid systems did not follow each other closely, the allowable motions for each system were closely related. As shown in Figures 4.7 and 4.8, it was clear that the time series matched well even though the Poincaré Sections were not similar, which still indicated that responses of the reference and hybrid systems were correlated in the example. Thus, knowing the response of the hybrid system will give an approximation of how the reference system will respond. Again, as long as the exact trajectory is not required, i.e., one is satisfied that the system moves through the correct states at the correct sampling frequency, then hybrid simulation is still useful for understanding the response of the reference system. This information linked with the numerical error metrics agrees with the conclusion made in the first case: one needs to be fully aware of what one wants from hybrid simulation; exact matching may not be possible. It is possible for hybrid simulation to properly reproduce certain dynamical quantities, which can be just as useful.

3. Finally, for the third case where one system was periodic and the other chaotic, it proved not worth trying to compare the two responses. For the periodic system, the response will approach a periodic steady-state, whereas in the chaotic system, the response will be an aperiodic solution, indicating large differences in the behavior of the response.

All of the above analysis was concerned with a single value of the integral gain, $K_i$, specifically $K_i = 3$. Upon changing $K_i$, we now understand more about the nature of the hybrid response. In all cases, the error internal to the hybrid system, $E^2_{\text{h}}(\tau = 1000)$, decreased as $K_i$ was increased. Unfortunately, this does not directly translate to better tracking between the hybrid and reference systems, as shown in a comparison of Figures 4.14 and 4.15. In the case when both systems are periodic, as $K_i$ increases it is possible for the hybrid system to change from a counterclockwise rotation to a clockwise rotation and back. Notwithstanding, in almost all other instances, increasing $K_i$ produces a better hybrid result. However, one cannot simply increase the value of $K_i$; there are stability and physical constraints that determine the feasible range of $K_i$. Understanding how to effectively use the control parameters is of great importance. The research reported herein examined one very simple control system since the underlying set of outcomes is independent of this choice; better controllers will not obviate the need to understand chaotic trajectories in the nonlinear case.

In conclusion, the application of hybrid simulation to nonlinear systems requires an understanding of what one wishes to achieve, a knowledge of the three possible outcomes, and the application of multiple metrics to ensure fidelity.
REFERENCES


Appendix  \( \theta_p \) and \( d\theta_p/d\tau \) Plots

In the main body of the text we consistently compare the dynamical response of the \( C \) part of the hybrid system to the reference system. For completeness, sake, this appendix provides comparison plots using the dynamical response of the \( P \) part. All conclusions made from the plots in the main body of the text remain true.

![State space trajectories](image)

**Figure A.1** The state space trajectories for the reference and hybrid systems with \( \bar{\mu} = 1.114 \); compare to Figure 4.3.
Figure A.2  The angular velocity time series of the reference and hybrid systems for $\bar{\mu} = 1.2$; compare to Figure 4.4.

Figure A.3  A zoomed in plot of the angular velocity time series of the reference and hybrid systems for $\bar{\mu} = 1.2$; compare to Figure 4.5.
Figure A.4  The Poincaré Sections of the reference and hybrid systems for $\bar{\mu} = 1.2$; compare to Figure 4.6.

Figure A.5  The angular velocity time series of the reference and hybrid systems for $\bar{\mu} = 2.2$; compare to Figure 4.7.
Figure A.6  A zoomed in plot of the angular velocity time series of the reference and hybrid systems for $\mu = 2.2$; compare to Figure 4.8.

Figure A.7  The Poincaré Sections of the reference and hybrid systems for $\mu = 2.2$; compare to Figure 4.9.
Figure A.8 The Poincaré Sections of the reference and hybrid systems for $\bar{\mu} = 1.2$ and $K_i = 10$; compare to Figure 4.16.
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